Pressure Gradient Prediction of Multiphase Flow in Pipes

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Authors' contributions

This work was carried out in collaboration between all authors. Authors AAS and OMA designed the study. Author OMA performed the statistical analysis and wrote the protocol. Author AAS wrote the first draft of the manuscript while author UAJ wrote the final draft and managed literature searches. All authors read and approved the final manuscript.

Original Research Article

ABSTRACT

Pressure traverse in multiphase flow differs from single phase flow due to the differential flow rates of the different phases. Correlations developed to predict multiphase flow pressure traverse are mostly for vertical wells but Beggs and Brill model is one of the few models that is used for inclined pipes. The work seeks to show the improvement in the modification of the model. This project is based on studies carried out on multiphase fluid flow in pipes of any inclination using the Beggs and Brill flow model as the focus. Two cases were considered, the liquid holdup correction and Gas Liquid Ratio (GLR) variations in which the Beggs and Brill and Beggs and Brill Traverse models were compared. Due to the empirical nature of the Beggs and Brill model, pressure gradient predictions are far from accurate when compared with measured data in the field. This

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project seeks to reduce the error margin between predicted pressure gradient values and measured data. It was observed that for the same reservoir, fluid, and pipe properties, the Beggs and Brill Traverse Model is a better prediction tool than the Beggs and Brill model. Prediction errors were seen to increase with increase in length for GLR above 400 scf/stb while they were more accurate for pipes between 12,000 and 17,000 ft and pressures between 3,000 and 4,500 psi. However, the Beggs and Brill Traverse Model, is limited by the choice of correlations used in the computation of fluid properties.

ABBREVIATIONS

ANN = Artificial Neural Networks, API = American Petroleum Institute, BB = Beggs and Brill, D= Pipe diameter, ft, $(\partial P/\partial z) = \Pr essure\ gradient$, $d(W)$ = Irreversible Friction loss, ε = Relative roughness, ρ = Density, lbs/ft^3 , g = Gravitational acceleration, 32.2 ft/sec^2 , H = Holdup, dimensionless, GLR ` = Gas/liquid ratio, scf/stb, L = Length of pipe, ft, P = Pressure, psi, Scf = Standard cubic feet, ft^3, Stb = Stock tank barrel barrel, T = Temperature, ^{o}F, q = Flow rate, b/d, θ = Angle of inclination from the horizontal, degrees, Y = Specific Gravity, V = Mixture Velocity, ft/sec, Ψ = Inclination correction factor, λ = Fluid input fraction, WOR = Water/oil ratio, stb/stb, Z factor = Gas compressibility factor.

SUBSCRIPTS

c = Constant, F = Frictional, g = Gas, KE = Kinetic Energy, o = Oil, m = mixture, PE = Potential Energy, sep = Separator, Step = Incremental, tp= two-phase, w = Water.

1. INTRODUCTION

When a well is completed, fluid flow from the reservoir to the surface through pressure differential between the reservoir and the bottom of the hole (Bottom Hole Pressure, BHP), then between the bottom of the hole and the Wellhead (Wellhead Pressure, WHP). Flow in a wellbore could be either Single-Phase or Multiphase and also within circular pipe like the tubing and through the *annular space* between tubing and casing. When considering wellbore flow performance, the aim is to predict pressure as a function of position between the bottomhole location and the surface. Also, in most cases, the velocity profile and phase distribution in multiphase flow are desired [1].

1.1 Multiphase Flow Concepts

In multiphase flow, the volume in the pipe occupied by a phase is often different from its proportion of the total volumetric flow rate. For a typical two-phase upward flow of gas, *g*, and liquid, *l*, where the less dense gas phase, *g*, will flow faster than the denser liquid phase, *l*, there is a resultant *"*slip" or "hold up" effect due to differences in flow velocity causing the in-situ volume fraction of each phase to differ from the input volume fraction of the pipe. The denser phase is "held up" in the pipe relative to the lighter phase.

1.2 Pressure Traverse

The pressure gradient, *dP/dZ*, for compressible and slightly compressible fluids, along the length of a wellbore varies for single-phase and multiphase mixtures. Pressure gradient variation results from variations in pressure, temperature and angle of inclination along the pipe length. Due to these variations, the total pressure drop along the length of the pipe is calculated in a stepwise manner. The total length is divided into increments small enough that the flow properties, and thus, pressure gradient, are almost constant in each increment. The pressure drop in each increment usually is integrated to obtain the overall pressure drop *ΔP*. This stepwise calculation procedure is what is known as Pressure Traverse Calculation. The process usually is iterative since pressure, temperature, and other fluid properties vary. Pressure traverse calculations are performed either by fixing the length increment and finding the pressure drop over this increment or by fixing the pressure drop and finding the wellbore length interval over which this pressure drop would occur. In this work, Two-phase (gas and liquid) upward flow of fluids in circular pipes is being considered.

The "energy-loss factor" correlation proposed by Poettman and Carpenter [2] was based on relatively low-rate flow data which are not applicable to high-rate flow conditions. Consequently, Baxendell and Thomas [3] attempted to establish a satisfactory correlation for high rates up to 5000 b/d. Tek [4] presented a correlation for a two-phase problem to predict the pressure distribution in vertical multiphase flow strings well within the accuracy range usually desired by common engineering calculations. He presented two methods of correlation to predict pressure drops in multiphase flow through vertical pipes. A method which accurately predicts with precision of about 10%, two phase pressure drops in flowing and gas lift production wells over a wide range of well conditions was used by [5,6] and examined pressure gradients occurring in flowing and gas-lift wells based on pressure-balance equation. Ref. [7] investigated gas-liquid flow in inclined pipes to determine the effect of pipe inclination angle on liquid holdup and pressure loss. They developed correlations for liquid holdup and friction factor for predicting pressure gradient for two-phase flow in pipes at all angles for many flow conditions. Many experimental and theoretical studies have been conducted to determine the principles governing the flow of heterogeneous gas-liquid mixtures in vertical and inclined wells [2,3,4,8,9].

The basis for any fluid flow calculations is an energy balance for the flow in fluid between two points. A steady-state mechanical energy balance equation for one pounce mass of fluid may be expressed as:

$$\frac{dp}{\rho_{tp}} + \frac{g}{g_c}\,dh + \frac{V_m dV_m}{g_c} + d\left(W_f\right) = 0 \qquad (1)$$

Where $d\left(W_f\right)$ represents the irreversible friction losses for flow in pipe up and down an inclination;

$$dh = Sin\theta dZ \qquad (2)$$

- Where dh = vertical distance moved
- θ = angle of the pipe to the horizontal
- dZ = Axial distance moved

- Substituting Eq. 2 into Eq. 1 gives:

$$\frac{dp}{dZ} = -\left[\frac{g}{g_c}\rho_{tp}\sin\theta + \rho_{tp}\frac{V_m dV_m}{g_c dZ} + \rho_{tp}\frac{d(W_f)}{dZ}\right] \qquad (3)$$

Where the two-phase density term is defined by this expression:

$$\rho_{tp} = \rho_l H_l + \rho_g (1 - H_l) \qquad (4)$$

Where H is the holdup correlation that allows a model to account for phase separation along a pipe and defines density and friction factors for simultaneous flow of gas and liquid. Eq. 3 may be written as:

$$-\frac{dP}{dZ} = \left(\frac{dP}{dZ}\right)_{PE} + \left(\frac{dP}{dZ}\right)_{KE} + \left(\frac{dP}{dZ}\right)_F \qquad (5)$$

That is the total pressure drop is the sum of the pressure drops due to the potential energy change, kinetic energy change and friction loss. Where: $(\partial P/\partial z)_{PE}$ = Static gradient, the energy required to support the gas and liquid column present in the well. $(\partial P/\partial z)_{KE}$ = Acceleration gradient, the energy due to fluid flow in the well. $(\partial P/\partial z)_F$ = Friction gradient, the energy required to overcome the drag of the fluids on the walls of the well as well as that energy used to overcome slippage between the gas and liquid phases.

Prediction of pressure drop in oil wells started back in 1952 when Poettmann and Carpenter published their first predictive scheme [2]. Many more attempts have been made to predict the complex flow behaviour within an oil well but, no single correlation can successfully predict pressure drop accurately over the wide range of operating conditions encountered around the world. Three groups of model correlations are homogenous flow model, slip model and flow pattern models. Homogeneous flow model assumes that the multiphase mixture behaves like a homogeneous single-phase fluid [2,3,4,8,9]. The Slip Model assumes that the different phases tend to separate because of differences in density resulting in different flow velocities for each phase. One phase tends to flow faster than the other causing a phenomenon known as slippage [9,10,11]. In Flow Pattern Model correlations are required to predict the liquid holdup and friction factor and the flow pattern which exists must also be predicted. Hence the hold up and friction factor depend on which flow pattern exists at a point [6,12,5,13,14,7,15]. Only Ref. [7,15] methods were developed for angles other than vertical upward flow and consequently, are applicable in injection wells.

2. METHODOLOGY

2.1 Beggs and Brill Correlation

Beggs and Brill correlation [7] is one of the few published correlations capable of handling various flow directions. It was developed using 1inch and 1.5 inch sections of pipe that could be any angle of inclination to the horizontal plain. The correlation deals with both the friction pressure loss and the hydrostatic pressure difference. Flow properties along the wellbore

may change significantly in gas-liquid flow. For this reason, the pressure gradient for a particular location in the wellbore is calculated and the overall pressure drop is then obtained with a pressure traverse calculation procedure.

The empirical correlation of Beggs and Brill was developed from experimental data obtained in a small scale test facility. The range of parameters studied were: gas flow (0 to 300 scf/day) liquid flow rate (0 to 4.0 ft^3/min); average system pressure (35 to 95 psia); pipe diameter (1 to 1.5 inch); liquid holdup (0 to 0.870); pressure gradient (0 to 0.8 psi/ft.) inclination angle (-90° to +90°) and horizontal flow pattern. The fluids used for the experiment were air and water [16].

The Beggs and Brill correlation is based on the flow regime that would occur if the pipe were horizontal. Corrections are then made to account for holdup behaviour with pipe inclination. The Beggs and Brill model is the recommended technique for wellbore that is not near vertical. It is one of the correlations that can be used for any pipe inclination. For this work, the software Traverse based on the Beggs and Brill [7] correlation and the listed modifications was adopted for easier and more accurate computations. Fig A-1 in the Appendix is the computer flow diagram used for computation of the Beggs and Brill model adopted from Chaudhry [17].

Traverse software was used to compute pressure traverse along a wellbore with two-phase flow, using the Beggs and Brill empirical model and for all wellbore inclinations. The results generated are then exported to Microsoft Excel for analysis. Traverse software Input data for initialization include: Outlet Pressure, P_1, Inlet Pressure, P_2, Pressure increment, P_{step}, Temperature, T and Relative Roughness, ε. Table A-1 in the Appendix contains all the input data used for the computation with the following range: ID(3.958-6.184 in.); q_o (7,190-27,270stb/d); q_g (738-9,184Mscf/d); T(188.5-194.0°F); L (7,093-10,289 ft.); Θ (46.1-88.2°); GLR (322.9-336.79scf/stb); API (35.56-36.55°); Y_o(0.842-0.847); P_1 (2025-2616 psig); P_2 (136-373 psig).

3. RESULTS AND DISCUSSION

The test and gradient prediction results showed that the BB and modified BB model had average percentage errors of 12.20% and 7.14% respectively which means that the BB (modified) model is a better gradient prediction tool than the BB model. These average values are obtained from the Appendix Table A-2; a single plot to show the difference of the two models cannot be shown because the data were not obtained under the same GLR conditions. Two cases were considered for the study; Liquid Holdup Correction and effect of increasing Gas Liquid Ratio. Data used for this study were obtained from tests conducted in the Forties Field and Prudhoe Bay Flowlines [18]. The sample case is a vertical well with Gas-Oil Ratios ranging from 0 - 4000 scf/stb for the following flow conditions: Oil flow rate, q_o = 400bpd; WOR=1, Separator pressure, P_{sep} = 100psig, Average Temperature, T = 140°F, Gas specific gravity, Y_g = 0.65, Oil Specific gravity, Y_o= 35ºAPI, Water specific gravity, Y_w = 1.074, tubing size = 2.5 in. ID.

Beggs and Brill correlation was examined with respect to four parameters: Tubing Size, Oil Gravity, Gas-Liquid Ratio (GLR) and water cut. For the tubing sizes, the pressure losses were accurate for the range under consideration (1-1.5in.) but larger tubing sizes tend to result in over prediction of pressure losses. Oil gravity investigation shows a reasonable good performance over a broad spectrum of oil gravities. Gas liquid Ratio: there is a good prediction of pressure loss in shorter lengths and lower pressures but an over prediction is obtained with increasing GLR and pressures above 3000 Psia. Water cut: the accuracy of

the pressure profile prediction is good for water cut of less than 10%. The effect of pressure prediction on liquid holdup and increasing GLR on the Beggs and Brill and the modified model are being considered which generated the following results. The results of the Predictions for the two models were compared for the two cases. The first case is pressure gradient curves for liquid holdup at GLR of 336.8 and 323 scf/stb. The second case are different GLR: 1, 200, 400, 800, 1200 and 1600 scf/stb.

3.1 The Pressure Gradient Curves for Liquid Holdup at Two GLR Conditions

Fig. 1 shows the pressure gradient curves for liquid holdup at GLR of 336.8 scf/stb. The two models are in agreement from zero depth through 7000 ft and a pressure of zero to 2350 psig. There was no over prediction of the pressure gradient with the modified Beggs and Brill model. Fig. 2 is very similar to Fig. 1 as the GLR is less than 400 scf/stb. The two models are in agreement within the pipe length of zero to 7000 ft. and pressure of zero to 2200 psig.
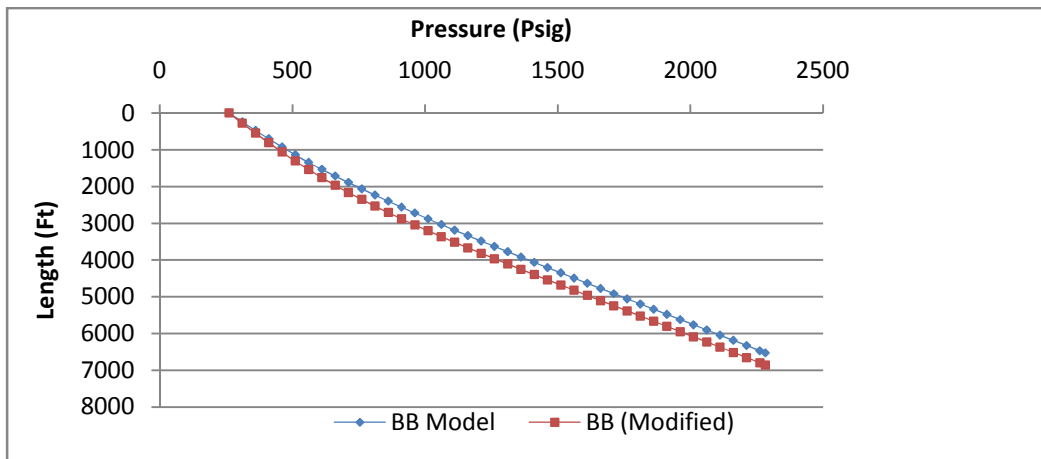

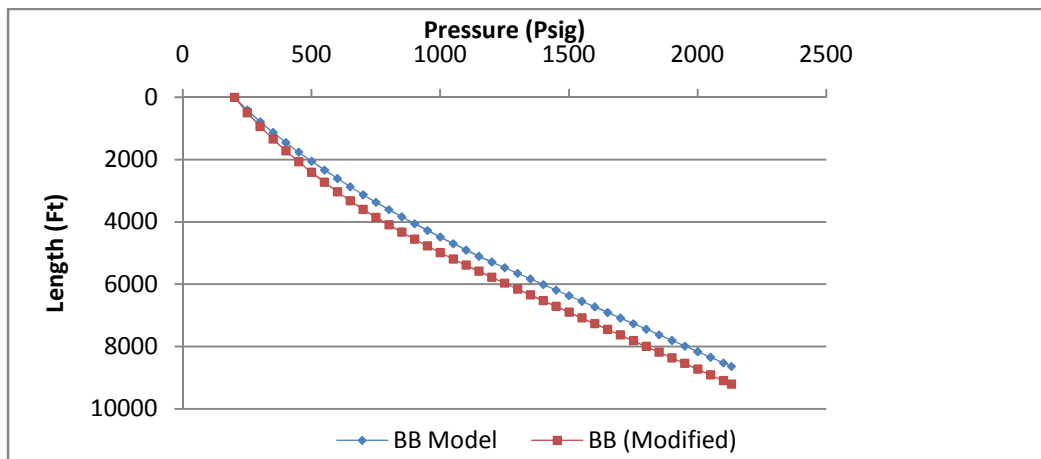**Fig. 1. Pressure gradient for liquid holdup at GLR of 336.8 scf/stb**

Fig. 2. Pressure gradient for liquid holdup at GLR of 323 scf/stb

3.2 Pressure Gradient Curves for Variation in Gas Liquid Ratios (GLR) were Examined from the Following Plots

Figs. 3-8 show result for different GLR for various pipe lengths against pressure. It was observed that for GLR of 1 scf/stb, BB (modified) and BB models are in agreement Fig. 3. For GLR > 200 scf/stb, BB (modified) model predicts pressure gradient more accurately for pipes of about 12000-14000ft and below. Prediction error increases with increase in length for a given GLR. For pipe lengths which are longer, the BB (modified) model overpredicts pressure gradient. Pressure ranges for which BB (modified) over predicts pressure gradients is 2800-4500 psi. and the pipe length range is 12000-17000 ft. Below these ranges, the BB (modified) model predicts pressure gradient more accurately. For GLR of 200scf/stb both models agree at a pressure of 4500 psi and length of 12305 ft. Fig. 4.
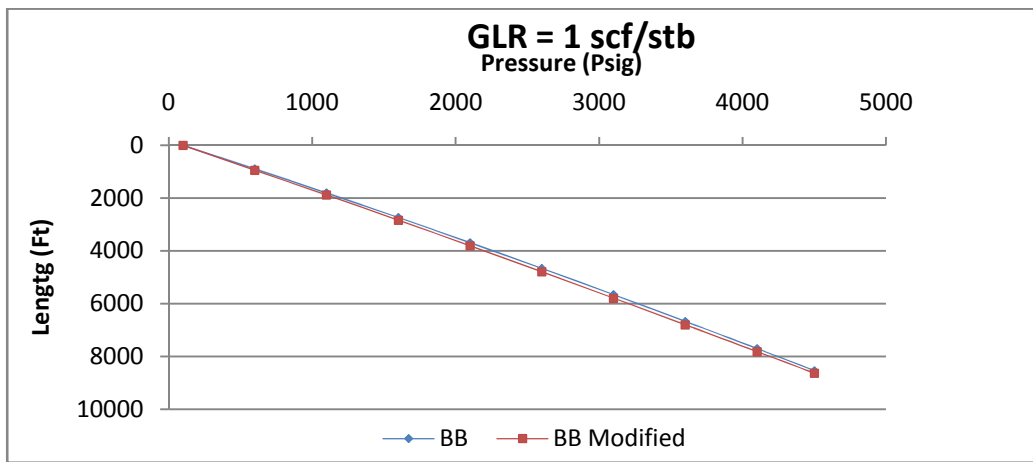

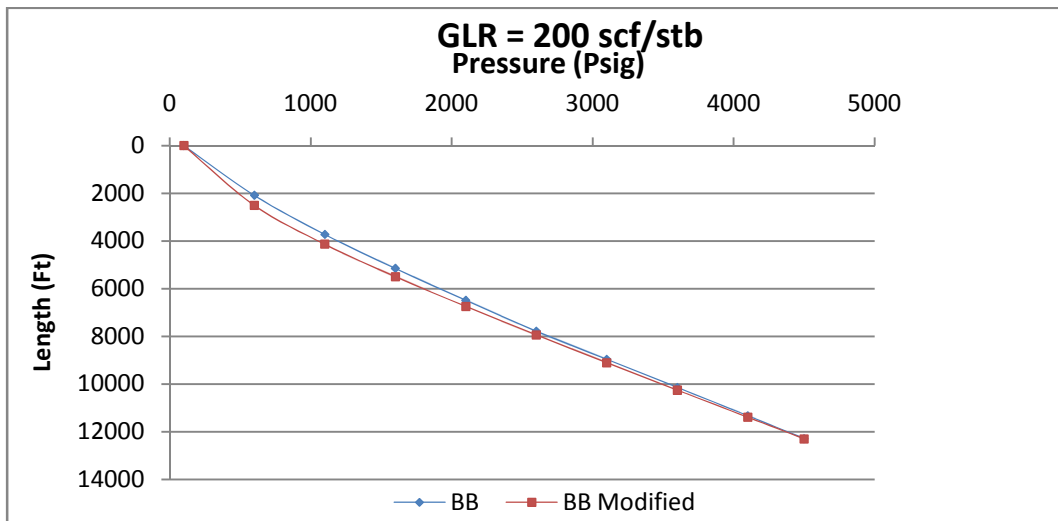**Fig. 3. Pressure gradient for GLR = 1scf/stb**

Fig. 4. Pressure gradient for GLR = 200scf/stb

For GLR of 400 scf/stb they agree at a pressure of 3400 psi and length of 12500 ft. Fig. 5. For GLR of 800 scf/stb they agree at pressure of 3400 psi and length of 14590 ft, Fig. 6. At GLR of 1200 scf/stb they agree at 3200 psi and length of 17000 ft Fig. 7. And finally at GLR of 1600 scf/stb, the point of agreement is 2800 psi and 1600 ft. Fig. 8.
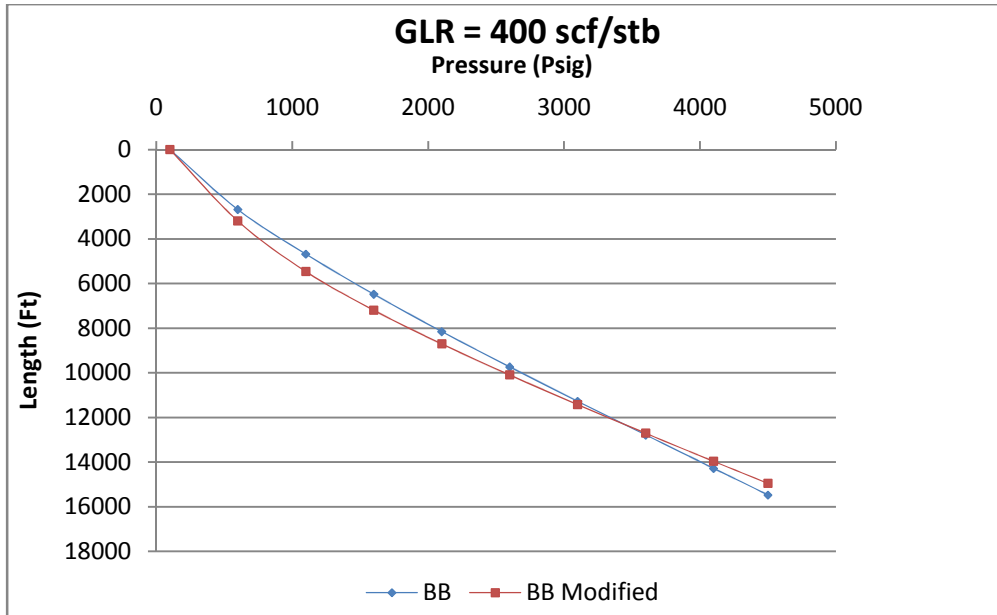

Fig. 5. Pressure gradient for GLR = 400scf/stb
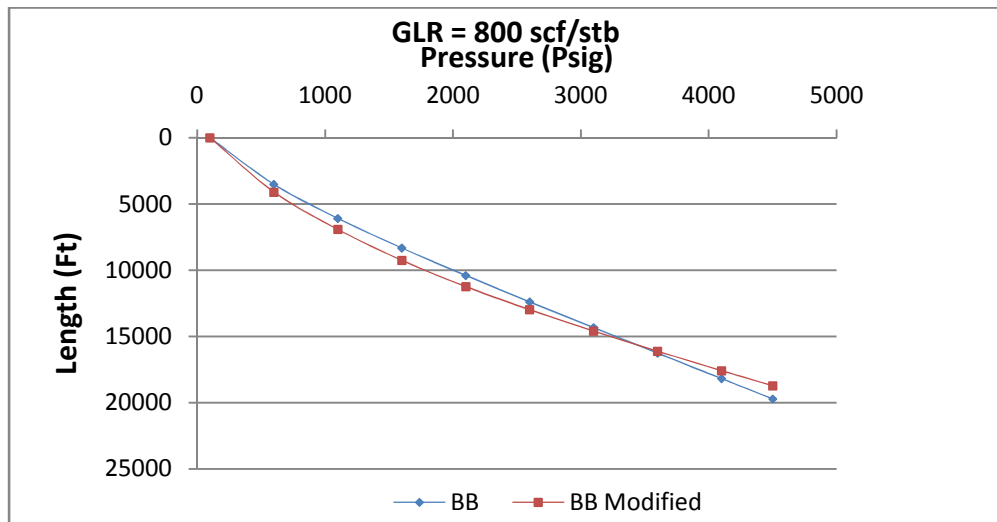
Fig. 6. Pressure gradient for GLR = 800 scf/stb
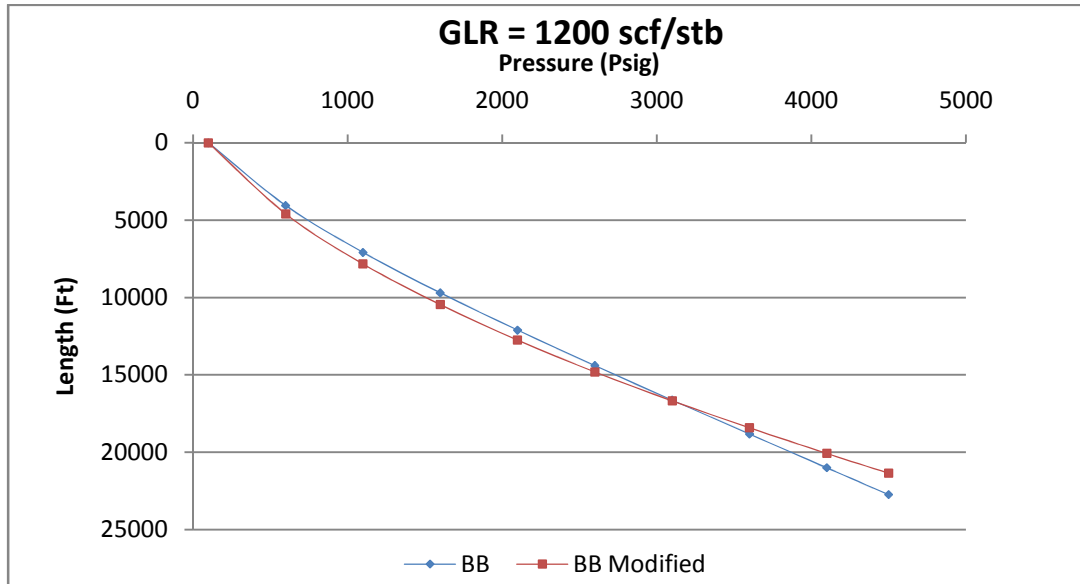
Fig. 7. Pressure gradient for GLR = 1200 scf/stb
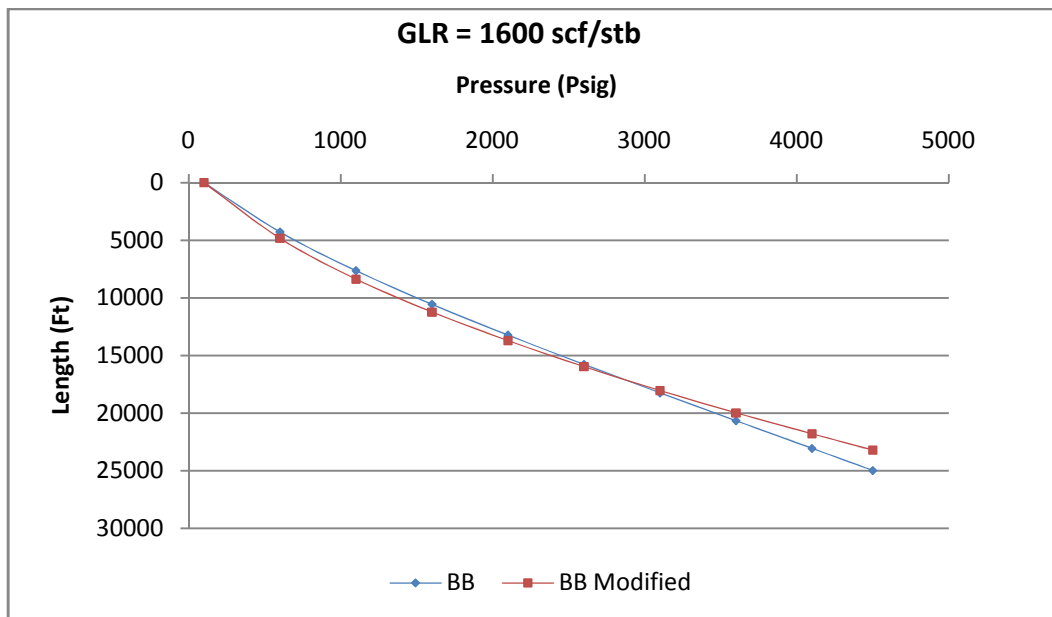
Fig. 8. Pressure gradient for GLR = 1600 scf/stb

Accurate prediction of pressure drop in vertical multiphase flow is needed for effective design of tubing and optimum production strategies and this was achieved with the aid of Artificial Neural Network (ANN) using five correlations [19]. The complexity of the pressure drop calculation of two-phase flow systems is due to the variations in the gas and liquid flow rates across the two-phase flow stream [20]. Two phase flow occurs during the production of oil and gas in the wellbore. Modeling this phenomenon is important for monitoring well

productivity and designing surface facilities. The outcome of such research facilitates a more accurate simulation of multiphase flow in the wellbores and pipes which can be applied to the surface facility design and well performance optimization [21].

4. CONCLUSION

The modified Beggs and Brill Traverse Model give more accurate predictions than the Beggs and Brill Model. However, a major limitation of the model is from the correlations used.

Modified Beggs and Brill Traverse model predicts pressure gradient more accurately at lower pressures and shorter pipe lengths (0 -2000 psig and 0 – 1400 ft)) while it over predicts it at higher pressures and longer pipe lengths (Pressures >3500 psig and Pipe length >1400 ft).

GLR greater than 400 scf/stb will show the tendency for the modified Beggs and Brill model to over predict pressure gradient while GLR less than 400 scf/stb will give more accurate output.

It is necessary to predict pressure drop in vertical multiphase flow in order to effectively design tubing and optimum production strategies.

Further studies are required to determine the effect of water cut above 10% in the pipe on predicted pressure gradient values.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX

Table A-1. Input data

Test	I.D.(in.)	q_o(stb/d)	q_g(Mscf/d)	T (deg.F)	Yg	L (ft)	θ(deg)	GLR(scf/stb)	APIgrav	Y_o	P_1(psig)	P_2 (psig)
1	6.184	18,900	6366	188.5	1.122	7877	67.9	336.83	36.55	0.842	2311	226
2	6.184	26,800	9026	188.5	1.122	7388	88.3	336.79	36.55	0.842	2418	249
3	6,184	16200	5456	188.5	1.122	8103	62.7	336.79	36.55	0.842	2423	320
4	6,184	15,600	5254	188.5	1.122	8379	56.2	336.79	36.55	0.842	2195	220
5	6.184	10,600	3570	188.5	1.122	8255	61.1	336.79	36.55	0.842	2282	246
6	6.184	23,540	7928	188.5	1.122	9180	48.8	336.79	36.55	0.842	2225	252
7	6.164	20,990	7069	188.5	1.122	9199	48.8	336.78	36.55	0.842	2200	249
8	6.134	24,200	8151	188.5	1.122	7746	68.6	336.82	36.55	0.842	2453	300
9	6.184	25,205	8489	188.5	1.122	7480	68.3	336.80	36.55	0.842	2282	261
10	6.184	19,600	6601	188.5	1.122	8543	56.7	336.79	36.55	0.842	2224	234
11	3.958	7190	2322	194.0	1.122	8045	62.8	322.95	35.56	0.847	2053	136
12	6,184	15,120	4884	194.0	1.122	7596	69.9	323.02	35.56	0.847	2616	307
13	6,184	22,235	7182	194.0	1.122	8349	56.4	323.00	35.56	0.847	2139	205
14	3.958	2,284	738	194.0	1.122	8715	56.4	323.12	35.56	0.847	2192	199
15	6.184	7,090	2290	194.0	1.122	7500	69.6	322.99	35.56	0.847	1950	179
16	6.184	19,750	6379	194.0	1.122	6899	88.2	322.99	35.56	0.847	2069	215
17	6.184	6,390	2064	194.0	1.122	9599	47.8	323.00	35.56	0.847	2131	200
18	6.134	12,340	3986	194.0	1.122	7093	70.3	323.01	35.56	0.847	1946	213
19	6.184	13,860	4477	194.0	1.122	8166	61.8	323.02	35.56	0.847	2129	205
20	6.184	17,800	5749	194.0	1.122	8674	55.5	322.98	35.56	0.847	2354	281
21	3,958	6,540	2112	194.0	1.122	8600	56.6	322.94	35.56	0.847	2082	154
22	6,184	14,650	4732	194.0	1.122	7349	70.2	323.00	35.56	0.847	2025	201
23	6,184	15,400	4974	194.0	1.122	8045	61.9	322.99	35.56	0.847	2218	242
24	6.164	12,500	4037	194.0	1.122	7901	61.9	322.96	35.56	0.847	2029	186
25	3.956	7,520	2429	194.0	1.122	9459	49.0	323.01	35.56	0.847	2234	180
26	6.184	21,656	6995	194.0	1.122	7999	60.4	323.01	35.56	0.847	2488	373
27	3.958	7,940	2565	194.0	1.122	10289	43.7	323.05	35.56	0.847	2326	216
28	3.958	8,300	2681	194.0	1.122	7900	62.8	323.01	35.56	0.847	2080	188
28	6.134	27270	9184	188.5	1.122	8169	56.6	336.78	36.55	0.842	2281	277
30	6.184	11,000	3705	188.5	1.122	9714	46.1	336.82	36.55	0.842	2308	305

Table A-2. Test and prediction results

Test	Measured			BB		BB (modified)	
S/N	Inlet P2 (psig)	Outlet P1(psig)	Length (ft)	Length (ft)	% error	Length (ft)	% error
1	2311	226	7877	7094.614	9.93	7607.821	3.42
2	2418	249	7388	6449.682	12.70	7272.074	1.57
3	2423	320	8103	7209.242	11.03	7575.690	6.51
4	2195	220	8379	7728.769	7.76	8096.588	3.37
5	2282	246	8255	7531.184	8.77	7963.143	3.54
6	2225	252	9180	7920.768	13.72	8248.693	10.14
7	2200	249	9199	8006.883	12.96	8337.129	9.37
8	2453	300	7746	6800.094	12.21	7129.197	7.96
9	2282	261	7480	6527.016	12.74	6853.804	8.37
10	2224	234	8543	7489.237	12.33	7834.011	8.30
11	2053	136	8045	6870.916	14.59	7244.408	9.95
12	2616	307	7596	7373.935	2.92	7784.238	-2.48
13	2139	205	8349	7193.331	13.84	7576.773	9.25
14	2192	199	8715	7756.340	11.00	8317.409	4.56
15	1950	179	7500	6432.106	14.24	6977.241	6.97
16	2069	215	6899	5893.825	14.57	6282.971	8.93
17	2131	200	9599	8635.212	10.04	9207.968	4.07
18	1946	213	7093	6124.558	13.65	6563.795	7.46
19	2129	205	8166	7126.118	12.73	7571.879	7.28
20	2354	281	8674	7646.587	11.84	8044.662	7.26
21	2082	154	8600	7398.153	13.97	7782.768	9.50
22	2025	201	7349	6387.475	13.08	6817.000	7.24
23	2218	242	8045	7077.978	12.02	7491.111	6.88
24	2029	186	7901	7001.754	11.38	7465.163	5.52
25	2234	180	9459	8254.259	12.74	8599.729	9.08
26	2488	373	7999	6984.630	12.68	7333.605	8.32
27	2326	216	10289	8939.927	13.11	9268.078	9.92
28	2080	188	7900	6434.352	18.55	6799.729	13.93
29	2281	277	8169	6928.159	15.19	7239.800	11.37
30	2308	305	9714	8773.141	9.69	9067.435	6.66
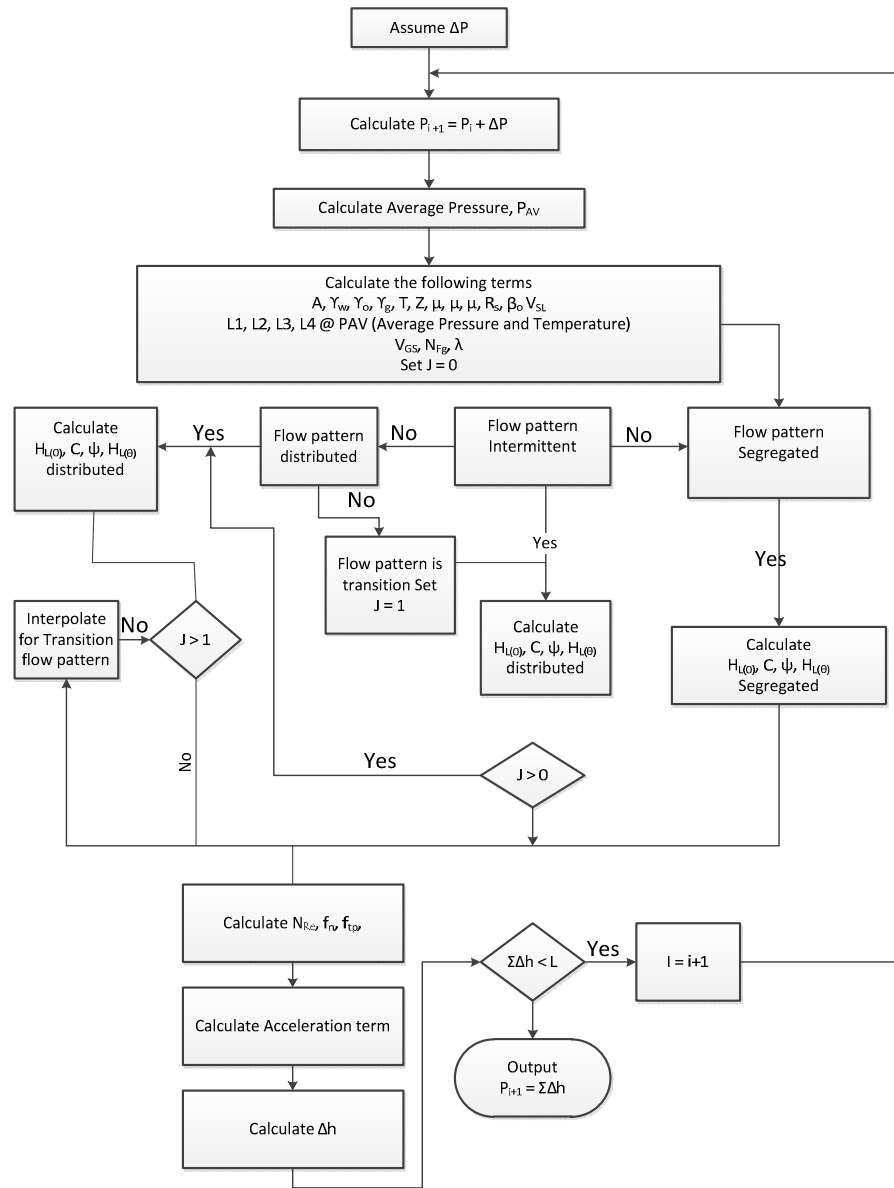
Fig. A-1. Computer flow diagram for the Beggs and Brill Method
(Source: Chaudhry 2004)