Physical Science International Journal
4(4): 542-564, 2014

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Study on Nonlinear Ion-acoustic Solitary Wave Phenomena in Slow Rotating Plasma

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Author's contribution

The whole work was carried out by the author GCD.

Original Research Article

Received 15th March 2013
Accepted 12th November 2013
Published 6th January 2014

ABSTRACT

Nonlinear waves have been an important subject in the field of astroplasmas under the action of Coriolis force because of rotation could be the progenitor of many heuristic feature on waves. Our main interest is to study the nonlinear ion-acoustic wave in a rotating plasma. Pseudopotential analysis has been used to derive the Sagdeev-like wave equation which, in turn, becomes the tool to study the different nature of nonlinear plasma waves. Special methods have been developed successfully to derive different kinds of solitary wave solutions. Main emphasis has been given to the interaction of Coriolis force to the changes of coherent structures of solitary waves e.g. Compressive and rarefactive solitary waves along with their explosions or collapses. It has shown that the variation of rotation affects the nonlinear wave modes and causeway exhibits shock waves, double layers, sinh-wave, and formation of sheath structure in dynamical system. It has shown that the rotation, however small in magnitude, generates a narrow wave packet with the generation of high energy therein which, in turn, yields the phenomena of radiating soliton. It finds that the Coriolis force might be the cause in blowing up the ion-acoustic pulses and could be related the phenomena of solar burst. Thus the work has the potential interest to study the nonlinear waves in astroplasmas where in Coriolis force is present with a view to rekindle the soliton dynamics in space plasmas.

Keywords: Nonlinear wave; solitons; shock wave; double layers; coriolis force.

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1. INTRODUCTION

Studies on nonlinear solitary waves have been receiving tremendous momentum in various plasma environments in laboratory, space as well as in astrophysical plasmas because of its having potential importance in processes of plasma energization. Since its observations in water wave Scott [1], study on nonlinear wave have been carrying out through the augmentation of Korteweg-de Vries equation [2] (called as K-dV equation). Washimi and Taniuti [3] were probably the pioneers who derived theoretically the well known nonlinear K-dV equation in plasma and finds successfully the solitary waves (or solitons) what exactly observed in water wave. During the same decade, another pioneer method by Sagdeev [4] has derived the nonlinear wave phenomena in terms of an energy integral equation and analyzed rigorously soliton dynamics along with other nature of nonlinear waves in plasmas. Both have made unique platforms in scientific community and bridges successfully many theoretical observations in plasma experiments [5,6] as well as with the satellite observations in astroplasmas [7,8]. Many authors have studied then soliton dynamics in various plasma models among which Das [9] observed first a new nature of solitary wave in plasma causes by the presence of an additional negative ion and makes a heuristic milestone in soliton dynamics. The observations yield latter successfully in auroral ionosphere and magnetosphere by the Freja scientific space satellites Wu et al. [7] as well as in laboratory plasmas Watanabe [10], Lonngren [11], Cooney et al. [12]. Parallel works have studied also this novel features in different plasma constituents with multiple electrons in discharge phenomena Jones et al. [13], Hellberg et al. [14] and have shown the plasma constituent effects on the evolution of new features as similar to those have been observed theoretically by Das [9] as well as in laboratory plasmas (Watanabe [10], Lonngren [11], Cooney et al. [12] with negative ions. Many thorough advancements have been derived the occurrences of nonlinear ion-acoustic solitary waves of different kinds e.g. compressive and rarefactive solitons, double layers by many authors Raadu [15], Das et al.[16], followed by the new findings as of spiky and explosive solitary waves Das et al. [17], Nejoh et al.[18] as well as experimental evidences in multiple electron plasmas (Jones et al. [13], Hellberg et al. [14], Nishida et al. [19]). Again interest has been widened in presence of magnetic field which yields the formation of compressive and rarefactive solitons (Kakutani et al.[20], Kawahara [21] but with the effective variation on dispersiveness causes by the interaction of magnetic field. However, fewer observations have been made to show the role of dispersive effect on the existences of different solitons. Actual argument lies on the derivation of nonlinear wave in unmagnetized plasma which does not ensure the variation of dispersive effect and thus could not sustain such behaviour in solitary waves. But the magnetized plasma exhibits the occurrences of compressive and rarefactive solitons (Kakutani et al.[20], Kawahara [21]) which arises due to the effect of embedded magnetic field. Again several solitary wave modes have been investigated by many authors (Haas [22], Sabry et al .[23], Chatterjee et al. [24]) in quantum plasma configurations. Totality of soliton dynamics in plasmas depend on the nature of nonlinearity and dispersive effects. Both the nature find the typical role in plasmas explored in astrophysics, space plasmas and astroplasmas as well as in laboratory plasmas and concluded that plasma contaminated with an additional negative charge could exhibit many different nature on solitary waves.

Again, during last several years, there has been a flurry of theoretical studies on solitary waves as of dust acoustic waves(DAW), dust magnetosonic waves in plasmas contaminated with negatively dust charged grains Goertz [25], Goertz & Morfil [26]. In fact study has been acquiring a great significance and subsequent studies showed many applications in understanding the salient features of acoustic modes because of new and its vital role finding in astrophysical and space environments. Since its theoretical concept on the occurrences of DAW in plasma, predicted probably first by Rao et al. [27], and supported by the experiments of Barkan et al. [28], studies have then growing interest in plasmas with having different configuration of dust charged grains. In planetary rings, earth's magnetosphere, interstellar clouds, over the Moon's surface [29-32]. Numerous investigations on nonlinear wave phenomena have been studied theoretically relying on the experiments and satellite observations, but we are very much reluctant to cite all papers here. Recent works in different plasma models appear in laboratory and space plasmas [33] that too in unmagnetized or magnetized plasmas with temperature effect [34], nonlinear phenomenon as of sheath formation in inhomogeneous plasma and ionization effect [35,36], in astroplasmas with electron-positron-ion-plasmas [37-39] especially observable in the pulsar magnetospheres [40], dust charging variation effect [41], nonlinear phenomena in relation to the observations of spokes in the Saturn's B ring [42] are to be quoted. Results have derived many aspects of scientific values on nonlinear waves boosting with an uneven competition between theory and experiments as well as with the satellite observations in astroplasmas. We further for new features on nonlinear waves in astroplasmas under the action of Coriolis force appears due to the slow rotation of the medium. It is very much necessary to consider the plasma model under the interaction of rotation. It is observed that the heavenly body under slow rotation, however small it might be, shows interesting findings in astrophysical environments [43]. Because of rotation, two major forces known as Coriolis force and centrifugal force, Chandrasekhar [43], Greenspan [44] play very important role in the dynamical system. But, because of slow rotation approximation, centrifugal force in the dynamics could be ignored, and could be a common applicable in the study of wave in many astroplasmas environments. Based on Chandrasekhar's proposal [45] on the role of Coriolis force in slow rotating stars, many workers have studied latter the nature of wave propagation in rotating space plasma environments. Lehnert [46]'s study on Alfvén waves finds that the Coriolis force plays a dominant role on low frequency Alfvén waves leading to the explanation of solar sunspot cycle. Earlier knowledge pointed out that the force generated from rotation, however small in magnitude, has the effective role in slow rotating stars [45,46] as well as in cosmic phenomena [47]. Latter, from the theoretical point of view, linear wave propagation had been studied elaborately in rotating plasma Bajaj and Tandon [48], Uberoi and Das [49] and references therein , and the results on wave propagation in lower ionospheric plasmas conclude that the role of rotation cannot be ignored otherwise observations might be erroneous. Further, it has shown that the Coriolis force has a tendency to produce an equivalent magnetic field effect as and when the plasma rotates [49]. Interest has then widened well to theoretical and experimental investigations because of its great importance in rotating plasma devices in laboratory and in space plasmas too. But, earlier works were limited to study the linear wave in simple plasmas. Whereas, all the observations with nonlinear waves indicate that the plasma-acoustic modes might expect new features in rotating plasmas related to such problems in astrophysical environments. Das and Nag [50] have studied the nonlinear wave phenomena with due effect of rotation as in astrophysical problems observable in slow rotating stars Chandrasekhar [45], Lehnert [46] as well as in cosmic physics Alfvén [47] and in ideal plasma model [49]. Study evaluates that the rotation plays the progenitor of various nature of nonlinear wave as of the formation of rarefactive and compressive, bursting or collapses of soliton pulses as similar to those observed in multicomponent plasmas earlier [7,8,16,17,40]. Variation of Coriolis force creates a narrow wave packet of soliton with the creation of high electric force and magnetic force and, as a result of which, density depression occurs causing the radiation-like phenomena coined as soliton radiation [51,52]. Latter Mamun [40] has shown this nature of small amplitude waves generated in highly rotating neutron stars or pulsar and concludes that the variation of rotation causes the soliton radiation termed as pulsar radiation. Moslem et al. [53] and Kourakis et al. [54] executed such observations convincingly in pulsar magnetospheres.

To study the totality on existence of nonlinear wave propagation in rotating plasma, we have considered a simple unmagnetized plasma rotating with an uniform angular velocity. Sagdeev Potential (SP)-like wave equation has been derived by the use of quasi-potential method, and thereafter wave equation has been analyzed with the variation of nonlinear effects and rotation. Investigations will be structured as append : Sec.2.1 describes the basic equations governing the plasma dynamics under the action of Coriolis force and thereafter nonlinear Sagdeev-like wave equation has been derived. To derive the properties and propagation of different pulse excitations, modified sech-method (or tanh-method) has been employed to solve wave equation as for solitons, double layers, shock waves(in secs. 2.2-2.7). Results are summarized in the concluding Sec.3.
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2.1 Basic Equations and Derivation of Nonlinear Wave Equation

To study the nonlinear solitary wave propagation, we consider a plasma consisting of isothermal electrons (under the assumption Te >>Ti) and positive ions. Here nonlinear acoustic wave propagation has been taken unidirectional (say along x-direction). We assume the plasma is rotating with an uniform angular velocity, Ω around an axis making an angle θ with the propagation direction. Further the plasma is having the influence of Coriolis force generated from the slow rotation approximation. Other forces might have effective role in the dynamics but all have been neglected because of having the aim to know the effect of Coriolis force in isolation. The basic equations governing the plasma dynamics are the equations of continuity and motion, and, following Uberoi and Das [49] can be written (with respect to a rotating frame of reference) in the normalized forms as

$$\frac{\partial n}{\partial t}+\frac{\partial n v_x}{\partial x}=0 \tag{1}$$

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}=-\frac{\partial \Phi}{\partial x}+\eta v_y sin\theta \tag{2}$$

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}=\eta(v_z cos\theta - v_x sin\theta) \tag{3}$$

$$\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}=-\eta v_y cos\theta \tag{4}$$

where the normalized parameters are defined as $n = n_i / n_0$, $x = x / \rho$, $v_{x,y,z} = (v_i)_{x,y,z} / C_s$, $t = t\, \omega_{ci}$, $\rho = C_s / \omega_{ci}$, $C_s = (kT_e/m_i)^{1/2}$, $\omega_{ci} = eH/m_i$ with $\eta=2\Omega$. ω_{ci} and ρ denote respectively the ion-gyro frequency and ion-gyro radius, C_s is the ion acoustic speed. $H = 2\Omega m_\alpha/q_\alpha$ has been produced due to the rotation, m_i is the mass of ions moving with velocity $v_{x,y,z}$, and n be the density.

Basic equations are supplemented by Poisson equation which relates the potential Φ with the mobility of charges as

$$\frac{\lambda_d^2}{\rho^2}\left(\frac{\partial^2\Phi}{\partial x^2}\right) = n_e - n\text{; where } \lambda_d = \left(\frac{\varepsilon_0 k T_e}{n_0 e^2}\right)^{1/2} \text{ is the Debye length} \tag{5}$$

For the sake of mathematical simplicity, equations for electrons are simplified to Boltzman relation as

$$n_e = \exp(\Phi) \tag{6}$$

where $\Phi = e\phi/kTe$ is the normalized electrostatic potential and n_e is the electron density normalized by n_0 (= n_{i0} = n_{e0}).

Now to derive the Sagdeev potential equation, pseudopotential method has been employed which needs to describe plasma parameters as the function of ξ [$\xi = \beta$ (x −Mt)] with respect to a frame moving with M (Mach number) and β^{-1} is the width of the wave. Now using these transformations along with appropriate boundary conditions at $|\xi|$□ ∞ given as [50]

(i) $v_\alpha \to 0$ (α = x,y,z) (7a)

(ii) $\Phi \to 0$ (7b)

(iii) $\frac{d\Phi}{d\xi} \to 0$ (7c)

(iv) $n \to 1$ (7d)

basic Eqs. (1) – (4) are reduced to the following ordinary differential equations

$$-M\frac{\partial n}{\partial t} + \frac{\partial n v_x}{\partial \xi} = 0 \tag{8}$$

$$-M\frac{\partial v_x}{\partial \xi} + v_x\frac{\partial v_x}{\partial \xi} = -\frac{\partial \Phi}{\partial \xi} + \eta v_y sin\theta \tag{9}$$

$$-M\frac{\partial v_y}{\partial \xi} + v_x\frac{\partial v_y}{\partial \xi} = \eta(v_z cos\theta - v_x sin\theta) \tag{10}$$

$$-M\frac{\partial v_z}{\partial \xi} + v_x\frac{\partial v_z}{\partial \xi} = -\eta v_y cos\theta \tag{11}$$

Now integrating equations once, along with the boundary conditions, Eq.(8) evaluates v_x as

$$v_x = M\left(1-\frac{1}{n}\right) \tag{12}$$

The substitution of v_x into Eqs.(9) and (10) gives

$$v_y = \frac{1}{\eta} sin\theta\left[1-\frac{M^2}{n^3}\frac{dn}{d\Phi}\right]\frac{d\Phi}{d\xi} \tag{13}$$

$$\frac{dv_y}{d\xi} = (n-1)\eta sin\theta - \eta\left(\frac{n}{M}\right)v_z cos\theta \tag{14}$$

Again use of v_y in Eq.(10), v_z evaluates as

$$v_z = Mcot\theta\left(\frac{1}{n}-1\right)+\left(\frac{cot\theta}{M}\right)\int_0^{\Phi} nd\Phi \tag{15}$$

We, substituting Eqs.(13) and (15) in Eq.(14), obtain the nonlinear wave equation as

$$\beta^2\frac{\partial}{\partial\xi}\left[A(n)\frac{\partial\Phi}{\partial\xi}\right] = \eta^2(n-1) - \frac{n\eta^2 cos^2\theta}{M^2}\int_0^{\Phi} nd\Phi \equiv -\frac{dV(\Phi,M)}{d\Phi} \tag{16}$$

where $A(n) = 1-\frac{M^2}{n^3}\frac{dn}{d\Phi}$ and $V(\Phi,M)$ which could be regarded as modified

Sagdeev potential. Multiplying both sides of Eq.(16) with A(n) and thereafter mathematical manipulation with once integrating in the limit Φ = 0 to Φ, Eq.(16) evaluates as

$$\frac{1}{2}\frac{\partial}{\partial\Phi}\left[A(n)\frac{\partial\Phi}{\partial\xi}\right]^2 = A(n)\left\{\eta^2(n-1) - \frac{n\eta^2 cos^2\theta}{M^2}\int_0^{\Phi} nd\Phi\right\} \tag{17}$$

A(n), which is a function of plasma constituents, plays the main role in finding the different nature of nonlinear wave phenomena. This is the desired equation to derive the sheath formation along with different acoustic modes in plasmas. But, due to the presence of A(n), solution of Eq.(17) cannot be evaluated analytically, and consequently as for the desired observations in astrophysical problems, we make a crucial approximation of having small amplitude acoustic modes. Mathematical simplicity has been followed by the quasineutrality condition in plasmas. This condition is based on the assumption that the electron Debye length is much smaller than the ion-gyro-radius, and, following Baishya and Das [55], ion density approximates as

$$n = \exp(\Phi) \tag{18}$$

and A(n) can be modified explicitly as

$$A(n) = 1 - M^2 \exp(-2\Phi) \tag{19}$$

Now Eq. (17), with the substitution of Eqs.(18) and (19), reads as

$$\frac{1}{2}A(n)^2\left(\frac{d\Phi}{d\xi}\right)^2 = \eta^2\left[F(\Phi) - \Phi - \frac{BF(\Phi)^2}{2} + M^2\left\{B\Phi + \frac{1 - BF(\Phi)}{F'(\Phi)} - \frac{1}{2F'(\Phi)^2} - \frac{1}{2}\right\}\right] \tag{20}$$

with

$$V(\Phi, M, \theta) = -\eta^2\left[F(\Phi) - \Phi - \frac{BF(\Phi)^2}{2} + M^2\left\{B\Phi + \frac{1 - BF(\Phi)}{F'(\Phi)} - \frac{1}{2F'(\Phi)^2} - \frac{1}{2}\right\}\right] \tag{21}$$

and $F(\Phi) = \int_0^\Phi n d\Phi, \quad F'(\Phi) = n, \quad B = \frac{cos^2\theta}{M^2}$

From the set of equations, dΦ/dξ can be evaluated from Eq.(20), and leads to a nonlinear equation in F(Φ). But to solve the modified nonlinear equation, some typical numerical values of plasma parameters are to be needed. F(Φ) has been expanded in power series of Φ up to the desired order which, in turns, exhibits the evolution of different nature of solitary waves.

2.2 Derivqation of Soliton Solution with Lowest Order Nonlinearity in Φ

First, we consider $\Phi \ll 1$ i.e. small amplitude wave approximation and Eq. (20) modifies as

$$\beta^2 A\frac{d^2\Phi}{d\xi^2} = A_1\Phi + A_2\Phi^2 \tag{22}$$

where $A_1 = \eta^2\left(1 - \frac{cos^2\theta}{M^2}\right)$ and $A_2 = \frac{\eta^2}{2}\left(1 - \frac{3cos^2\theta}{M^2}\right)$

and correspondingly A(n), following Baishya and Das [55] and Das et al. [56], finds as

$$A(n) = 1 - M^2 \exp(-2\Phi) \approx 1 - M^2 \tag{23}$$

To analyze the existences of nonlinear acoustic waves, sech-method based on which wahas been used to derive soliton solution in the form of sech(ξ) or might be in any other hyperbolic function and extended successfully in the astrophysical problems [57]. Thus we have, in contrast to steady state method, used an alternate method called as sech-method of having the desire on solitary wave solution in the form of sech(ξ) nature [58]. It is true that the K-dV equation, under the small amplitude approximation, derives soliton solution in the form of sechξ or tanhξ. We, for the need of present method, introduce a transformation Φ(ξ) = W(z) with z = sechξ, which, in fact, has wider application in complex plasma. Nevertheless, one can use some other procedure to get the nature of soliton solution of the wave equation. But, since the sech-method is comparatively a wider range [52,57], and has an easier success and merit as well. Using this transformation, Eq.(22) has then reduced to a Fuchsian-like nonlinear ordinary differential equation as

$$\beta^2 Az^2(1-z^2)\frac{d^2W}{dz^2}+\beta^2 Az(1-2z^2)\frac{dW}{dz}-A_1W-A_2W^2=0 \qquad (24)$$

Eq. (24) has a regular singularity at z = 0 and encourages the fundamental procedure of solving this differential equation by series solution technique and follows the most favourable straightforward technique known as Frobenius method (Courant & Friedricks [59]. Accordingly, we assume the solution for W(z) to be a power series in z as :

$$W(z)=\sum_{r=0}^{\alpha} a_r z^{(\rho+r)} \qquad (25)$$

Which enable to find recurrence relation as

$$\beta^2 Az^2(1-z^2)\sum_{r=0}^{\infty}(\rho+r)(\rho+r-1)a_r z^{(\rho+r-2)}+\beta^2 Az(1-2z^2)\sum_{r=0}^{\infty}(\rho+r)a_r z^{(\rho+r-1)}$$
$$-A_1\sum_{r=0}^{\infty}a_r z^{(\rho+r)}-A_2\left(\sum_{r=0}^{\infty}a_r z^{(\rho+r)}\right)^2=0 \qquad (26)$$

The nature of roots from the indicial equation determines the nature of soliton solution of the differential equation. The problem is then modified to find the values of a_r and ρ. The procedure is quite lengthy as well as tedious. To avoid such laborious procedure, we adopt a catchy way [57] to find the series for W(z). We truncate the infinite series (26) into a finite one with (N+1) terms along with ρ = 0. Then the actual number N in series W(z) has been determined by the leading order analysis in Eq.(26) i.e. balancing the leading order of the nonlinear term with that of the linear term of the differential equation. The process determines *N* = 2 and *W* (*z*) becomes

$$W(z)=a_0+a_1z+a_2z^2 \qquad (27)$$

Substituting expression (27) in Eq.(24) and, with some algebra, the recurrence relation determines the following expressions

$$-A_1a_0+A_2a_0^2=0 \qquad (28)$$

$$-\beta^2Aa_1-A_1a_1+2A_2a_0a_1=0 \qquad (29)$$

$$4\beta^2Aa_2-A_1a_2+A_2a_1^2+2A_2a_0a_2=0 \qquad (30)$$

$$-2\beta^2 A\, a_1 + 2 A_2\, a_1\, a_2 = 0 \tag{31}$$

$$-6\, \beta^2 A\, a_2 + A_2\; a_2^2 \;\; = 0 \tag{32}$$

From these recurrence relations, we, based on some mathematical simplification, following Das and Sarma. [57], the values of a’s and β are evaluated *as*

$a_0 = 0$, $a_1 = 0$, $a_2 = \left(\dfrac{3A_1}{2A_2}\right)$, $\beta = \sqrt{\dfrac{A_1}{4A}}$

and consequently the solution obtains as

$$\Phi(x,t) = \left(\frac{3A_1}{2A_2}\right) sech^2\left(\frac{x - Mt}{\delta}\right) \tag{33}$$

where $\delta = \sqrt{\dfrac{4A}{A_1}}$ is the width of the wave.

The solution represents solitary wave profile and fully depends on the variation of A_1 and A_2.

3. RESULTS AND DISCUSSIONS

Study describes the derivation of nonlinear wave equation as Sagdeev potential like equation in rotating plasmas. Soliton profile derives from the first order approximation on Sagdeev equation, and fully depends on the variation of A_1 and A_2 along with variation of Mach number, M and θ i.e. for different magnitudes of rotation. Different plasma configurations have the different values in M. Its variation has the restriction by the plasma configuration. However, we, without loss of generality, have considered the Mach number greater than one for the numerical estimation. We plot the variation of A_1 and A_2 in Fig.1 for some typical plasma parameters of varying Mach number, M with different, θ. Out of which, variation of A_1 shows be positive always and causeway the soliton profile yields a schematic variation by the variation of A_1.

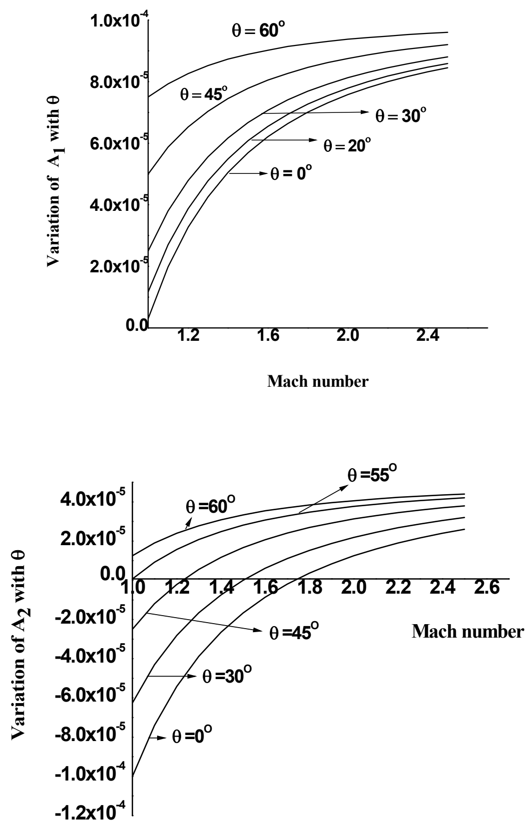

Fig. 1. Variation of $A_1$and A_2 with Mach number for different angles of rotation

Thus the amplitude depends crucially on the variation of A_2 as it could be positive or negative depending on θ and M, and thereby highlights compressive soliton in the case of A_2 being positive while it shows the rarefactive nature for A_1 and A_2 having opposite signs. Fig. 2. shows that rarefactive soliton could be observed in the case of small Mach number (i.e. when $A_2 < 0$) and it, with increasing of M and θ, changes from rarefactive to compressive soliton leaving behind a critical point at which A_2 goes to zero and existences of soliton pulse breaks down. Thus the Coriolis force introduces a critical point even in a simple plasma at which A2 goes to zero, and the formation of soliton will disappear. Coriolis force shows a destabilizing effect on the formation of soliton in plasma-acoustic modes.

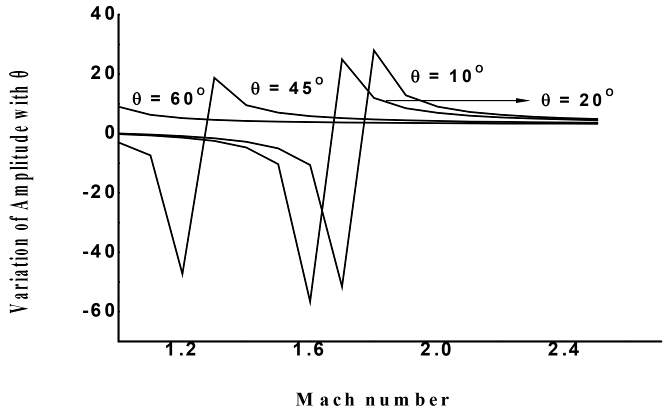

Fig. 2. Variation of Amplitude with Mach number for different angles of rotation

Again, at the neighborhood of critical point, the width of the solitary wave narrows down (amplitude will be large) because of which soliton collapses or explodes depending respectively on the conservation of energy in solitary wave profile. Now the explosion of the soliton depends on the amplitude growth wherein soliton does not maintain the energy conservation. Otherwise the case of preserving the energy conservation leads to a collapse of soliton. Again it describes the fact that, due to formation of a narrow wave packet, there is a generation of high electric force and consequently high magnetic force within the profile of soliton. Because of high energy, electrons charge the neutral and other particles as a result density depression occurs and phenomena term as soliton radiation has been seen. Such phenomena on solitons and radiation do expect similar occurrences of solar radio burst [50,57]. Finally, it concludes that the rotation, however small in magnitude, plays important role as the progenitor of showing all new observations in soliton pulses even in a simple fully ionized plasma coexisting with electrons and ions.

3.1 Derivqation of Soliton Solution with Second Order Nonlinearity in Φ and Results

In order to get rid of singular observations on soliton propagation or properly to say to know more about the nonlinear solitary waves derivable from the Sagdeev wave equation, we consider next higher order effect (i.e. third order effect) in the expansion of Φ and derives Eq.(17) as

$$\beta^2 A\frac{d^2\Phi}{d\xi^2} = A_1\Phi + A_2\Phi^2 + A_3\Phi^3$$

with $$A_3 = \frac{\eta^2}{6}\left(1 - \frac{7cos^2\theta}{M^2}\right) \quad (34)$$

Eq.(20), under a linear transformation as F = ν Φ + μ with ν =1 and $\mu = \left(\frac{A_2}{3A_3}\right)$, derives a special type of nonlinear wave equation known as Duffing equation of the form

$$\beta^2 A \frac{d^2F}{d\xi^2} - B_1 F + B_2 F^3 = 0 \tag{35}$$

where $B_1 = A_1 - 2 A_2 \mu + 3 A_3 \mu^2$, $B_2 = - A_3$ are used along with a relation $A_1 - A_2 \mu + A_3 \mu^2 = 0$ and must be followed to get a stable solution of the wave equation. Now to get the results on acoustic modes, Duffing equation has been solved again by tanh-method. That needs, as before, a transformations Φ(ξ) = W(z) with z = tanh ξ to be used to Duffing equation causeway it gets a standard Fuschian equation as

$$\beta^2 A\left(1-z^2\right)^2 \frac{d^2F}{d\xi^2} - 2\beta^2 Az(1-z^2)\frac{dF}{d\xi} - B_1 F + B_2 F^3 = 0 \tag{36}$$

Forbenius series solution method derives a trivial solution with N = 1, which does not ensure to derive the nonlinear solitary wave propagation in plasmas. This necessitates the consideration of an infinite series which after a straightforward mathematical manipulation derives the solution as

$$F(z) = a_0\left(1-z^2\right)^{\frac{1}{2}} \tag{37}$$

Following the earlier procedure along with the substitution of Eq.(37),Eq.(36), after similar mathematical manipulation(Das and Sarma [57]), evaluates the soliton solution as

$$\Phi(x,t) = -\frac{A_2}{3A_3} \pm \sqrt{\left(\frac{3B_1}{B_2}\right)} sech\left(\frac{x-Mt}{\delta}\right) \tag{38}$$

Where $B_1 = A_1 - 2 A_2 \mu + 3 A_3 \mu^2$ and $B_2 = - A_3$

The solution depends on the variation of B_1, B_2 and thus on A_2, A_3 which are controlling by the variation of rotation and Mach number, M. Thus to know the characteristics of solitary wave, B_1 and B_2 are plotted in Fig.3 with the variation of Mach number, M andθ. It is evident that the soliton existences and its propagation fully depends on the variation of rotation. For slow rotation, both B_1 and B_2 are negative and confirm the evolution of solitary wave propagation otherwise, for opposite signs in B_1 and B_2, wave equation fails to exhibit soliton dynamics. The (±) signs represent respectively compressive and rarefactive solitons appeared in the same region. The required condition for the existence of soliton propagation must be as $B_1 < 0$, i.e. $A_1 + 3 A_3 \mu^2 < 2 A_2 \mu$, otherwise non-existences lead the solution as of a shock wave occurring for high rotation. Thus the consideration of slow rotation justifies to the findings of solitary wave propagation in astroplasmas.

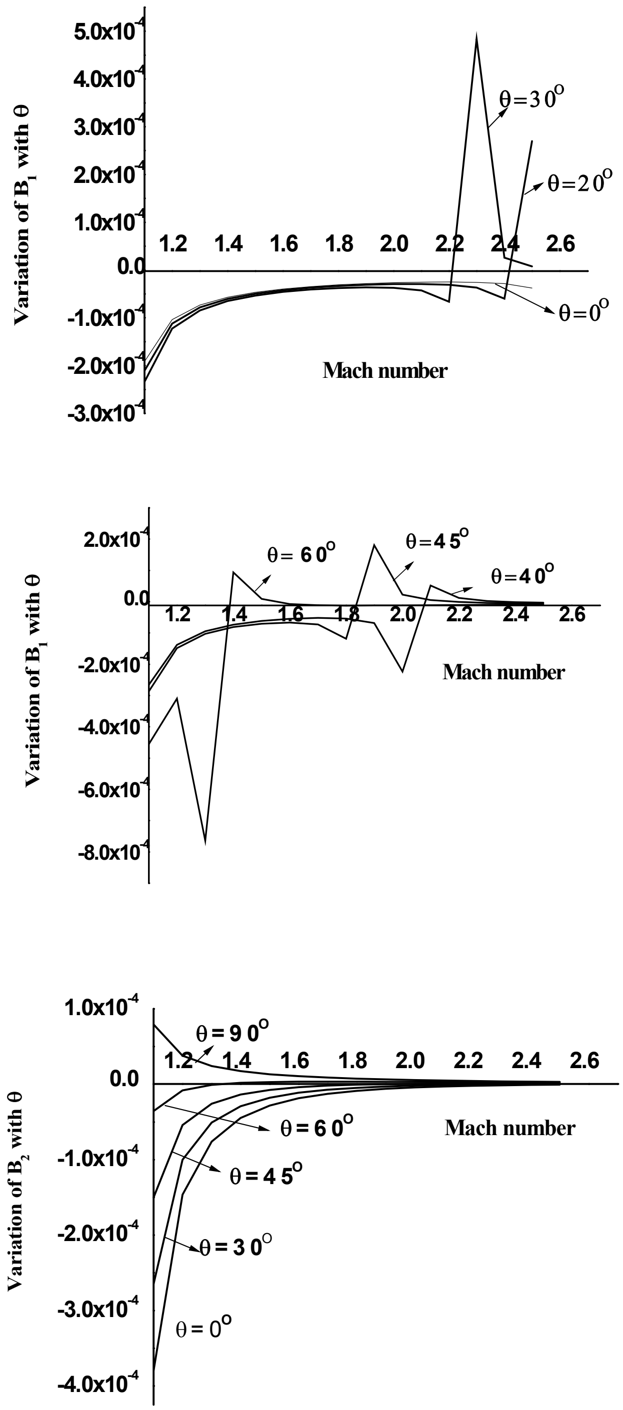

Fig. 3. Variation of B_1 and B_2 with Mach number for different angles of rotation

3.2 Derivqation of Soliton Solution with Next Higher Order Nonlinearity in Φ and Results

Now to avoid the singular behaviour in soliton propagation, wave equation Eq.(17) again approximated with next higher order term as:

$$\beta^2 \left(\frac{d\Phi}{d\xi} \right)^2 = A_1 \Phi^2 + \frac{2}{3} A_2 \Phi^3 + \frac{1}{2} A_3 \Phi^4 \tag{39}$$

The procedure of tanh-method is not taken up as our intension is to use an alternate procedure to find the soliton propagation. The reason of not using the same tanh-method for solving the nonlinear wave equation as it seems to be needed an appropriate transformation for getting a standard form [57,60]. Using some mathematical simplification along with Ψ = 1/Φ , Eq.(39) has been modified as

$$\beta \left(A_1\Psi^2 - 2/3\, A_2\, \Psi - 1/2 A_3\right)^{-1/2} d\Psi = ½\, d\xi \tag{40}$$

The straightforward mathematical manipulation derives the solution as

$$\Phi = \left[-\frac{A_2}{3A_3} \pm \left(\frac{A_2{}^2}{9A_1{}^2} - \frac{A_3}{2A_1} \right)^{\frac{1}{2}} cosh\left(\frac{x - Mt}{\delta} \right) \right]^{-1} \tag{41}$$

where $\delta = \frac{\beta}{\sqrt{A_1}}$

Solution depends on the variation of A_1, A_2 and A_3 which are functions of angular velocity, Mach number and angle of rotation. It has already shown that A_1 is always positive with the variation of M and θ i.e. for different magnitudes of rotation controlling the strength of rotation. Now, because of having varying values of A3, which can be positive or negative (shown in Fig.4). the expression C_r =($2A_2^2 - 9A_1A_3$) has to be controlled to be positive for the existences of nonlinear solitary wave otherwise the negative value of ($2A_2^2 - 9A_1A_3$) leads to a shock wave. Again based on the some typical case where A1< A3, Wave equation (41) can be expanded as a series and along with limiting case A3 → 0 the solution (41) reduces to the soliton solution of $sech^2$(~) profile) as similar to the profile given by Eq.(33). In alternate case when A2→ 0 , solution deduce the soliton in the form of sech(~) profile (as similar to solution given by Eq.(38). These properties of nonlinear wave equation have discussed expeditiously elsewhere Devi et al. [60] and thus we are very much reluctant to repeat all here. Now from the discussions it is clear that the plasma parameters has to be controlled along with the effect of Coriolis force i.e. rotation and M to get the different soliton features which are quite different from the observations could be found in simple plasma (where compressive soliton exists). All new findings are due to Coriolis force generated in rotating plasmas, and concludes that the observations in astroplasmas without rotation will not be having full information rather it might get erroneous conclusions.

Again Eq.(39) can be furthered as of simpler Sagdeev potential equation as

$$\beta^2 \left(\frac{d\Phi}{d\xi}\right)^2 + V(\Phi) = 0 \tag{42}$$

The Sagdeev potential like equation could reveal the double layers which has important dynamical features in plasmas. To derive, Eq.(42) has been transformed as

$$\beta \left(\frac{d\Phi}{d\xi}\right) = p\Phi\left(\Phi - \Phi_r\right) \tag{43}$$

Where the new parameters have redefined as

$$p = \sqrt{\frac{A_3}{2}} \text{ and } \Phi_r = \left(\frac{-2A_2}{3A_3}\right)$$

along with the double layer condition $2A_2^2 = 9A_1A_3$, for $A_3 > 0$.
Following tanh-method[57], double layer solution has been obtained as

$$\Phi(\xi) = \frac{1}{2}\Phi_r\left[1 + \tanh\frac{(x - Mt)}{\delta}\right] \tag{44}$$

Fig. 4 shows that for lower value of the Mach number and A_3 takes only negative values for slow rotation, while it flips over to positive value with the increase of rotation. This may influence the formation of double layers in the rotating plasma what exactly be studies interest. Thus for plasma parameters controlled by the variation on Coriolis force and Mach number, double layer solution might coexist with other solitary waves provided the higher order nonlinearity in the dynamical system is incorporated. Moreover the control might require necessary condition on A1, 2, A3 along with the necessary condition on $(2 A_2^2 – 9A_1A_3)$.

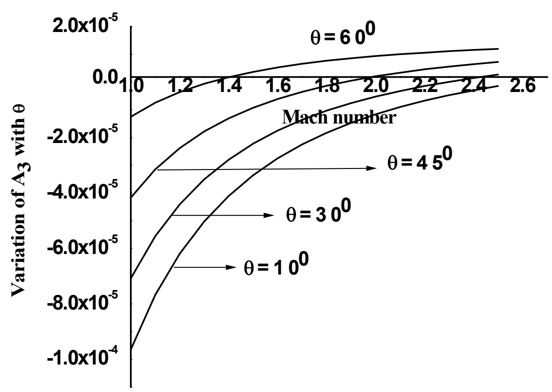

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Fig. 4. Variation of A_3 with Mach number for different angles of rotation

3.3 Derivqation of Soliton Solution With Next Higher Order Nonlinearity in Φ and Results

In order to have further investigations on nonlinear wave phenomena derivable from Eq.(17), we consider next higher order nonlinearity in Φ, and Eq.(17) derives as

$$\beta^2 \left(\frac{d\Phi}{d\xi}\right)^2 = A_1\Phi^2 + A_2\Phi^3 + A_3\Phi^3 + A_4\Phi^4 \quad (45)$$

where, $A_1 = \eta^2\left(1 - \frac{cos^2\theta}{M^2}\right)$, $A_2 = \frac{\eta^2}{2}\left(1 - \frac{3cos^2\theta}{M^2}\right)$ and $A_3 = \frac{\eta^2}{6}\left(1 - \frac{7cos^2\theta}{M^2}\right)$

and $A_4 = \frac{\eta^2}{24}\left(1 - \frac{15cos^2\theta}{M^2}\right)$

Using the transformation F = $\nu\Phi + \mu$ with ν =1 and $\mu = \frac{A_3}{4A_4}$ Eq.(45) has been simplified as

$$a\frac{d^2F}{d\xi^2} - bF + cF^4 = 0 \quad (46)$$

where $a = \beta^2$, $b = A_1 - 2A_2\mu + 3A_3\mu^2 - 4A_4\mu^3$, and $c = -A_4$, supported by two additional conditions $4A_1\mu - 4A_2\mu^2 + 3A_3\mu^3 = 0$ and $2A_2 - 3A_3\mu = 0$

Eq. (46) resembles very much to Painleve equation. To follow the proposed tanh-method, the process encounters a problem of getting N = 2/3 by balancing the order of linear and nonlinear terms. Thus the alternate choice the solution to be some higher order of sech-nature. Thereby solution has been obtained as

$$\Phi(x,t) = -\frac{A_3}{4A_4} \pm \left(\frac{A_1 - 2A_2\mu + 3A_3\mu^2 - 4A_4\mu^3}{-2A_4}\right)^{\frac{1}{3}} sech^{\frac{2}{3}}\left(\frac{x - Mt}{\delta}\right) \quad (47)$$

The mathematical analysis reveals that, Sagdeev potential equation with higher-order nonlinearity admits the compressive solitary wave or double layers depending on the nature of the expression under the radical sign which are functionally dependable on rotation and Mach number.

Fig. 5 shows that slow rotation maintains the existences of the solitary wave propagation while the increases in rotation magnitude (signified by higher values of rotational angle, θ) the amplitude shows a discontinuity, which might explain the explosion or collapse in solitary wave. In such phenomena, there must be either conservation of energy (collapse of solitary wave), or dissipation of energy (as in case of explosion) which may be related as the similar occurrences of solar flares, sunspots and other topics of astrophysical interest [7,8,25,51,52,61].

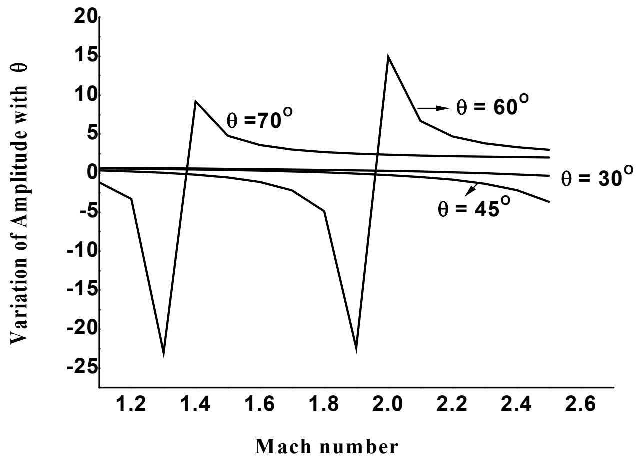

Fig. 5. Variation of amplitude of the solitary wave with Mach number

The procedure ensures that continuation could be interesting in finding the features of soliton propagation in a wide range of configurations, along with the existences of narrow region in which a shock like wave is expected and then the study has to be furthering by the use of higher order effect in nonlinearity.

3.4 Derivqation of Soliton Solution with n-th Order Nonlinearity in Φ and Results

To generalize the analysis, Sagdeev potential equation is expanded up to the n-th order nonlinearity and following [57] the solution is obtained as

$$\Phi(x,t) = -\frac{A_{n-1}}{nA_n} \pm \left(\frac{M}{-A_n}\right)^{\frac{1}{n-1}} sech^{\frac{2}{n-1}}\left(\frac{x-Mt}{\beta}\right) \tag{48}$$

where $\beta = M^{1/2}$ and M is a linear combination of A_1, A_2, …………, A_n

Eq. (48) gives shock wave solution depending on the sign of the quantity under the radical.

Now to find out the higher order solution of Sagdeev potential equation with other possible acoustic modes, we integrate Eq. (17) to obtain

$$\beta^2\left(\frac{d\Phi}{d\xi}\right)^2 = A_1\Phi^2 + \frac{2}{3}A_2\Phi^3 + \frac{1}{2}A_3\Phi^4 + \frac{2}{5}A_4\Phi^5 \tag{49}$$

Next with suitable mathematical transformation and use of proper boundary conditions, Eq.(49) can be transformed to the following form

$$\beta^2 \left(\frac{d\Phi}{d\xi} \right)^2 = \alpha \Phi^2 (p - \Phi)^3 \tag{50}$$

Comparing Eqs. (35) and (34) we obtain the relations $\alpha = \frac{2}{5} A_4$ and $p = \frac{5A_3}{12A_4}$, which are supported by the condition $A_3^2 = \frac{16}{5} A_2 A_4$

Finally the solution comes out with a new feature of showing sinh-nature.

$$\Phi(\xi) = p \left(sinh^2 \left[\left(\frac{p}{p - \Phi} \right)^{\frac{1}{2}} \mp \frac{\sqrt{\alpha}}{2} p^{\frac{3}{2}} \xi \right] \right) \tag{51}$$

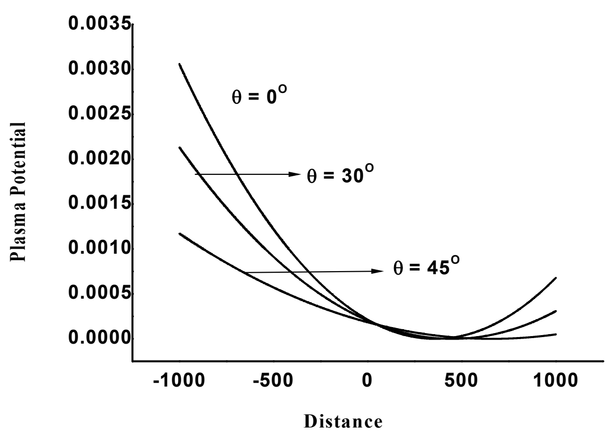

Fig. 6. Variation of nature of the Sinh- wave for different angles of rotation

Fig. 6 shows the analysis of the fourth order nonlinear approximation in Sagdeev potential equation and derives new wave propagation with the nature of having identically to sin-hyperbolic curve. The wave is also influenced by the interaction of rotation parameters and the magnitude of the wave shows an increase with the decrease in value of θ and thereby shows the influence of slow rotation on the existences of nonlinear solitary waves.

4. CONCLUSIONS

Overall studies exhibit the evolution of different nature of nonlinear waves showing the effective interaction of Coriolis force. The model is taken under the approximation of slow rotation which are appropriate to rely on astrophysical plasmas, and concludes that the present studies could be an advanced theoretical knowledge as well. It has shown that small amplitude approximation in Sagdeev wave equation derives compressive or rarefactive solitary waves and slow rotational effect is the progenitor of solitary waves even in simple fully ionized plasma. There exists a critical point at which A_2 equals to zero and causeway derives rarefactive nature of soliton when $A_2 < 0$ otherwise a changes occur from the rarefactive to compressive soliton profile bifurcated by the critical point at which existences break down. At the neighborhood of this critical point, solitary wave grows to be large forming a narrow wave packet and, because of which, the soliton either collapses or explodes depending on the conservation of energy in the wave packet. Because of which, there is a generation of high electric force and consequently high magnetic force within the narrow wave packet as a result density depression occurs and exhibits soliton radiation resembles this phenomenon bridging with the occurrences of solar radio burst [8,61], soliton radiation [51,52] as well as in plasma environments of pulsar magnetosphere [40] finds at the neighbourhood of a critical point occurs due to rotation of the plasma.

Further with the variation of nonlinear effect along, interaction of slow rotation derives many other plasma-acoustic modes like double layers, shock waves and sin-hyperbolic wave profile in the dynamical system. It has been observed that the Mach number does not show any new observation on the existences on solitary wave rather it reflects schematic variation on the nature of the soliton wave, Coriolis force interaction, however small might be, exhibits different salient features of acoustic modes. The results emerging from the present studies is quite different as compare to the observations made in simple non-rotating plasmas and reflects that the wave phenomena in astroplasmas must consider the rotational effect otherwise the studies will not give full observations rather it misses many acoustic modes in observations.

We have shown, in comparison to a non-rotating plasma, rotation brings all kinds of nonlinear plasma waves and rotational effect is a progenitor of compressive and rarefactive solitons, double layers, shock waves along with soliton radiation similar to those could be found in pulsar magnetosphere as well as in the high rotation neutron stars. The complete solution of the Sagdeev potential equation i.e. without having any approximation on nonlinearity, derives a special feature of nonlinear wave phenomena known as sheath in plasmas. Fewer observations have been made among them recent works on showing sheath formation in dusty plasmas [62], in rotating plasmas (Das and Chakraborty [63]) deserve the merit. Study has shown the sheath formation over the Earth's Moon surface [63], and thereafter finds the dynamical behaviours of dust grains levitation into sheath. It predicts the important role of Coriolis force in the problems of astroplasmas without which the results are likely to be erroneous. They have discussed also the formation of nebulons i.e. formation of dust clouds over the Moon's surface and bridges a good agreement with some observations given by NASA Report [64].

ACKNOWLEDGEMENTS

Author gratefully acknowledges the financial support provided by ISRO- RESPOND Research Program, India (Project No. PF/ 2008-2009/ 7763-7769) to the research works. Author also thanks the Reviewers for their fruitful suggestions and critical comments in bringing the paper in present form.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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