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Estimation of Seasonal Variances in Descriptive Time Series Analysis

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

This study presents the Buys-Ballot estimation procedure of row and seasonal variances in time series decomposition when trend component of time series is linear. Therefore, the main objective of this study is to obtain the Buys-Ballot estimates of row and seasonal variances for the mixed model. The method adopted in this study is Buys-Ballot procedure developed for choice of appropriate model for decomposition of any study series. The statistical software (MINITAB 17.0 version) is also adopted in this study. Results of the Buys-Ballot estimates for mixed model indicate that, the row variance is a function of trend parameters and seasonal component of the original series. It is for column variance, a constant multiple of the square of the seasonal component.

Keywords: Time series decomposition; trend-cycle component; mixed model; seasonal variance; Buys-Ballot estimate; choice of model.

1. INTRODUCTION

Time series decomposition method is primarily used for the assessment of trends in repeated measurements taken at equally spaced time interval and their relationships with other trends or events, taking into consideration of an account of the temporal structure of such time series

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data. An important area in descriptive time series analysis is the decomposition of a time series into a set of non-observable components that can be associated to different types of temporal variations Dagum [1]. Descriptive time series analysis involve the separation of an observed time series into components representing trend (long term direction), the seasonal (calendar related movements), cyclical (long term oscillations) and irregular (short term fluctuations) components. Description includes the examination of trend, seasonality, cycles, trend and scale and so on that may influence the series. This is also very vital preliminary to modelling, when it has to be decided whether and how to seasonally adjust, to transform, and to deal with outliers and whether to fit a model. In the examination of trend, seasonality and cycles, a time series is often described as having trends, seasonal indices, cyclic pattern and random component Iwueze and Nwogu [2].

For short period of time, the cyclical component is superimposed into the trend (Chatfield [3]) and the observed time series $\left(X_t, t = 1, 2, ..., n\right)$ can be decomposed into the trend-cycle component (M_t), seasonal component (S_t) and the residual/error component (e_t). Hence, the decomposition models are

Additive Model:

$$X_t = M_t + S_t + e_t \tag{1}$$

Multiplicative Model:

$$X_t = M_t \times S_t \times e_t \tag{2}$$

and Mixed Model

$$X_t = M_t \times S_t + e_t . \tag{3}$$

For additive model (1) the assumption is that, the residual/error term e_t is the Gaussian white noise $N\left(0, \sigma_1^2\right)$ and sum of the seasonal component over a complete period/year is zero

$$\left(\sum_{j=1}^{s} S_j = 0\right) \tag{4}$$

Also, for multiplicative model (2), the assumption is that the residual/error term e_t is the Gaussian white noise $N\left(0, \sigma_1^2\right)$ and the sum of seasonal component over a complete period/year

$$s\left(\sum_{j=1}^{s} S_j = S\right) \tag{5}$$

while for mixed model (3) e_t is the Gaussian white noise $N\left(0, \sigma_1^2\right)$ and sum of seasonal component over a complete period/year is

$$s\left(\sum_{j=1}^{s} S_j = S\right) \tag{6}$$

Oladugba et al. [4] presented briefdescription of time series decomposition models (additive and multiplicative seasonality). They pointed out that, the seasonal fluctuation displays constant amplitude with respect to the trend in additive seasonality while amplitude of the seasonal fluctuation depends on trend in multiplicative model.

1.1 Buys-Ballot Procedure for Time Series Decomposition

Iwueze and Nwogu [2] pointed out that, when a time series data contains seasonal affect with period s, we expect observations separated by multiples of s to be similar: X_t should be similar to $X_t \pm is$, $i = 1,2,3, \ldots\ m$. To analyze the time series data, it is important to arrange the series in a two – way tables (Table 1), according to the period and season, including the totals and averages. Such two – dimensional tables that exhibits within period pattern, thatare the same from period to period are referredas Buys – Ballot table for seasonal time series. The Buys - Ballot table (Table 1) helps in the assessment/estimation of the trend cycle component and seasonal indices of time series data. The row averages $(\bar{X}_{i.})$ estimate trend, and the differences $(\bar{X}_{.j} - \bar{X}_{..})$ or the ratio $\left(\frac{\bar{X}_{.j}}{\bar{X}_{..}}\right)$ between the column averages $(\bar{X}_{.j})$ and the overall average $(\bar{X}_{..})$ estimate the seasonal indices.

This study considers the estimation of seasonal/column variances in descriptive time series analysis. The emphasis is to obtain the Buys-Ballot estimates of row and seasonal/column variances, which take into consideration the mixed model structure and linear trend component.

2. MATERIALS AND METHODS

The method adopted in this study is Buys-Ballot procedure developed for choice of appropriate model for decomposition of any study data. The method is based on row, column and overall means arranged in the Buys-Ballot Table with m rows and s columns, m represent the length of periodic interval, s is the number of columns. For details of Buys-Ballot procedure see Wei [5], Iwueze and Nwogu [2,6] and [7], Iwueze and Ohakwe [8], Nwogu et al. [9], Dozie [10], Dozie et al. [11], Dozie and Ijomah [12].

Table 1. Buys - Ballot table for Seasonal time series

Rows/ Period (i)	Columns (season) j								
	1	**2**	⋯	j	...	s	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
1	X_1	X_2	⋯	X_j	⋯	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_1$
2	X_{s+1}	X_{s+2}	⋯	X_{s+j}	⋯	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_2$
3	X_{2s+1}	X_{2s+2}	⋯	X_{2s+j}	⋯	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_3$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	⋯	$X_{(i-1)s+j}$	⋯	X_{is}	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
m	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	⋯	$X_{(m-1)s+j}$	⋯	X_{ms}	$T_{m.}$	$\bar{X}_{m.}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	⋯	$T_{.j}$	⋯	$T_{.s}$	$T_{..}$		
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	⋯	$\bar{X}_{.j}$	⋯	$\bar{X}_{.s}$		$\bar{X}_{..}$	
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	⋯	$\hat{\sigma}_{.j}$	⋯	$\hat{\sigma}_{.s}$			$\hat{\sigma}_x$

In this arrangement each time period *t* is represented in terms of the year/period (i) and column/season (j) Therefore, the row, column and overall totals, averages and variances are defined as

$$T_{i.} = \sum_{j=1}^{s} X_{(i-1)s+j}, \qquad \bar{X}_{i.} = \frac{T_{i.}}{s}, \qquad \hat{\sigma}_{i.}^2 = \frac{1}{s-1}\sum_{j=1}^{s}\left(X_{ij} - \bar{X}_{i.}\right)^2$$

$$T_{.j} = \sum_{i=1}^{m} X_{(i-1)s+j}, \qquad \bar{X}_{.j} = \frac{T_{.j}}{m}, \qquad \hat{\sigma}_{.j}^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(X_{ij} - \bar{X}_{.j}\right)^2$$

$$T_{..} = \sum_{i=1}^{m}\sum_{j=1}^{s} X_{(i-1)+j}, \qquad \bar{X}_{..} = \frac{T_{..}}{n}, n = ms, \qquad \hat{\sigma}_{x}^2 = \frac{1}{n-1}\sum_{i=1}^{m}\sum_{j=1}^{s}\left(X_{ij} - \bar{X}_{..}\right)^2$$

2.1 Buys-Ballot Procedure for Estimation Row Variance

The Buys-Ballot estimates are developed for short period of time in which the trend and cyclical components are jointly combined and restricted to a case in which the trend is a straight line. The length of periodic interval is taken to be s. Thus, row variance is obtained for the mixed model (3). Recall, for the mixed model, $\sum_{j=1}^{s} S_{t+j} = s$, $S_{t+j} = S_j$ and $e_t \sim N(0,1)$. When arranged in a Buys-Ballot table, with m-rows and s-columns $t = (i-1)s + j$, $i = 1,2,...,m$, $j = 1,2,...,s$. Therefore X_t in (3) becomes

$$X_{(i-1)s+j} = M_{(i-1)s+j} \times S_{(i-1)s+j} + e_{(i-1)s+j} \tag{7}$$

For convenience, let $X_{ij} = X_{(i-1)s+j}$, $M_{ij} = M_{(i-1)s+j}$ and $e_{ij} = e_{(i-1)s+j}$. Hence,

$$M_{ij} = a + b[(i-1)s + j] \tag{8}$$

and

$$X_{ij} = M_{ij} S_j + e_{ij}$$

$$= \{a + b[(i-1)s + j]\} S_j + e_{ij} \tag{9}$$

The row variance is

$$\hat{\sigma}_i^2 = \frac{1}{s-1} \sum_{j=1}^{s} \left(X_{ij} - \overline{X}_{i.} \right)^2 \tag{10}$$

$$(s-1)\hat{\sigma}_i^2 = \sum_{j=1}^{s} \left(M_{ij} S_j + e_{ij} - \frac{1}{s} \sum_{j=1}^{s} M_{ij} S_j - \overline{e}_{i.} \right)^2$$

$$M_{ij} S_j = [a + bs(i-1)] S_j + bjS_j$$

$$\frac{1}{s} \sum_{j=1}^{s} M_{ij} S_j = [a + bs(i-1)s] + \frac{b}{s} \sum_{j=1}^{s} jS_j \ .$$

Hence,

$$(s-1)\,\hat{\sigma}_i^2 = \sum_{j=1}^{s} \left\{ \left(M_{ij} S_j + e_{ij} - \left(\frac{1}{s} \sum_{j=1}^{s} M_{ij} S_j + \overline{e}_{i.} \right) \right) \right\}^2$$

$$= \sum_{j=1}^{s} \left\{ [a + bs(i-1)] S_j + b\, jS_j + e_{ij} - \left([a + bs(i-1)s] + \frac{b}{s} \sum_{j=1}^{s} jS_j + \overline{e}_i \right) \right\}^2$$

$$= \sum_{j=1}^{s} \left\{ [a + bs(i-1)](S_j - 1) + b\,(jS_j - \frac{1}{s} \sum_{j=1}^{s} jS_j) + (e_{ij} - \overline{e}_i) \right\}^2$$

$$= \sum_{j=1}^{s} [a + bs(i-1)]^2 \left(S_j - 1 \right)^2 + b^2 \sum_{j=1}^{s} \left(jS_j - \frac{1}{s} \sum_{j=1}^{s} jS_j \right)^2$$

$$+ 2b[a + bs(i-1)] \sum_{j=1}^{s} \left(S_j - 1 \right) \left(jS_j - \frac{1}{s} \sum_{j=1}^{s} jS_j \right) + \sum_{j=1}^{s} \left(e_{ij} - \overline{e}_i \right)^2$$

$$+ 2 \sum_{j=1}^{s} \left(e_{ij} - \overline{e}_i \right) \left\{ b[a + bs(i-1)](S_j - 1) + \left(jS_j - \frac{1}{s} \sum_{j=1}^{s} jS_j \right) \right\}.$$

Therefore,

$$\hat{\sigma}_i^2 = [a + bs(i-1)]^2 \frac{\sum_{j=1}^{s} \left(S_j - 1 \right)^2}{s-1} + b^2 \frac{\sum_{j=1}^{s} \left(jS_j - \frac{1}{s} \sum_{j=1}^{s} jS_j \right)^2}{s-1}$$

$$+ 2b[a + bs(i-1)] \frac{\sum_{j=1}^{s} \left(S_j - 1 \right) \left(jS_j - \frac{1}{s} \sum_{j=1}^{s} jS_j \right)}{s-1} + \frac{1}{s-1} \left[\sum_{j=1}^{s} e_{ij}^2 - s\overline{e}_i^2 \right]$$

$$+2\sum_{j=1}^{s}\left(e_{ij}-\bar{e}_{i}\right)\left\{b\left[a+bs(i-1)\right]\left(S_{j}-1\right)+\left(jS_{j}-\frac{1}{s}\sum_{j=1}^{s}jS_{j}\right)\right\}.$$

$$E\left(\hat{\sigma}_{i}^{2}\right)=\left[a+bs(i-1)\right]^{2}\operatorname{var}\left(S_{j}\right)+b^{2}\operatorname{var}\left(jS_{j}\right)$$

$$+2b\left[a+bs(i-1)\right]\operatorname{cov}\left(S_{j},jS_{j}\right)+\sigma^{2} \tag{11}$$

2.2 Buys-Ballot Procedure for Estimation Seasonal Variance

As in the method for estimation of seasonal variance of the trend-cycle component. The Buys- Ballot procedure for estimation of seasonal variance is restricted to a case in which the trend component of time series is linear. The length of periodic interval is taken to be s. Hence, the seasonal variance is obtained for the mixed model

$$\hat{\sigma}_{j}^{2}=\frac{1}{m-1}\sum_{i=1}^{m}\left(X_{ij}-\bar{X}_{.j}\right)^{2} \tag{12}$$

$$=\frac{1}{m-1}\sum_{i=1}^{m}\left\{\left[a+bs(i-1)+bj\right]S_{j}+e_{ij}-\left[a+b\left(\frac{n-s}{2}\right)+bj\right]S_{j}-\bar{e}_{j}\right\}^{2}$$

$$=\frac{1}{m-1}\sum_{i=1}^{m}\left\{\left[a+bs(i-1)-a-b\left(\frac{n-s}{2}\right)\right]S_{j}+bjS_{j}-bjS_{j}+\left(e_{ij}-\bar{e}_{.j}\right)\right\}^{2}$$

$$=\frac{1}{m-1}\sum_{i=1}^{m}\left\{bs\left[(i-1)-\frac{m-1}{2}\right]S_{j}+\left(e_{ij}-\bar{e}_{.j}\right)\right\}^{2}$$

$$(m-1)\hat{\sigma}_{j}^{2}=\sum_{i=1}^{m}\left\{(bs)^{2}\left[(i-1)-\frac{m-1}{2}\right]^{2}S_{j}^{2}+2bs\left[(i-1)-\frac{m-1}{2}\right]S_{j}\left(e_{ij}-\bar{e}_{.j}\right)\right\}+\left(e_{ij}-\bar{e}_{.j}\right)^{2}$$

$$=(bs)^{2}S_{j}^{2}\sum_{i=1}^{m}\left[(i-1)-\frac{m-1}{2}\right]^{2}+2bsS_{j}\sum_{i=1}^{m}\left[(i-1)-\frac{m-1}{2}\right]\left(e_{ij}-\bar{e}_{.j}\right)+\sum_{i=1}^{m}e_{ij}^{2}-m\bar{e}_{j}^{2}$$

$$(m-1)E\left(\hat{\sigma}_{j}^{2}\right)=(bs)^{2}S_{j}^{2}\left[\sum_{i=1}^{m}(i-1)^{2}-(m-1)\left(\frac{m-1}{2}\right)^{2}\right]+m\sigma^{2}-m\frac{\sigma^{2}}{m}$$

$$=(bs)^{2}S_{j}^{2}\left[\frac{m(m-1)(2m-1)}{6}-\frac{m(m-1)^{2}}{4}\right]+(m-1)\sigma^{2}$$

$$=(m-1)\left\{(bs)^{2}S_{j}^{2}\left[\frac{m(2m-1)}{6}-\frac{m(m-1)}{4}\right]+\sigma^{2}\right\}$$

$$=(m-1)\left\{(bs)^{2}\frac{m(m+1)}{12}S_{j}^{2}+\sigma^{2}\right\}$$

$$=(m-1)\left[\frac{b^{2}n(n+s)}{12}S_{j}^{2}+\sigma^{2}\right]$$

$$E\left(\sigma_j^2\right) = \frac{1}{m-1}\sum_{i=1}^{m}\left(X_{ij} - \overline{X}_{.j}\right)^2$$

$$= b^2 \frac{n(n+s)}{12} S_j^2 + \sigma^2 \qquad (13)$$

Table 2. Summary of row, column and overall variances of Buys-Ballot table for mixed model

Variances	Linear trend-cycle component: $M_t = a + bt$, $t = 1, 2, ..., n = ms$
	Mixed model
$\hat{\sigma}_{i.}^2$	$\left\{\left[(a + bs(i-1)) + bC_1\right]^2 + \mathrm{var}\left[\left[a + bs(i-1)\right]S_j + bjS_j\right]\right\} + \sigma_1^2$
$\hat{\sigma}_{.j.}^2$	$\frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2$
$\hat{\sigma}_x^2$	$\frac{n}{n-1}\left\{\begin{array}{l}\frac{b^2(n^2-s^2)}{12} + \left[a^2 + 2ab\left(\frac{n-s}{2}\right) + \frac{b^2(n-s)(2n-s)}{6}\right]\mathrm{Var}(S_j) \\ + 2b\left[a + b\left(\frac{n-s}{2}\right)\right]\mathrm{Cov}(S_j, jS_j) + b^2\,\mathrm{Var}(jS_j)\end{array}\right\} + \sigma_1^2$

The Buys-Ballot estimates of row, column and overall variances for the mixed model is given in Table 2. It is noted from Table 2, that the row variance of the Buys-Ballot table is a functions of trend parameters and seasonal component of the original time series. Also, the expected value of the row variance involves sum of squares and cross-products of trend parameters and seasonal components. The seasonal/column variance depends on slope and seasonal component only. The focus of attention is on the column variances $\left(\hat{\sigma}_j^2\right)$ of the Buys-Ballot table. Thus, model structure for decomposition methods required that the model is known and formally described. Among the variances, the column variances which depends only on slope and seasonal component for the mixed model, will aid the choice of model, because it is the only one that is easily amenable to statistical test. For the purposes of choosing an appropriate time series model for decomposition of the study series, an analyst only needs to look at the seasonal/column variances $\left(\sigma_{.j}^2\right)$ of the time series. The seasonal/column variance for the mixed model is $b^2 \frac{n(n+s)}{12} S_j + \sigma_{i.}^2$. Thus, the test for choice between mixed and multiplicative models may be reduced to identify the mixed model whose seasonal/column variance is simply a function of slope and seasonal component only.

Nwogu et al. [9] and Dozie et al. [11] provided chi-squared test as the basis for choice between mixed and multiplicative models in time series decomposition. Therefore, the problem of choosing an appropriate time series model for decomposition when trend cycle component is linear reduces to that of testing null hypothesis

H_0: $\sigma_j^2 = \sigma_{0j}^2$

and the appropriate time series model is mixed, against the alternative

H_1: $\sigma_j^2 \neq \sigma_{0j}^2$

and the appropriate time series model is not mixed, where

$\sigma_j^2 = (j = 1, 2, ..., s)$ is the actual variance of the jth column.

$$\sigma_{0j}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2 \qquad (14)$$

and σ_1^2 is the error variance, assumed equal to 1.

The test statistic

$$\chi_c^2 = \frac{(m-1)\sigma_j^2}{\sigma_{0j}^2} \quad (15)$$

follows the chi-square distribution with $m-1$ degrees of freedom, m represents the number of year/period and s is the number of columns. The interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ contains the statistic (15) with 100 (1- α)% degree of confidence.

3. ANALYSIS

The purpose of this section is to present empirical example to illustrate the validity of the proposed chi-square test when the trend component is linear. The empirical example consists of simulated time series data from the mixed model. Each time series data has been arranged as monthly data (s = 12) and for 10 years (m = 10) in the Buys-Ballot Table while the time plot of simulated time series is given in Fig. 1.

3.1 Simulations Results using the Mixed Model

The simulated time series used consists 100 data sets of 120 observations each simulated from the mixed model: $X_t = (a+bt)\times S_t + e_t$, using the MINITAB 17.0 version software. The trend-cycle component is used with *a=1*, *b=0.2*, $e_t \sim N(0,1)$ and *S*$_1$*=0.92, S*$_2$*=0.87, S*$_3$*=0.99, S*$_4$*=0.96, S*$_5$*=0.98, S*$_6$*=1.13, S*$_7$*=1.26, S*$_8$*=1.18, S*$_9$*=1.05, S*$_{10}$*=0.93, S*$_{11}$*=0.80, S*$_{12}$*=0.93, S = 12.00.* The results of the calculated values of the statistic from the simulated timeseries data are given in Table 3. The critical values at 5% level of significance and which for $m-1=9$ degrees of freedom, equal to (2.7 and 19.0). The null hypothesis that the time series data admits mixed model is rejected, if the calculated value of the test statistic is not within the interval, otherwise, do not reject the null hypothesis. When compared with the interval (2.7 and 19.0), 99 out of the 100 calculated values of test statistic from the stimulated time series data contained in Table 3 lie within the interval. This indicates that the proposed test identified mixed time series model clearly in 99 percent of times. This result further confirm the validity of the proposed test, when the trending curve is linear.

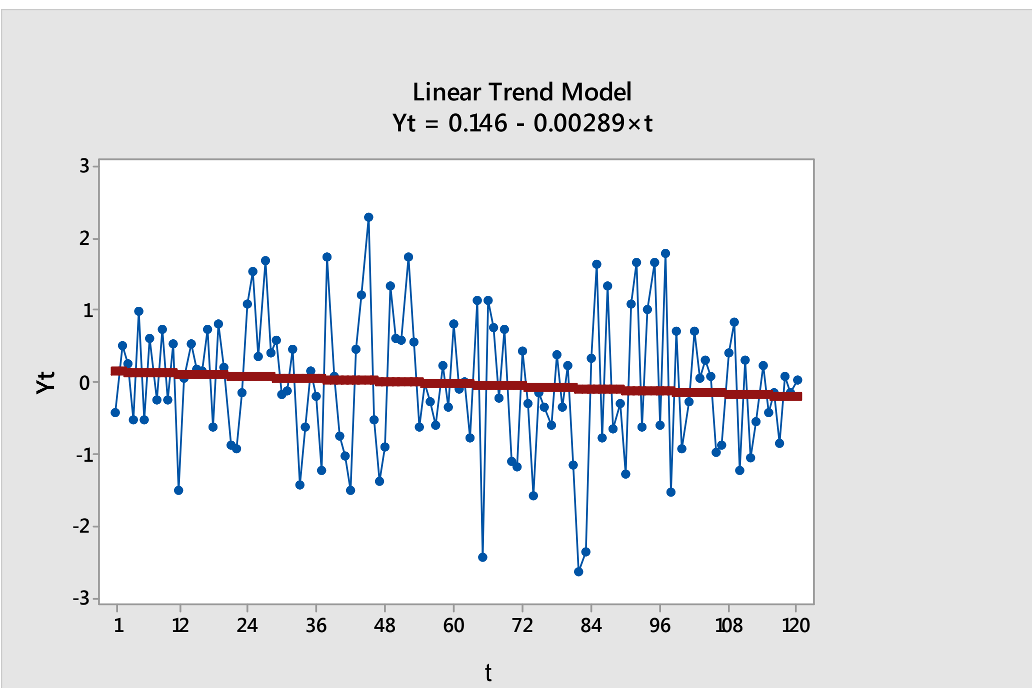

Fig. 1. Time plot of the simulated series

Table 3. Calculated Chi-Square for mixed model

Col	Series									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	9.7030	8.3378	8.8326	8.5060	9.6699	9.6388	9.1650	9.6931	7.0242	8.7591
2	8.2490	9.0725	10.3767	10.9261	9.2569	8.9195	7.9636	9.3989	10.6750	10.4958
3	8.5977	9.8364	7.8754	8.8338	8.9669	8.6298	10.2171	10.2222	8.5836	9.6242
4	8.9772	8.3298	9.4209	9.6255	10.0175	7.7497	8.7574	9.4730	8.4233	9.2049
5	8.9402	10.4199	9.5047	9.7199	8.5451	7.9427	10.6137	7.7128	8.4362	8.4254
6	9.0287	8.3659	10.4049	7.8284	7.7388	9.0531	9.0257	9.1088	9.9519	9.1118
7	9.6877	9.0744	9.4345	10.2101	10.1725	9.9103	8.5550	8.1717	9.0800	8.5356
8	8.5097	10.6823	8.3249	10.5502	9.3769	8.5896	10.0440	9.6341	9.4408	10.0922
9	10.4321	8.4610	8.0583	6.0611	9.6294	8.5410	8.1663	8.5630	8.0476	9.3160
10	8.6774	8.0925	7.2423	8.4937	9.8090	10.7988	8.0742	9.5039	9.5628	8.2780
11	9.3674	9.0833	10.6779	8.6402	8.4780	8.2573	8.2936	8.5495	9.9820	8.7411
12	7.9085	8.3595	8.8407	9.4225	5.9134	10.4224	8.6652	7.8754	8.7822	7.1425
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ *degrees of freedom are 2.7 and 19.0*

Calculated Chi-Square for mixed model cont'

Col	Series									
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	9.6939	9.6812	11.2890	7.8937	8.8311	10.8291	8.4564	8.1722	8.6506	9.7686
2	9.3136	9.7152	8.2521	10.0819	8.4783	9.3746	10.5106	9.1172	9.7605	8.2439
3	9.7731	8.3800	8.8122	10.6055	9.3073	8.5353	10.5772	9.1856	8.7013	8.3760
4	8.1619	9.2736	9.2167	9.3167	9.2862	8.7353	8.6010	9.3607	10.1157	9.0052
5	7.9400	8.5100	8.1450	8.9255	8.1535	8.5075	7.6286	7.3245	9.2858	10.3793
6	10.2383	9.5309	7.7630	8.2256	10.8533	9.8446	8.8866	11.1474	8.9900	9.2056
7	7.4789	9.2220	9.5023	9.7359	8.3534	9.2614	8.9940	8.9669	9.9913	9.2990
8	10.6669	7.4814	8.6176	8.6354	9.4443	8.2233	9.2810	9.1737	8.9823	9.1720
9	8.2764	8.5176	8.2645	10.0158	8.2099	9.1167	8.3737	9.1706	8.4103	7.8303
10	9.2051	7.8395	10.6764	8.8229	8.8234	9.0889	9.3363	8.0078	9.3268	8.5678
11	7.8485	11.7465	7.7588	7.6919	10.6874	7.5890	9.9976	10.1156	7.8242	9.2952
12	9.3546	9.0022	10.4922	7.8286	7.9816	8.8269	7.7865	8.2118	7.2791	9.4001
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ *degrees of freedom are 2.7 and 19.0*

Calculated Chi-Square for mixed model cont'

Col	Series									
	21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
1	9.30159	8.4574	8.3671	10.6684	7.6925	8.7545	8.5611	9.2311	8.4948	8.3142
2	8.11775	10.5118	11.6535	10.0250	8.9997	8.9807	10.3242	10.0915	7.7348	10.5062
3	9.15745	10.5784	7.6997	6.9715	9.3588	10.8256	7.8573	9.4748	10.8365	9.2667
4	8.61506	8.6020	10.3876	9.6487	9.3676	8.8734	9.0186	7.3221	8.1431	8.9273
5	8.11981	7.6295	9.5533	8.8282	7.5219	9.4306	9.0810	9.5459	7.9090	9.1875
6	9.67501	8.8876	8.1293	8.6525	10.0796	8.0334	7.9133	9.1432	7.9294	9.4852
7	9.01450	8.9950	9.4067	9.4858	8.1759	9.0241	8.4581	9.1535	9.5517	8.8962
8	9.13922	9.2820	8.6244	8.4325	8.6744	7.9083	9.4320	8.2775	9.9468	8.7883
9	8.62370	8.3747	10.3370	9.4893	8.1273	9.1616	9.0755	9.5858	9.1852	8.5255
10	9.46844	9.3374	8.3334	7.5962	9.6441	8.2396	10.0809	9.2287	9.0403	8.1030
11	9.60340	9.9988	7.4274	9.9227	11.3073	10.0354	8.3357	9.3521	10.4331	9.5711
12	9.30979	7.7874	8.4342	8.7823	9.6657	8.8830	10.5474	7.9761	8.8209	8.7869
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ degrees of freedom are 2.7 and 19.0

Calculated Chi-Square for mixed model cont'

Col	Series									
	31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
1	5.0921	9.2683	8.3058	8.8025	9.8439	9.3048	9.8602	9.4615	9.9628	7.51051
2	10.8037	9.0258	9.3027	7.6594	9.1421	9.4500	8.8337	10.8048	8.5905	8.96473
3	5.1566	8.2966	8.6602	10.0093	10.4079	10.9372	7.9933	8.4811	9.1333	8.48896
4	12.2505	8.2388	10.7586	7.5951	8.2335	9.7990	7.9485	8.2859	8.6502	9.37817
5	10.4191	7.7784	9.3609	9.1837	8.4052	9.8697	9.4147	9.5667	10.2568	9.83323
6	16.4968	8.9418	9.8435	8.8705	8.6785	8.6987	8.1723	8.1233	8.8561	8.09127
7	10.3324	9.1654	9.7053	8.9460	7.7973	7.5498	9.6533	7.8750	8.5088	8.88711
8	2.5788	9.0405	8.6404	8.9234	9.7541	9.7436	8.3248	9.2933	8.4451	9.49254
9	3.3121	10.2094	8.8420	9.0033	8.8627	9.0485	10.1016	10.2332	8.2809	9.40239
10	14.5750	11.3157	7.6662	10.3105	9.0078	8.0175	10.4073	9.2558	9.5250	9.05847
11	21.9050	7.9632	8.0743	9.0855	8.9073	7.5443	8.7072	7.7058	9.1173	9.22200
12	8.9651	8.8024	10.4922	9.6875	8.5660	8.4163	8.5443	9.5107	8.7707	9.31121
Decision	Reject	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ degrees of freedom are 2.7 and 19.0

Calculated Chi-Square for mixed model cont'

Col	Series									
	41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
1	8.75211	11.3780	8.7958	9.3699	8.1065	9.0606	8.3965	9.5222	11.2509	8.7612
2	7.04422	10.2314	9.7269	8.2030	8.1288	9.3381	8.6352	7.2416	8.3501	6.5975
3	9.57227	8.2535	9.9282	8.4946	8.6386	8.5259	7.5001	9.0012	9.3119	9.6954
4	9.98518	8.8195	10.1069	8.1599	9.7817	7.7283	8.6365	9.9487	7.4243	8.8219
5	9.00121	8.81646	9.3141	8.3400	8.7389	11.0398	8.1156	9.9087	11.2627	8.7557
6	8.4657	7.5993	8.5820	9.8015	8.1303	8.8530	9.7526	10.1308	9.2156	10.3952
7	9.3855	8.1237	7.4729	9.4654	8.2942	9.3282	9.6651	8.9043	9.2156	9.6353
8	8.9993	9.5734	8.2158	9.3095	10.8014	9.1478	10.2870	9.9186	8.2559	8.5633
9	9.5681	8.4433	9.5463	9.3127	9.2610	8.5357	8.0612	7.7301	7.7546	8.9187
10	9.2969	9.6251	9.4751	8.6219	9.2789	11.1484	9.5309	7.8244	8.5753	8.6135
11	7.6070	8.9823	9.7411	8.1828	10.2921	7.8570	8.9468	8.4156	8.4391	9.4896
12	9.6108	8.9950	8.1432	10.3053	8.6330	8.0557	10.1141	9.3609	9.1447	9.7649
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ degrees of freedom are 2.7 and 19.0

4. CONCLUSION

This study has examined the estimation procedure of seasonal variances in descriptive time series analysis. Results of the Buys-Ballot estimates for mixed model show that, the column/seasonal variance is, a constant multiple of the square of the seasonal component. It is for row variance a function of trend parameters and seasonal component of the original time series. Also, using empirical examples, the proposed chi-squared test statistic identified the mixed time series model clearly in 99 out of the 100 stimulations. This further confirmed the validity of the proposed test when trending curve is linear. In considering the mixed time series model, the error terms are assumed to be (i) uncorrelated; and (ii) normal distribution with mean zero and constant variance. Further studies are therefore recommended for cases in which these assumptions are not met.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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