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A Production Inventory Model for Fractionally Time-Dependent Demand Rate with Weibull Deterioration and Partially Backlogged Items

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This article presents a report on a creation stock model that the point is to foster an ideal creation and stock procedure that boosts the general benefit while thinking about the impact of expansion on the decaying things over the long haul. The exponential demand rate is thought to accurately reflect real-world demand fluctuations over time. The model gives a manager's realistic representation of the production and inventory system by considering these aspects, allowing them to make informed decisions. To enhance the creation and stock strategy, a numerical structure is created, consolidating different expense parts, for example, holding

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cost, arrangement cost, creation cost, and lack cost. The goal is to find the ideal creation amount and reorder point that limit the all-out cost and expand the general benefit. The proposed model's analytical results show a complex connection between the optimal production quantity, reorder point, and other relevant parameters. According to the findings, production inventory models must consider both the exponential demand rate and the inflation rate of deteriorating goods. The proposed model offers a pragmatic methodology for upgrading the creation and stock choices, at last improving the productivity and functional effectiveness of organizations managing crumbling things within the sight of expansion. This paper has followed an analytical approach to diminish the entire inventory cost. Finally, a sensitivity analysis was performed to study the effect of changes of different parameters of the model on the optimal policy. Moreover, in order to approve the determined models, we have clarified mathematical models and examined affectability.

Keywords: Production; inventory; fractionally time dependent demand; deterioration; partially backlogging.

2010 Mathematics Subject Classification: 90B05, 90B31.

1 Introduction

The introductory section consists of two parts. The first part consists of the motivation of research work, whereas the second part is a reported literature review.

1.1 Motivation (general problem description)

Inventory is the most fascinating and profoundly investigated subject of production and activity of executives. Inventory, being a fundamental piece of our lives since the start of development, is found all around as family inventory, social inventory and business inventory. Inventory bears the cost of adaptability**.** Several industries, including business, manufacturing, retail, infrastructure, and distribution, are greatly impacted by the nature of inventory. Demand is a key factor in real-world situations, particularly in retail settings, when creating an efficient inventory strategy. Scholars have distinguished between many forms of item demand, such as quadratic, time-dependent, stock-dependent, exponential, linear, increasing, or decreasing, or constant. Over time, nevertheless, it has become clear that these demand categories would not be sufficient to ensure the seamless running of organisations. The original push to include scientific methods into inventory investigations seems to have coincided with the expansion of engineering specialties, especially industrial engineering, and manufacturing businesses. Concerns about inventory first surfaced in the industrial sector, since products were manufactured in relatively large numbers. Aggarwal [1] developed an inventory model for deteriorating products called economic order quantity (E.O.Q.), in which the seller permits the customer to defer payment. Amutha and Chandrasekaran [2] introduced a model for inventory management that addresses degrading products characterized by quadratic demand and deteriorating items without shortages. Singh [3] investigated production inventory challenges involving depleting commodities characterized by selling price and time-dependent increasing pattern in demand. Nobil et al. [4] proposed a generalised a multi-product single machine EPQ inventory model with imperfect production is extended under production capacity. This model considers the temporal worth of money, allows for shortages and entirely backorders, and has a time-proportional demand rate. Goyal [5] developed mathematical techniques for figuring out the economic order quantity for an item for which the supplier permits a certain delay in collecting the payments generated by him. During this endeavor, Fallahi et al. [6] have formulated transportation policies are primary concerns of managers in inventory and production planning issues tailored for deteriorating goods. This model incorporates the systemic influence of inflation and encompasses demand dynamics contingent on both price and time. Singh et al. [7] explained an inventory model for fading gadgets with an ordinary deterioration rate. This paper develops a deterministic inventory model for spoiling items in which shortages are not allowed for declining things. Kumar [8] ascertain the optimal strategies for various inventory models distinguished by diverse characteristics, with a particular focus on minimizing total costs.

The discussion includes subjects like shortages, the destruction rate, and the consumption rate depending on starting stock. There are deterioration indications everywhere in the never-ending stream of life, and they vary according on the subject under examination. The most significant inventory models consider a steady rate of degradation that persists over an extended length of time. Several models study this phenomenon. Scholars have been examining how inventory systems are affected by deterioration, and their research will be useful for future investigations in this field. Sharma et al. [9] developed investigate the continuous-production inventory problem for a single product at a three-level facing a constant deterioration. It had the following characteristics: a demand rate that is a linear function of the selling price; unit cost impacting quantity discount schemes. Teng et al. [10], who include the cost of missed sales in addition to the non-constant purchase cost. To help with decision-making, Zhou and Yang's work [11] develops a deterministic inventory model with an inventory-dependent demand rate and two separate warehouses: an owned warehouse (OW) and a rental warehouse (RW).

An important factor in an inventory management system is the type of demand function. An inventory management system's operating dynamics change significantly depending on the demand function that is used. Thorough understanding and accurate modelling of the selected demand function are necessary for holding cost optimisation, guaranteeing sufficient stock availability, and refining all-encompassing inventory strategies. As a result, the type of demand function that is used has a significant influence on the effectiveness and flexibility of an inventory management system, especially when it comes to adjusting to the nuances of consumer wants and individual product characteristics. Singh [12] presented a deterministic inventory model intended for goods that degrade gradually and for which demand is quadratic. In this area, Skouri [13] uses two replenishment algorithms to build an inventory model with a partial backlog of unfilled demand, a time-dependent degradation rate based on the Weibull distribution, and a general ramp-type demand rate. There are no shortages in the first approach, but there are shortages in the first method. An inventory model that took inflation and a demand rate based on stock levels for a unique item into account was presented by Singh and Mukherjee [14]. Developing an ideal plan with a preset time horizon to reduce the expenses related to inventory and production rates was the purpose of the method and ealistic features of reusable items inventory systems, such as the presence of multiple products and operational constraints stated by Fallahi et al. [15]. A study by Chung & Huang [16] shows that the total yearly variable cost function has a variety of convexities. Furthermore, a model has been developed to outline the best ordering approach in the context of allowable shortages and reasonable payment lateness.

2 Literature Review

The essential exploration regions applicable to this paper are underlined here. Depreciation of currency immediately increases the price of raw materials and commodities, resulting in higher overall inventory costs. Ghoreishi et al. [17] provide an economic ordering policy model for non-instantaneously deteriorating commodities that considers variables like allowable payment delays, and customer returns. The model also incorporates demand that is impacted by selling prices and inflation. Dual-tier production inventory models with exponential demand and a deterioration rate dependent on time were proposed in a study by Kumar et al. [18]. The economic order quantity model, which treats the scheduling time as a variable, was examined by Rajan and Uthayakumar [19]. Within the framework of a permitted payment delay, the model assumes that the holding cost is described as an exponentially rising function and that the demand rate is a continuous function of time. The findings are summarized in the below Table 1.

This table summarizes the comparative analysis of various researchers who employed different model types to investigate demand patterns, production levels, inflation, and related factors. The research outcomes indicate diverse insights, with each model type offering unique advantages in understanding the complexities of these economic phenomena.

Now we study an important Production Inventory Model with taking different type of parameters of Table 1, such as cubical time dependent demand function, deteriorating items under inflationary environment. backlogging also include.

In this paper we use following 7 Sections: -

Section 2- Notations and Assumptions of our Model displayed in this Section.
Section 3- Mathematical Development and Solution of our Model are in this Section.
Section 4- Numerical Example are given in this section of our Model.
Section 5- Sensitivity of our problem shown by a table, which was made by changes values of parameters used in this Model.
Section 6- Graphs of observations of model given in this Section.
Section 7- Graphical Conclusion and Practical Implication is given in this section.
Section 8- Conlusion of our Model is cleared in this Section.

Table 1. Summarized Analysis of Some Authers Research Work

Writers	Type of Model	Demand Design	Level of Production	Degradation Rate
Amutha & Chandrasekaran [2]	EPQ	Constant	Yes	No
Singh [3]	EOQ	Time dependent	No	Constant
Hossen [8]	EOQ	Price and Time dependent	No	Constant
Kumar and Inaniyan [7]	EOQ	Quadratic	No	Pareto Type
Kumar et al. [8]	EOQ	Cubical Polynomial	No	Pareto Type
Meena et al. [20]	EOQ	Selling Price Dependent	No	Weibull
Sharma et al. [21]	EOQ	Ramp Type Function	No	Weibull
Sharma & Singh [22]	EPQ	Linear time function	Two	Constant
Singh et al. [23]	EPQ	Quadratic Time Dependent	Three	Cost
Singh & Sharma [24]	EOQ	Price Dependent	No	Constant
Taleizadeh et al. [25]	EOQ	Quadratic function of Time	No	Three-parameter Weibull distribution
Sunita & Kumar [26]	EOQ	Exponentially Time Varying	No	Pareto Type

3 Assumption and Notations

In this section, we define the preliminary aspects necessary to develop the proposed model and formulate the related optimization problem. In particular, this section introduces the following assumptions and symbolizations that are used.

3.1 Assumptions

- Demand Rate is fractionally time dependent function.
- The Deterioration Rate is Weibull Distribution function of time.
- Partially Backlogging is permitted.
- Lead Time is assumed to be zero.
- Holding Cost is constant.
- Backorder Cost is constant.
- Deterioration Cost is constant.
- Cost of Lost Sale is constant.
- Ordering Cost is constant.
- Purchase Cost is constant.
- The replenishment rate is assumed to be limitless and immediate.

3.2 Notations (symbolizations)

- C_S: Ordering (Set up) Cost per unit.
- C_B: Backorder Cost per unit per unit time.
- C_P: Purchase Cost per unit.
- C_L: Cost of Lost Sale per unit.
- C_D: Deterioration Cost per unit.
- C_H: Unit Holding Cost per unit time.
- θ(t): Deterioration Rate. Where $\theta(t) = \alpha\beta t^{\beta-1}$ $\alpha \geq 0$, $\beta \geq 0$
- T: Cycle Length.
- $B_G(t)$: Backlogging Rate, $B_G(t) = e^{-\delta(T-t)}$, δ is Backlogging Parameter and δ > 0 where $M \leq t \leq T$
- D(t) - The Demand Rate, $D(t) = a + bt^{\frac{m}{n}}$ where $\alpha \geq 0$, $b \geq 0$
- L- Time at which inventory level fulfil.
- M- Time at which inventory level reaches to zero.

- T- Time at which shortages in inventory level reaches too highest.
- I_1 - Inventory Level at production time during interval [0, L].
- I_2 - Inventory Level during time interval [L, M].
- I_3 - Inventory Level during shortages in time interval [M, T].
- Q* - Order Quantity during the cycle length T.
- I(P) - Maximum Inventory Level during [0, M].
- I(N) - Maximum Inventory Level during Shortage period [M, T].
- K- Rate of Production per unit time.
- TAC - Total Cost during complete cycle time.

4 Mathematical Formulations of the Proposed Model

As shown in Fig. 1, The manufacturing process begins at t=0 and continues until the stock amount reaches its peak at t=L. During this time interval, the inventory level is denoted by I_1. Due to degradation and client demand, production halts at time t=L, and the deterioration of products starts. The inventory level gradually decreases and reaches zero at time t=M. After time M, the effect of backlogging comes into play, and a complete cycle of inventory is completed by time t=T. The Inventory level during interval [L M] is denoted by I_2 and the inventory level during time interval [M T] is denoted by I_3.

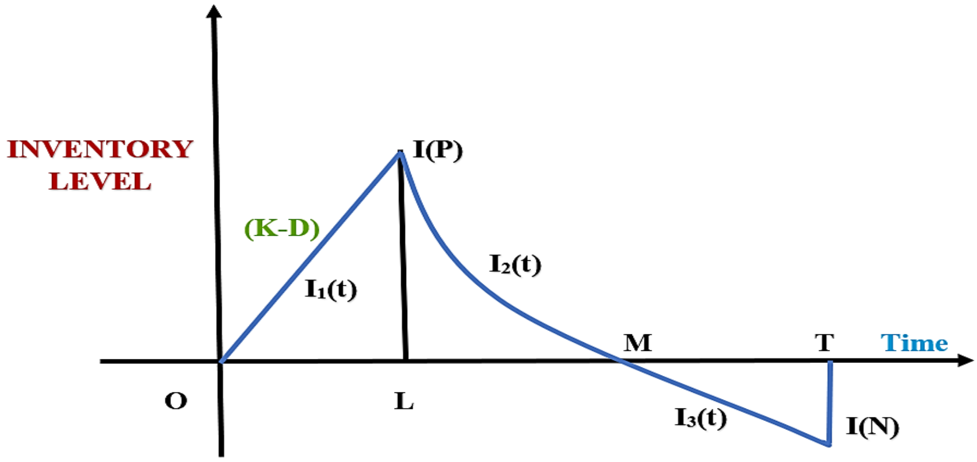

Fig. 1. Production Inventory Model with Shortage Allowed

4.1 Mathematical formulation of the model 1

The differential equation during the time interval [0, L] is presented as follows:

$$\frac{d}{dt}\left(I_1(t)\right) = K - D\left(t\right) = K - \left(a + bt^{\frac{m}{n}}\right) \text{ where } 0 \le t \le L \tag{1}$$

The differential equation during the time interval [L, M] is presented as follows: -

$$\frac{d}{dt}\left(I_2(t)\right) + \alpha\beta t^{\beta-1} I_2(t) = -D\left(t\right) = -\left(a + bt^{\frac{m}{n}}\right) \text{ where } L \le t \le M \tag{2}$$

With Boundary Conditions the interval [0, L]

$$I_1(t) = 0 \text{ at t=0 and } I_1(t) = I(P) \text{ at t=L} \tag{3}$$

And Boundary Conditions in the interval [L, M]

$$I_2(t) = I(P) \text{ at t=L and } I_2(t) = 0 \text{ at t=M} \tag{4}$$

Solution of equation (1) with help of boundary conditions given in (3), we get $I_1(t) = (K-a)t - \dfrac{bt^{\frac{m}{n}+1}}{\left(\frac{m}{n}+1\right)}$ (5)

And $I(\text{P}) = (K-a)L - b\dfrac{L^{\frac{m}{n}+1}}{\left(\frac{m}{n}+1\right)}$ (6)

Result number (5) and (6) are the solutions of equation number (1)

Solution of equation (2) with help of boundary conditions given in (4), we get

$$I_2(\text{t}) = e^{-\alpha t^\beta}\left[\begin{array}{l} a(M-t) + \dfrac{b}{\left(\frac{m}{n}+1\right)}\left(M^{\frac{m}{n}+1} - t^{\frac{m}{n}+1}\right) + \dfrac{a\alpha}{\beta+1}\left(M^{\beta+1} - t^{\beta+1}\right) + \dfrac{\alpha b}{\frac{m}{n}+\beta+1}\left\{M^{\frac{m}{n}+\beta+1}\right. \\ \left. -t^{\frac{m}{n}+\beta+1}\right\} + \dfrac{a\alpha^2}{2(2\beta+1)}\left(M^{2\beta+1} - t^{2\beta+1}\right) + \dfrac{b\alpha^2}{2\left(2\beta+\frac{m}{n}+1\right)}\left(M^{2\beta+\frac{m}{n}+1} - t^{2\beta+\frac{m}{n}+1}\right) \end{array}\right] \tag{7}$$

And

$$I(\text{P}) = e^{-\alpha L^\beta}\left[\begin{array}{l} a(M-L) + \dfrac{b}{\left(\frac{m}{n}+1\right)}\left(M^{\frac{m}{n}+1} - L^{\frac{m}{n}+1}\right) + \dfrac{a\alpha}{\beta+1}\left(M^{\beta+1} - L^{\beta+1}\right) + \dfrac{\alpha b}{\frac{m}{n}+\beta+1}\left\{M^{\frac{m}{n}+\beta+1}\right. \\ \left. -L^{\frac{m}{n}+\beta+1}\right\} + \dfrac{a\alpha^2}{2(2\beta+1)}\left(M^{2\beta+1} - L^{2\beta+1}\right) + \dfrac{b\alpha^2}{2\left(2\beta+\frac{m}{n}+1\right)}\left(M^{2\beta+\frac{m}{n}+1} - L^{2\beta+\frac{m}{n}+1}\right) \end{array}\right] \tag{8}$$

Result number (7) and (8) are the solutions of equation number (2)

4.2 Mathematical formulation of the model 2 (shortages)

The differential equation during the time interval [M, T] is presented as follows: -

$$\frac{d}{dt}\left(I_3(t)\right) = -D(t).\text{B}_G(t) = -\left(a + bt^{\frac{m}{n}}\right).e^{-\delta(T-t)} \text{ where } M \le t \le T \tag{9}$$

With Boundary Conditions in this interval [M,T]

$$I_3(t) = 0 \text{ at t = M and } I_3(t) = I(N) \quad \text{at t = T} \tag{10}$$

Solution of equation (9) with help of boundary conditions given in (10), we get

$$I_3(t) = e^{-\delta T}\left[\begin{array}{l} a(M-t) + \frac{a\delta}{2}\left(M^2 - t^2\right) + \frac{a\delta^2}{6}\left(M^3 - t^3\right) + \frac{b}{\frac{m}{n}+1}\left(M^{\frac{m}{n}+1} - t^{\frac{m}{n}+1}\right) \\ + \frac{b\delta}{\frac{m}{n}+2}\left(M^{\frac{m}{n}+2} - t^{\frac{m}{n}+2}\right) + \frac{b\delta^2}{2\left(\frac{m}{n}+3\right)}\left(M^{\frac{m}{n}+3} - t^{\frac{m}{n}+3}\right) \end{array}\right] \tag{11}$$

Using boundary condition $-I_3(T) = I(N)$, we get the negative Inventory

$$I(\mathrm{N}) = e^{-\delta T}\left[\begin{array}{l} a(M-T) + \frac{a\delta}{2}\left(M^2 - T^2\right) + \frac{a\delta^2}{6}\left(M^3 - T^3\right) + \frac{b}{\frac{m}{n}+1}\left(M^{\frac{m}{n}+1} - T^{\frac{m}{n}+1}\right) \\ + \frac{b\delta}{\frac{m}{n}+2}\left(M^{\frac{m}{n}+2} - T^{\frac{m}{n}+2}\right) + \frac{b\delta^2}{2\left(\frac{m}{n}+3\right)}\left(M^{\frac{m}{n}+3} - T^{\frac{m}{n}+3}\right) \end{array}\right] \tag{12}$$

Total inventory, $Q^* = I(P) + I(N)$

$$Q^* = \left[\begin{array}{l} e^{-\alpha L^{\beta}}\left\{ a(M-L) + \frac{b}{\left(\frac{m}{n}+1\right)}\left(M^{\frac{m}{n}+1} - L^{\frac{m}{n}+1}\right) + \frac{a\alpha}{\beta+1}\left(M^{\beta+1} - L^{\beta+1}\right) + \frac{\alpha b}{\frac{m}{n}+\beta+1}\left(M^{\frac{m}{n}+\beta+1} \right.\right. \\ \left.\left. - L^{\frac{m}{n}+\beta+1}\right) + \frac{a\alpha^2}{2(2\beta+1)}\left(M^{2\beta+1} - L^{2\beta+1}\right) + \frac{b\alpha^2}{2\left(2\beta+\frac{m}{n}+1\right)}\left(M^{2\beta+\frac{m}{n}+1} - L^{2\beta+\frac{m}{n}+1}\right)\right\} \\ + e^{-\delta T}\left\{ a(M-T) + \frac{a\delta}{2}\left(M^2 - T^2\right) + \frac{b}{\frac{m}{n}+1}\left(M^{\frac{m}{n}+1} - T^{\frac{m}{n}+1}\right) + \frac{b\delta}{\frac{m}{n}+2}\left(M^{\frac{m}{n}+2} - T^{\frac{m}{n}+2}\right) \right. \\ \left. + \frac{a\delta^2}{6}\left(M^3 - T^3\right) + \frac{b\delta^2}{2\left(\frac{m}{n}+3\right)}\left(M^{\frac{m}{n}+3} - T^{\frac{m}{n}+3}\right)\right\} \end{array}\right] \tag{13}$$

4.3 Cost calculation of proposed model

To calculate cost of the proposed model we take different types of costs, which are follows:

1. **Ordering Cost (O.C.)** $= C_S$
2. **The Back Order Cost (B.C.)** $= -C_B\left[\int_M^T I_3(t)\,dt\right]$

$$= -C_B e^{-\delta T}\left[\begin{array}{l} a\left(MT-\frac{T^2}{2}\right)+\frac{a\delta}{2}\left(M^2T-\frac{T^3}{3}\right)+\frac{a\delta^2}{6}\left(M^3T-\frac{T^4}{4}\right)+\frac{b}{\frac{m}{n}+1}\left(M^{\frac{m}{n}+1}T\right. \\ \left.-\frac{T^{\frac{m}{n}+2}}{\frac{m}{n}+2}\right)+\frac{b\delta}{\frac{m}{n}+2}\left(M^{\frac{m}{n}+2}T-\frac{T^{\frac{m}{n}+3}}{\frac{m}{n}+3}\right)+\frac{b\delta^2}{2\left(\frac{m}{n}+3\right)}\left(M^{\frac{m}{n}+3}T-\frac{T^{\frac{m}{n}+4}}{\frac{m}{n}+4}\right) \\ -\frac{aM^2}{2}-\frac{a\delta M^3}{3}-\frac{a\delta^2M^4}{8}-\frac{b}{\frac{m}{n}+2}M^{\frac{m}{n}+2}-\frac{b\delta}{\frac{m}{n}+3}M^{\frac{m}{n}+3}-\frac{b\delta^2}{2\left(\frac{m}{n}+4\right)}M^{\left(\frac{m}{n}+4\right)} \end{array}\right] \tag{14}$$

3. The Holding Cost (H.C.) $= \int_0^L \mathrm{C}_H .\{\mathrm{I}_1(t)\}\,dt + \int_L^M \mathrm{C}_H .\{\mathrm{I}_2(t)\}\,dt$

$$C_H\left[\begin{array}{l} \left\{(K-a)\frac{L^2}{2}-\frac{bL^{\frac{m}{n}+2}}{\left(\frac{m}{n}+1\right)\left(\frac{m}{n}+2\right)}\right\}+\left\{\frac{aM^2}{2}+\frac{bM^{\left(\frac{m}{n}+2\right)}}{\left(\frac{m}{n}+2\right)}+\frac{a\alpha}{(\beta+2)}M^{\beta+2}+\frac{\alpha b}{\left(\frac{m}{n}+\beta+2\right)}\right. \\ .M^{\left(\frac{m}{n}+\beta+2\right)}+\frac{a\alpha^2}{2(2\beta+2)}M^{(2\beta+2)}+\frac{b\alpha^2}{2\left(2\beta+\frac{m}{n}+2\right)}M^{\left(2\beta+\frac{m}{n}+2\right)}-\frac{a\alpha}{(\beta+1)(\beta+2)}M^{(\beta+2)} \\ -\frac{\alpha b}{\left(\frac{m}{n}+\beta+2\right)}.\frac{M^{\left(\frac{m}{n}+\beta+2\right)}}{(\beta+1)}-\frac{a\alpha^2}{(2\beta+2)}\frac{M^{(2\beta+2)}}{(\beta+1)}-\frac{\alpha b}{\left(\frac{m}{n}+2\beta+2\right)}\frac{M^{\left(\frac{m}{n}+2\beta+2\right)}}{(\beta+1)}-M^{(3\beta+2)} \\ \frac{a\alpha^3}{2(3\beta+2)(\beta+1)}-\frac{b\alpha^3}{2(\beta+1)\left(3\beta+\frac{m}{n}+2\right)}M^{\left(3\beta+\frac{m}{n}+2\right)}-a\left(ML-\frac{L^2}{2}\right)-\frac{b}{\left(\frac{m}{n}+1\right)} \\ \left(L.M^{\frac{m}{n}+1}-\frac{L^{\left(\frac{m}{n}+2\right)}}{\left(\frac{m}{n}+2\right)}\right)-\frac{a\alpha}{(\beta+1)}\left(L.M^{\beta+1}-\frac{L^{(\beta+2)}}{(\beta+2)}\right)-\frac{\alpha b}{\left(\frac{m}{n}+\beta+1\right)}\left(L.M^{\left(\frac{m}{n}+\beta+1\right)}\right. \\ \left.-\frac{L^{\left(\frac{m}{n}+\beta+2\right)}}{\left(\frac{m}{n}+\beta+2\right)}\right)-\frac{a\alpha^2}{2(2\beta+1)}\left(L.M^{(2\beta+1)}-\frac{L^{(2\beta+2)}}{(2\beta+2)}\right)-\frac{b\alpha^2}{2\left(\frac{m}{n}+2\beta+1\right)}\left(L.M^{\left(\frac{m}{n}+2\beta+1\right)}\right. \\ \left.-\frac{L^{\left(\frac{m}{n}+2\beta+2\right)}}{\left(\frac{m}{n}+2\beta+2\right)}\right)+a\alpha\left(\begin{array}{l} M.\frac{L^{(\beta+1)}}{(\beta+1)} \\ -\frac{L^{(\beta+2)}}{(\beta+2)} \end{array}\right)+\frac{\alpha b}{\left(\frac{m}{n}+1\right)}\left(M^{\left(\frac{m}{n}+1\right)}.\frac{L^{\beta+1}}{(\beta+1)}-\frac{L^{\left(\frac{m}{n}+\beta+2\right)}}{\left(\frac{m}{n}+\beta+2\right)}\right) \\ +\frac{a\alpha^2}{(\beta+1)}\left(\frac{L^{(\beta+1)}.M^{(\beta+1)}}{\beta+1}-\frac{L^{(2\beta+2)}}{(2\beta+2)}\right)+\left(M^{\left(\frac{m}{n}+\beta+1\right)}.\frac{L^{\beta+1}}{(\beta+1)}-\frac{L^{\left(\frac{m}{n}+2\beta+2\right)}}{\left(\frac{m}{n}+2\beta+2\right)}\right) \\ \frac{\alpha b}{\left(\frac{m}{n}+\beta+1\right)}+\frac{a\alpha^3}{2(2\beta+1)}\left(\frac{L^{(\beta+1)}.M^{(2\beta+1)}}{\beta+1}-\frac{L^{(3\beta+2)}}{(3\beta+2)}\right)+\frac{b\alpha^3}{2\left(\frac{m}{n}+2\beta+1\right)} \\ \left.\left(\frac{L^{\beta+1}.M^{\left(\frac{m}{n}+2\beta+1\right)}}{\beta+1}-\frac{L^{\left(\frac{m}{n}+3\beta+2\right)}}{\left(\frac{m}{n}+3\beta+2\right)}\right)\right\} \end{array}\right] \tag{15}$$

4. The deterioration cost for the inventory during [0, M]

(D.C.) $= C_D\left[Q^* - \int_0^L Ddt - \int_L^M Ddt\right]$

$$= C_D\left[\begin{array}{l} -\dfrac{bL^{\frac{m}{m}+1}}{\left(\frac{m}{n}+1\right)} + (K-a)L + e^{-\alpha L^{\beta}}\left\{a(M-L) + \dfrac{b}{\left(\frac{m}{n}+1\right)}\left(M^{\left(\frac{m}{n}+1\right)} - L^{\left(\frac{m}{n}+1\right)}\right) + \dfrac{a\alpha}{\beta+1}\right. \\ \left(M^{\beta+1} - L^{\beta+1}\right) + \dfrac{\alpha b}{\left(\frac{m}{n}+\beta+1\right)}\left(M^{\frac{m}{n}+\beta+1} - L^{\frac{m}{n}+\beta+1}\right) + \dfrac{a\alpha^2}{2(2\beta+1)}\left(M^{2\beta+1} - L^{2\beta+1}\right) \\ \left. + \dfrac{b\alpha^2}{2\left(2\beta+\frac{m}{n}+1\right)}\left(M^{2\beta+\frac{m}{n}+1} - L^{2\beta+\frac{m}{n}+1}\right)\right\} - e^{-\delta t}\left\{a(M-T) + \dfrac{a\delta}{2}\left(M^2 - T^2\right) + \right. \\ \dfrac{a\delta^2}{6}\left(M^3 - T^3\right) + \dfrac{b}{\left(\frac{m}{n}+1\right)}\left(M^{\frac{m}{n}+1} - T^{\frac{m}{n}+1}\right) + \dfrac{b\delta}{\left(\frac{m}{n}+2\right)}\left(M^{\frac{m}{n}+2} - T^{\frac{m}{n}+2}\right) + \dfrac{b\delta^2}{2\left(\frac{m}{n}+3\right)} \\ \left.\left(M^{\frac{m}{n}+3} - T^{\frac{m}{n}+3}\right)\right\} - \left(aM + \dfrac{bM^{\frac{m}{n}+1}}{\left(\frac{m}{n}+1\right)}\right) \end{array}\right] \tag{16}$$

5. **Purchase Cost (P.C.) = C_P. Q***

$$= C_P\left[\begin{array}{l} e^{-\alpha L^{\beta}}\left\{a(M-L) + \dfrac{b}{\left(\frac{m}{n}+1\right)}\left(M^{\left(\frac{m}{n}+1\right)} - L^{\left(\frac{m}{n}+1\right)}\right) + \dfrac{a\alpha}{\beta+1}\left(M^{\beta+1} - L^{\beta+1}\right) + \dfrac{\alpha b}{\left(\frac{m}{n}+\beta+1\right)}\right. \\ \left.\left(M^{\frac{m}{n}+\beta+1} - L^{\frac{m}{n}+\beta+1}\right) + \dfrac{a\alpha^2}{2(2\beta+1)}\left(M^{2\beta+1} - L^{2\beta+1}\right) + \dfrac{b\alpha^2}{2\left(2\beta+\frac{m}{n}+1\right)}\left(M^{2\beta+\frac{m}{n}+1} - L^{2\beta+\frac{m}{n}+1}\right)\right\} \\ -e^{-\delta T}\left\{a(M-T) + \dfrac{a\delta}{2}\left(M^2 - T^2\right) + \dfrac{a\delta^2}{6}\left(M^3 - T^3\right) + \dfrac{b}{\left(\frac{m}{n}+1\right)}\left(M^{\frac{m}{n}+1} - T^{\frac{m}{n}+1}\right) + \dfrac{b\delta}{\left(\frac{m}{n}+2\right)}\right. \\ \left.\left(M^{\frac{m}{n}+2} - T^{\frac{m}{n}+2}\right) + \dfrac{b\delta^2}{2\left(\frac{m}{n}+3\right)}\left(M^{\frac{m}{n}+3} - T^{\frac{m}{n}+3}\right)\right\} \end{array}\right] \tag{17}$$

6. **Lost Sale Cost (L.S.C.)** $= -C_L\left[\int_M^T \left(1 - e^{-\delta(T-t)}\right).Ddt\right]$

$$= C_L\left[\frac{\delta T^2}{2} - \frac{\delta^2 T^3}{6} - \delta\left(TM - \frac{M^2}{2}\right) + \frac{\delta^2 T^2 M}{2} + \frac{\delta^2 M^3}{6} - \frac{\delta^2 TM^2}{2}\right] \quad (18)$$

7. Total Average Inventory Cost (T.A.C.) = $C_T(T_3)$ = $\frac{1}{T_3}$ (Total Cost)

$$= \frac{1}{T_3} \text{ [O.C.+ H.C. +D.C.+ P.C.+B.C.+L.S.C.] (19)}$$

5 Numerical Example

5.1 Numerical example number 1

If we take a numerical example and utilise some of the values of the parameters that our inventory model uses as: - C_S= 150 units/year, C_B= 100, C_P= 120, C_L=75, C_D =120, C_H=130, α=0.5, β=1.5, δ=2.5, a=2, b=3, m=3.5, n=2.75, L =1.5, M =3.65, K=150; after that we place these values in equation number (13) and equation number (19), and We use the mathematical software MATLAB (R2021a) to solve this issue, and the results show the following optimum values shown in Table 2.

Table 2. Optimal Value of Given Parameter

T*	Q*	TAC*
4.17	**277.88**	**21584**

From above example we tried to show the mathematical approach of the given system of inventory model.

5.2 Numerical example number 2

If we take a numerical example and utilise some of the values of the parameters that our inventory model uses as: C_S= 200 units/year, C_B= 120, C_P= 130, C_L=95, C_D =105, C_H=115, α=0.4, β=1.3, δ=2.1, a=2.2, b=3.1, m=2.5, n=1.2, L =2, M =4.5, K=210; after that we place these values in equation number (13) and equation number (19), and We use the mathematical software MATLAB (R2021a) to solve this issue, and the results show the following optimum values shown in Table 3.

Table 3. Optimal value of Given Parameter

T*	Q*	TAC*
5.27	**122.33**	**11048.5**

From above example we tried to show the mathematical approach of the given system of inventory model.

6 Sensitivity Analysis

For sensitivity analysis of this Model, we change values of parameters one by one and announce the effects on T*, Q* and TAC*. Rate of changes (in percentage) in values of parameters are taken -20 %, -10%, +10% and +20%. Following table show result of above changes.

Variation of T*, Q* and TAC* w.r.t. C_S, C_B, C_P, C_L, C_D, C_H, α, β, δ, a, b, m, n, L, M and K.

C_S= 150 units/year, C_B= 100, C_P= 120, C_L=75, C_D =120, C_H=130, α=0.5, β=1.5, δ=2.5, a=2, b=3, m=3.5, n=2.75, L =1.5, M =3.65, K=150

Table 4. Effect of Changes of Various Parameters

Parameter	Change in Parameter	T*	Q*	TAC*
C_S =150	-20%	4.21	277.88	21580
	-10%	4.19	277.88	21582
	0%	4.17	277.88	21584
	10%	4.15	277.88	21586
	20%	4.13	277.88	21588
C_B =100	-20%	4.20	277.88	21584
	-10%	4.18	277.88	21584
	0%	4.17	277.88	21584
	10%	4.19	277.88	21584
	20%	4.20	277.88	21584
C_P = 120	-20%	4.13	280.92	18551
	-10%	4.15	279.90	20070
	0%	4.17	277.88	21584
	10%	4.19	276.86	23093
	20%	4.21	275.84	24598
C_L =75	-20%	4.21	277.88	21584
	-10%	4.19	277.88	21584
	0%	4.17	277.88	21584
	10%	4.16	277.88	21584
	20%	4.14	277.88	21584
C_D=120	-20%	4.14	279.00	20076
	-10%	4.16	278.44	20830
	0%	4.17	277.88	21584
	10%	4.18	277.34	22336
	20%	4.19	276.82	23087
C_H = 130	-20%	4.24	274.14	21376
	-10%	4.20	276.14	21486
	0%	4.17	277.88	21584
	10%	4.14	279.43	21671
	20%	4.11	280.82	21571
α=0.5	-20%	4.24	281.60	21861
	-10%	4.20	279.73	21721
	0%	4.17	277.88	21584
	10%	4.13	276.06	21450
	20%	4.10	274.27	21320
β=1.5	-20%	4.23	306.91	23541
	-10%	4.20	291.41	22492
	0%	4.17	277.88	21584
	10%	4.14	265.96	20789
	20%	4.11	255.36	20086
δ= 2.5	-20%	4.09	234.42	18786
	-10%	4.13	256.26	20190
	0%	4.17	277.88	21584
	10%	4.20	299.31	22967
	20%	4.23	320.56	24342
a=2	-20%	4.24	276.37	21530
	-10%	4.20	277.25	21563
	0%	4.17	277.88	21584
	10%	4.14	278.32	21594
	20%	4.11	278.59	21597
b=3	-20%	5.01	228.25	19509
	-10%	4.53	260.66	20830
	0%	4.17	277.88	21584

Parameter	Change in Parameter	T*	Q*	TAC*
	10%	3.88	288.22	22038
	20%	3.65	294.28	22289
m=3.5	-20%	3.77	295.62	22437
	-10%	3.97	287.13	22035
	0%	4.17	277.88	21584
	10%	4.36	267.67	21079
	20%	4.56	256.19	20513
n= 2.75	-20%	3.72	290.62	22442
	-10%	3.92	282.13	22040
	0%	4.12	272.88	21589
	10%	4.31	262.67	21084
	20%	4.51	250.19	20518
L = 1.5	-20%	3.81	273.97	21046
	-10%	3.99	277.02	21390
	0%	4.17	277.88	21584
	10%	4.35	276.15	21614
	20%	4.54	271.22	21457
M= 3.65	-20%	4.46	177.01	21755
	-10%	4.27	242.78	22259
	0%	4.17	277.88	21584
	10%	4.10	302.69	20624
	20%	4.04	322.06	19610
K= 150	-20%	4.17	246.41	19678
	-10%	4.17	262.14	20631
	0%	4.17	277.88	21584
	10%	4.17	293.62	22536
	20%	4.17	209.36	23489

7 Graphs of Observations and Results

The graphical illustration is a method of breaking down mathematical information. It shows the connection between data, thoughts, and ideas in a chart. It is straightforward, and it is quite possibly the principal learning methodologies.

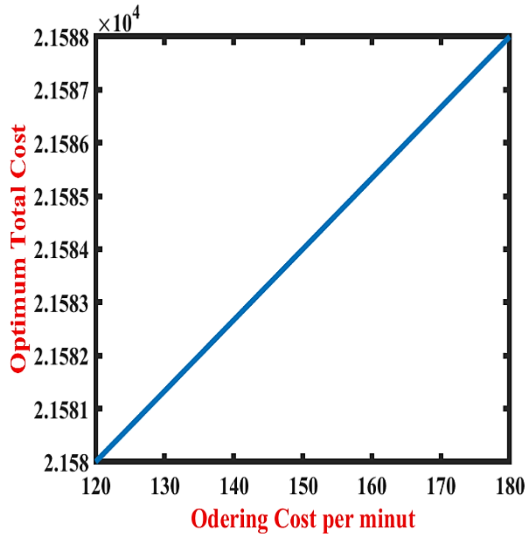

Fig. 2. Changes of optimum total cost with odering cost

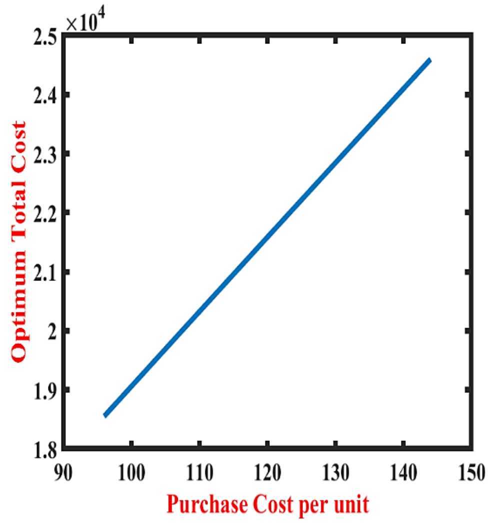

Fig. 3. Changes of optimum total cost with unit purchase cost

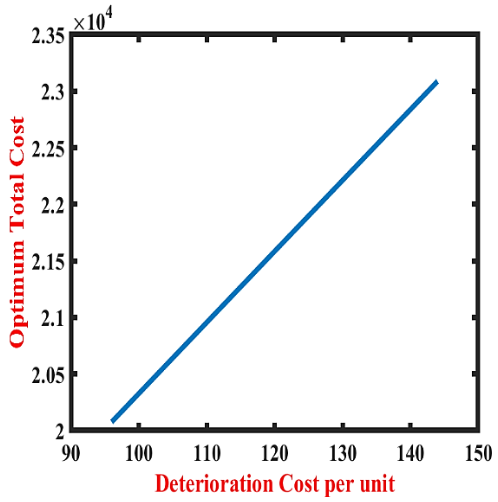

Fig. 4. Changes of optimum total cost with deteriorating cost

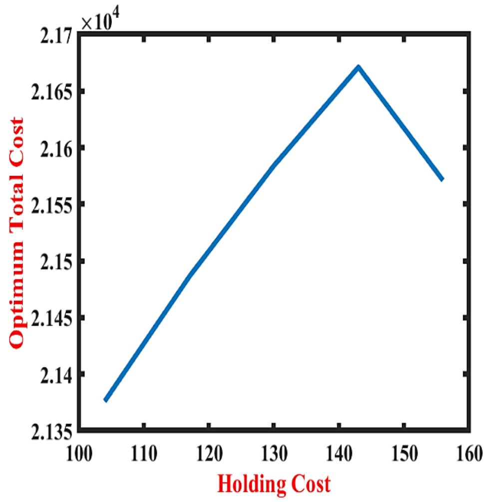

Fig. 5. Changes of optimum total cost with holding cost

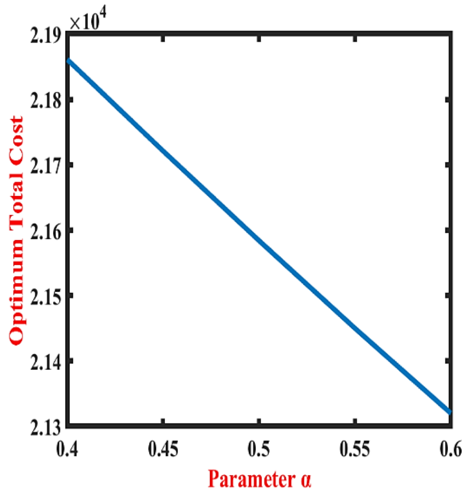

Fig. 6. Changes of optimum total cost with parameter α

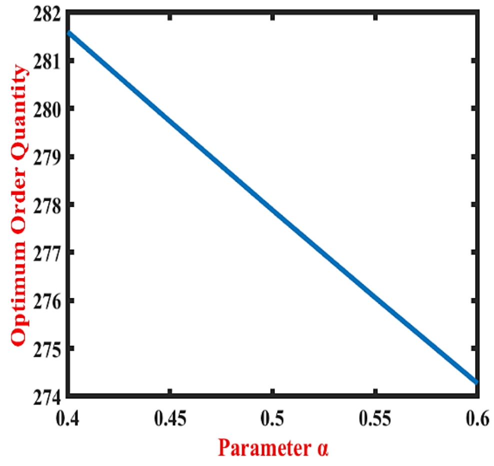

Fig. 7. Changes of optimum order quantity with parameter α

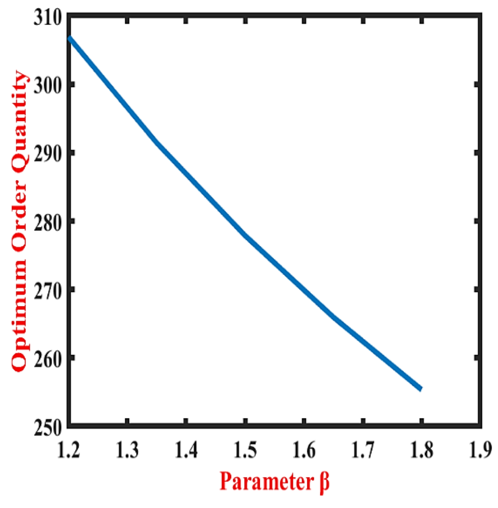

Fig. 8. changes of optimum order quantity with parameter β

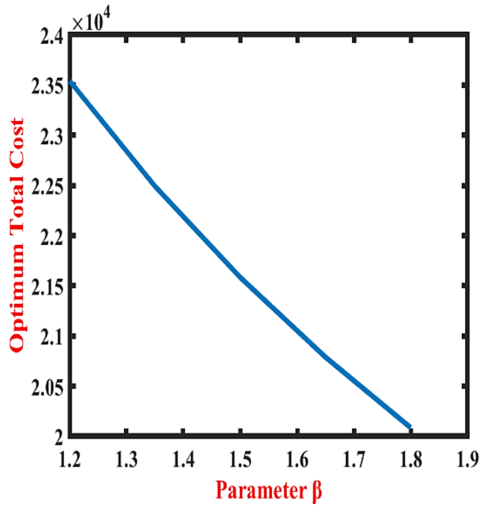

Fig. 9. Changes of optimum total cost with parameter β

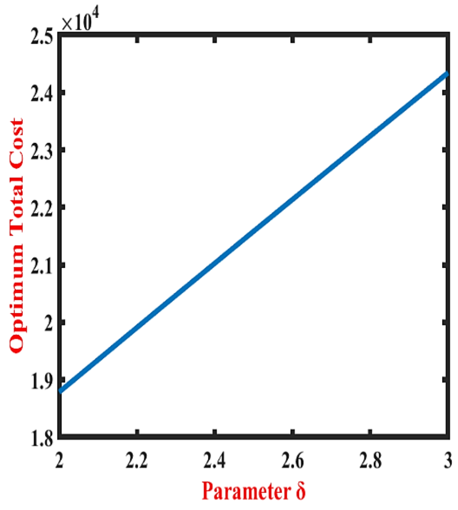

Fig. 10. Changes of optimum total cost with parameter δ

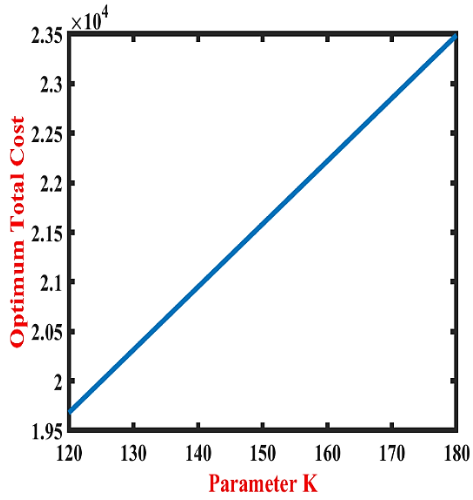

Fig. 11. Changes of optimum total cost with parameter K

8 Graphical Conclusion and Practical Implication of this Model

When we study of above graphs and table of our Production Inventory Model we find many different types results, here we discuss some important following conclusion:

(i) From Fig. 2, we observe that graph of Optimal Total Cost w.r.t. Ordering Cost per unit is increasing and graph is straight line.
(ii) From Fig. 3, we observe that graph of Optimum Total Cost is increasing with respect to Purchase Cost per unit and graph is straight line.
(iii) From Fig. 4, we find that Optimal Total Cost is also increasing with respect to Deterioration Cost per unit and graph is straight line.
(iv) It is seen that from Fig. 5, graph of Optimal Total Cost is increasing in starting and after some time it decreasing fastly with respect to Holding Cost.
(v) From Fig. 6 and Fig. 7, we find that Optimum Total Cost and Economic Order Quantity both are have same graph with respect to Parameter α. Graphs are increasing and straight line.
(vi) From Fig. 8 and Fig. 9, we find that Optimum Total Cost and Economic Order Quantity both are have same graph with respect to Parameter β. Both graphs are decreasing and like be a straight line.
(vii) From Fig. 10, we observe that Optimal Total Cost is increasing with respect to Parameter δ and graph is straight line.
(viii) When we see Fig. 11, we observe that Optimal Total Cost is increasing with respect to Parameter K and graph is straight line.

Practical Implication of Model: This model controls exact non-prompt weakening things like electronic parts, food things, and trendy items. To diminish production cost and ultimately boost benefit, it is of most extreme significance to direct buy choices through some thorough inventory strategy, so production lines don't run dry. Our model can lead them into dealing with the inventory framework in an advanced manner for this situation.

9 Conclusion

The planned model is exact for a dispatched item with a steady plan to a limited extent in time-subordinate interest. Because of this, we will get buyer fulfillment and procure more potential profit. Our development of the mathematical inventory paradigm involved the use of several crucial techniques, such as Fractionally Time Dependent Production, Deterioration, Demand, and Shortages. Using these approaches, our goal was to develop an inventory model with the objective of minimising the total average cost of inventory. After doing a thorough graphical study, we were able to identify a trend: the overall cost shows an increasing trajectory when the parameters α, δ, and K increase. On the other hand, there is an observed negative correlation between parameter β and the overall expense. Changes in the C_L (Lost Sale Cost Parameter) and C_B (Backorder Cost per unit per unit time) did not appear to have any effect on the optimal overall cost, suggesting its stability.

Therefore, businesses, professions, or organisations that face similar situations can apply this model to achieve favourable outcomes. Furthermore, people may improve the effectiveness and validity of this model in future settings by including additional viewpoints, circumstances, and specific norms or conventions.

For further research: This model can be expanded in numerous ways involving diverse demand rates such as constant, quadratic, Weibull deterioration with three parameters, cubic demand, time discounting, and rework of defective items. We can also generalize the model in several ways: the time value of money, quantity discounts, price discounts, and rework of faulty items.

Disclaimer (Artificial Intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of manuscripts.

Competing Interests

Authors have declared that no competing interests exist.

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