Mathematical Modeling of Treatment Effect on Tumor Growth and Blood Flow through a Channel with Magnetic Field

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Original Research Article

Abstract

In this research, we investigated the effect of tumor growth on blood flow through a micro channel by formulated the governing model with the assumption that blood is an incompressible, eclectrially conducting fluid which flow is caused by the pumping action of the heart and suction. The governing model was scaled using some dimensionless variables and the region of the tumor was obtained from Dominguez [1] which was incorporated in our model. The model is further reduced to an ordinary differential equation using a perturbation condition. However, the ordinary differential equation was solved using method of undermined coefficients, and the constants coefficients obtained via matrix method. Furthermore, the simulation to study the effect of the pertinent parameters was done suing computation software called Mathematica. It is seen in our investigation that the entering parameters such as magnetic field parameter, the Reynolds number, womersley number, oscillatory frequency parameter, and permeability parameter affect the blood velocity profile in decreasing and increasing fashion.

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Nomenclatures

(u^*, v^*, w^*)	: *Dimensional Velocity components;*
y^*	: *Dimensional radius of the artery;*
x^*	: *Dimensional axial distance;*
t^*	: *Dimensional time;*
B_0	: *Magnetic field parameter;*
$R(x)$	: *Tumor depleted region of the channel;*
R_0	: *Normal region of the channel;*
R_T	: *Treatment parameter;*
P_0	: *Diastolic blood pressure;*
P_1	: *Systolic blood pressure;*
$w_1(y)$	: *Perturbed unsteady state velocity;*
$w_0(y)$	: *Perturbed steady state velocity;*
$w(y,t)$	: *Fluid velocity profile;*
$\vec{B}$	: *Magnetic flux density;*
V_0	: *Suction velocity;*
$\vec{E}$	: *Electrical field intensity.*

Greek Symbols

v	: *Kinematic Viscosity of blood;*
μ_b	: *Dynamic viscosity of blood;*
ρ_b	: *Blood density;*
g	: *Acceleration due to gravity;*
δ_0	: *Initial height of tumor growth;*
δ	: *Height of tumor growth;*
σ	: *Electrical conductivity;*
ω	: *Oscillatory frequency;*
ε	: *Perturbation parameter;*
β_0	: *Constant term due to steady state;*
β_1	: *Constant term due to unsteady state.*

1 Introduction

Abnormal growth of cells that serves no purpose is basically what is referred to as Tumor. There are two types of tumor growth: the Benign tumor (non-cancerous) and Malignant (cancerous). The Benign tumor grows large but do not spread into, or invade nearby tissues or other parts of the body while a malignant tumor spreads into

or invade nearby tissues. This growth disrupts organ function and destroys organ cells blocking nutrient and oxygen supply allowing waste products to build up. Benign tumor only affects related tissues from which the growth begins from but malignant tumor grows without discretion.

Tumor growth models are important to create an engineering background for cancer treatment either by using the models to evaluation the treatment protocols. It describes the physiological processes and the measurements as well, Daniel and Levente (2017). Tumor growth is composed of dead tumor cells and living cells.

Over the years, there have been significant developments in theoretical, experimental, and clinical approaches to understanding the dynamics of the two types of tumor growth and their interactions with the immune system and bodily tissues, Unni and Seshaiyer [2]. It has led to the development of procedure for cancer therapy including virotherapy, immunotherapy, chemotherapy, targeted drug therapy and many more.

The cardiovascular system is made up of blood cells, blood vessels and the heart. The main function of the heart is to pump blood into circulation, to the tissues and organs of the human body through the blood vessels. According to Bunonyo and Amos [3], blood an essential ingredient of the vitality of the body system, and the major constituents are the red blood cells (erythrocytes), the platelet and the plasma fluid. Blood contains haemoglobin which has magnetic properties that are different depending on the oxidation state of haemoglobin. The body contains Proteins, LDL cholesterol, which for increasing quantity in the cardiovascular system can build up in various arteries, clogging and reducing their flexibility. Hardening of the arteries result to an atherosclerosis and that occludes normal blood flow because of the loss of the vessel flexibility and deposition of lipid, which caused the heart with hard pressure to push blood through to the downstream, Okpeta and Bunonyo [4].

Bio-magnetic fluid dynamics has many major applications such as Magnetic drug targeting, adjusting blood flow during surgery and transporting complex bio-waste fluids, cancer tumor treatment etc. Extensive research has been undertaken on the fluid dynamics of bio-magnetic fluids under the presence of an external magnetic field. The application of magneto-hydrodynamics in physiological flow is of growing interest to many researchers, which reported that blood is an electrically conducting fluid, Singh and Rathee [5]. Over several decades there are so many researchers that have worked on blood flow problems; a mathematical modeling of tumor growth in mice following low-level direct electric current was proposed by Cabrales et al. [6].

A numerical study of the nonlinear fractional mathematical model of tumor cells in presence of chemotherapeutic treatment was proposed by Kumar and Atangana [7]. An inverse problem formulation for parameter estimation of a reaction diffusion model of low grade gliomas was proposed by Gholami et al. [8]. They presented a numerical scheme for solving a parameter estimation problem for a model of low grade glioma growth. They estimated the spatial distribution of tumor concentration as well as the magnitude of anisotropic tumor diffusion.

Bonate [9] formulated tumor growth in oncology where oncology is the study of cancer. He developed mathematical models of preclinical and clinical tumor growth. Bru and Herrero [10] investigated tumor growth using the physical laws by modeling cancer processes. Their model described the growth of solid tumors which provides a procedure to extract information about the growth dynamics from the analysis of the geometrical properties of the interface tumor-host tissue. Dai and Liu [11] proposed an optimal control and pattern formation for a haptotaxis model of solid tumor invasion.

Duchting and vogelsaenger [12] proposed the aspects of modeling and simulating tumor growth and treatment. Their model is based on the hypothesis that the proliferation of malignant cells may be simulated by an unstable closed loop control circuit. Duchting and vogelsaenger [13] proposed progress in modeling and simulation of three- dimensional tumor growth and treatment. Their work showed how tumor treatment may be optimized in the long run using computer simulation experiments.

Hawkins-Daarud et al. [14] modeled tumor associated edema in gliomas during angiogenic therapy and its impact on manageable tumor. Gliomas is the most aggressive form of primary brain tumor. They evaluated virtual growth tumors with varying growth kinetics and observed tumors with lower proliferation rates will have the most reduction in swelling from their applied treatments.

Makinde et al. [15] studied the blood vessels formed in asthmatic airways are involved in inflammatory and airway remodeling processes in chronic asthma. Vascular endothelial cell growth factor (VEGF) and angiopoietin-1 (Ang-1) are primary angiogenic growth factors, involved in the formation of such blood vessels. VEGF has been reported to contribute to non-specific airway hyper-responsiveness, have chemotactic effects on eosinophils, and enhance airway smooth muscle cell proliferation. Makinde et al. [16] researched on hydromagnetic steady flow of a viscous conducting fluid in a channel with slip at the permeable boundaries. analytical solution where gotten for the governing nonlinear boundary values problem using perturbation method together with pade approximations technique based on computer extended series solution.

Mandel [17] investigated an unsteady analysis of non-Newtonian blood through the tapered arteries with stenosis. The non-Newtonian blood flow problem was solved numerically with finite difference scheme used to solve the unsteady nonlinear Navier strokes equation in cylindrical coordinates system governing flow with verifiable assumption to reduce the problem into two-dimensional flow. Iterative method has been shown in solving the equation numerically and it was known that the vascular wall deformability and non-Newtonian characteristics of the following blood affect the axial velocity profile.

Mathematical modeling, analysis and simulation of tumor dynamics with drug interventions were investigated by Unni and Seshaiyer [2]. Their results suggest that the model employed is a robust candidate for studying the dynamics of tumor cells and it helps to provide the dynamic interactions between the tumor cells, immune system and drug-response systems.

Miklavcic et al. [18] carried out a mathematical modeling of tumor growth in mice following electrotherapy and bleomycin treatment. The effect of bleomycin on tumor growth was obtained by introducing the influential parameter which transferred the bleomycin concentration in tumor tissue obtained from pharmacokinetic model to the effect on tumor growth.

Modeling of tumor cells regression in response to chemotherapeutic treatment was carried out by Ansarizadeh et al. [19]. They observed that the response of three different levels of immune system strength to the pulsed chemotherapy for which the tumor performs better if a chemotherapeutic drug injected near the invasive fronts of the tumor.

Morales-Delgado et al. [20] proposed an application of the Caputo-Fabrizio and Atangana-Baleanu fractional derivatives to mathematical model of cancer chemotherapy effect. They obtained approximate analytical solutions of a cancer chemotherapy effect model involving fractional derivatives with exponential kernel and with the general Mittag-Leffler function.

Stochastic modeling of tumor growth within organ during chemotherapy using bivariate birth, death, and migration processes was carried out by Padi et al. [21]. Their models are derived from stochastic differential equations and model behavior was analyzed with numerical data.

The development of a hybrid cellular automaton model to mimic the growth of avascular tumors, including the infusion of a bioreductive drug to study the effects of protein binding on drug transportation was reported by Kazmi et al. [22]. They conducted experiments with multicellular tumor spheroids and results showed good agreement with their predicted growth dynamics. Zhu et al. [23] researched on tumor growth under hyperthermia condition and came to the conclusion with both the tumor growth rate curve and corresponding average glucose concentration and obtaining numerical results illustrating the controlling effect on tumor growth under hyperthermia condition in the initial stage.

In this investigation, we introduced treatment on the tumor in order to correct the turbulence caused by the tumor and incorporate such into the blood momentum equation under an induced magnetic field on the blood flowing through a micro-channel.

2 Mathematical Formulation

In formulating mathematical model to investigate the tumor growth effect and treatment on blood flow through a micro channel, we modify the existing Navier-Stokes equation to incorporate the electromagnetic force and

permeability induced flow by considering the Darcy law for permeability and expand the equation further to suit our objectives. However, we consider the following assumptions:

2.1 Assumptions

In this section, the following assumptions were made:

1. Blood is a mixture of plasma and formed elements
2. Blood is an electrically conducting fluid
3. The flow is unidirectional
4. The flow is 2D but this study considered the axial directional flow
5. There is no-slip at the wall of the channel and symmetrical at the centre

3 Governing Equation

Following Misra and Adhikary [24] and Eldesoky [25], we present the blood momentum equation as follows:

$$\frac{\partial w^*}{\partial t^*} + V_0 \left(1 + \varepsilon e^{i\omega t}\right) \frac{\partial w^*}{\partial y^*} = \frac{1}{\rho} \frac{\partial P^*}{\partial x^*} + \upsilon \frac{\partial^2 w^*}{\partial y^{*2}} - \frac{\upsilon w^*}{k^*} - \sigma \frac{B_0^2 w^*}{\rho} \tag{3.1}$$

Following Bunonyo and Amos [3], we consider the region of the tumor growth is formulated as follows

$$y^* = \begin{cases} R_0 - \delta^* cos\left(2\pi \frac{x^*}{\lambda^*}\right) & \text{at } 0 \le x^* \le d_0 \\ R_0 & \text{at } d_0 \le x^* \le \lambda^* \end{cases} \tag{3.2}$$

where:

$$\delta = \delta_0 e^{\left(\frac{\lambda_1\left(1 - e^{(-\alpha_2 t)}\right)}{\alpha_2}\right)}, \text{ according to Dominguez [1]} \tag{3.3}$$

The boundary conditions are:

$$\left. \begin{array}{ll} \dfrac{\partial w^*}{\partial x^*} = 0 & \text{at} \quad y^* = 0 \\ w^* = 0 & \text{at} \quad y^* = R \end{array} \right\} \tag{3.4}$$

3.1 Dimensionless parameters

We introduce the following dimensionless parameters:

$$\left. \begin{array}{l} y = \dfrac{y^*}{R_0}, x = \dfrac{x^*}{\lambda}, \lambda = \dfrac{\lambda^*}{R_0}, \delta = \dfrac{\delta^*}{R_0}, w = \dfrac{w^*}{\upsilon w_0}, P = \dfrac{P^* R_0^2}{\mu \lambda w_0}, t = \dfrac{t^* R_0^2}{\upsilon}, \\ \dfrac{R_0^2}{k^*} = \dfrac{1}{k}, Re = \dfrac{V_0 R_0}{\upsilon}, \alpha^2 = \dfrac{\rho R_0^2 \omega}{\mu}, M^2 = \dfrac{\sigma B_0^2 R_0^2}{\mu}, t = \omega t^* \end{array} \right\} \tag{3.5}$$

Using the dimensionless variables in equation (3.5), the momentum equation governing the flow is reduced to:

$$\alpha^2 \frac{\partial w}{\partial t} + Re\left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial w}{\partial y} = \frac{\partial P}{\partial x} + \frac{\partial^2 w}{\partial y^2} - \frac{w}{k} - M^2 w \tag{3.6}$$

The region of tumor region is reduced to:

$$y = \begin{cases} 1 - \delta \cos\left(2\pi x\right) & \text{at } 0 \le x \le \frac{d_0}{\lambda} \\ 1 & \text{at } \frac{d_0}{\lambda} \le x \le R_0 \end{cases} \tag{3.7}$$

$$\delta = \delta_0 e^{\left(\frac{\lambda_1\left(1 - e^{(-\alpha_2 t)}\right)}{\alpha_2}\right)} \tag{3.8}$$

The corresponding boundary conditions are

$$\left.\begin{array}{lll} \dfrac{\partial w}{\partial x} = 0 & \text{at} & y = 0 \\[2mm] w = 0 & \text{at} & y = 1 - \delta \cos\left(2\pi x\right) \end{array}\right\} \tag{3.9}$$

4 Method of Solution

Since the flow is pulsatile because of the pumping action of the ventricles, we let the solution to be in the following form:

$$\left.\begin{array}{l} w\left(y,t\right) = w_0\left(y\right) + w_1\left(y\right)e^{i\omega t} \\[2mm] -\dfrac{\partial P}{\partial x} = P_0 + P_1 e^{i\omega t} \end{array}\right\} \tag{3.10}$$

Using equation (3.10) to simplify equation (3.6) – (3.9), we have:

Steady State for which $\varepsilon = 0$, the dimensionless momentum equation is:

$$\frac{\partial^2 w_0}{\partial y^2} - Re\left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial w_0}{\partial y} - \beta_0^2 w_0 = -P_0 \tag{3.11}$$

The oscillatory state which $\varepsilon \ne 0$, dimensionless momentum equation is:

$$\frac{\partial^2 w_1}{\partial y^2} - Re\left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial w_1}{\partial y} - \beta_1^2 w_1 = -P_1 \tag{3.12}$$

The tumor region is reduced to:

$$y = \begin{cases} 1 - \delta cos(2\pi x) & \text{at } 0 \le x \le \dfrac{d_0}{\lambda} \\ 1 & \text{at } \dfrac{d_0}{\lambda} \le x \le R_0 \end{cases} \tag{3.13}$$

$$\delta = \delta_0 e^{\left(\frac{\lambda_1 \left(1 - e^{(-\alpha t)}\right)}{\alpha_2} \right)} \tag{3.14}$$

$$\lambda_1 = -0.03, \alpha_2 = 0.3, t = 5, \delta_0 = 2, x = 0.13$$

The boundary conditions are

$$\left. \begin{array}{ll} \dfrac{\partial w_0}{\partial y} = 0, \dfrac{\partial w_1}{\partial y} = 0 & \text{at} \quad y = 0 \\ w_0 = 0, w_1 = 0 & \text{at} \quad y = 1 - \delta cos(2\pi x) + R_T \end{array} \right\} \tag{3.15}$$

where

$$\beta_1^2 = \left(\frac{1}{k} + M^2 + \alpha^2 i\omega \right), \beta_0^2 = \left(\frac{1}{k} + M^2 \right)$$

Solving equation (3.11), we let $w_0 = e^{m_1 y}$, so that we have the characteristics equation as:

$$p_0(m_1) = m_1^2 - Rem_1 - \beta_0^2 \tag{3.16}$$

The roots of the equation (3.16) are:

$$m_1 = \frac{Re + \sqrt{Re^2 + 4\beta_0^2}}{2} \quad \text{and} \quad m_2 = \frac{Re - \sqrt{Re^2 + 4\beta_0^2}}{2}$$

Then the general solution of equation (3.11) is:

$$w_0(y) = A_1 e^{m_1 y} + B_1 e^{m_2 y} + \frac{P_0}{\beta_0^2}, \tag{3.17}$$

Solving equation (3.12), we let $w_1 = e^{m_{11} y}$, so that we have the characteristics equation as:

$$p_{01}(m_{11}) = m_{11}^2 - \alpha_1 m_{11} - \beta_1^2 \tag{3.18}$$

where

$$\alpha_1 = Re\left(1 + \varepsilon e^{i\omega t}\right)$$

The roots of the equation (3.16) are:

$$m_{11} = \frac{\alpha_1 + \sqrt{\alpha_1^2 + 4\beta_1^2}}{2} \quad \text{and} \quad m_{21} = \frac{\alpha_1 - \sqrt{\alpha_1^2 + 4\beta_0^2}}{2}$$

Thence, the general solution of equation (3.11) is:

$$w_1(y) = A_2 e^{m_{11}y} + B_2 e^{m_{21}y} + \frac{P_1}{\beta_1^2}, \tag{3.19}$$

Solving equation (3.17) using the boundary condition in equation (3.15), we the following:

$$A_1 \left(\frac{Re + \sqrt{Re^2 + 4\beta_0^2}}{2} \right) + B_1 \left(\frac{Re - \sqrt{Re^2 + 4\beta_0^2}}{2} \right) = 0, \tag{3.20}$$

$$A_1 e^{\left(\frac{Re + \sqrt{Re^2 + 4\beta_0^2}}{2} \right)h} + B_1 e^{\left(\frac{Re - \sqrt{Re^2 + 4\beta_0^2}}{2} \right)h} = -\frac{P_0}{\beta_0^2}, \tag{3.21}$$

We can transform equations (3.20) and (3.21) into a matrix as follows:

$$\begin{pmatrix} \left(\frac{Re + \sqrt{Re^2 + 4\beta_0^2}}{2} \right) & \left(\frac{Re - \sqrt{Re^2 + 4\beta_0^2}}{2} \right) \\ e^{\left(\frac{Re + \sqrt{Re^2 + 4\beta_0^2}}{2} \right)h} & e^{\left(\frac{Re - \sqrt{Re^2 + 4\beta_0^2}}{2} \right)h} \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{P_0}{\beta_0^2} \end{pmatrix}, \tag{3.22}$$

Solving equation (3.22), we have the following constant coefficients as follows:

$$A_1 = \frac{m_2 P_0}{\beta_0^2} \left(\frac{1}{m_1 e^{m_2 h} - e^{m_1 h} m_2} \right) \tag{3.27}$$

$$B_1 = \frac{m_1 P_0}{m_2 \beta_0^2} \left(\frac{1}{e^{m_1 h} - \frac{m_1}{m_2} e^{m_2 h}} \right) \tag{3.29}$$

In a similar vein, to solve for the constant coefficients in equation (3.19) using the boundary conditions in equation (3.15) which we present the equations in matrix form as:

$$\begin{pmatrix} \left(\frac{\alpha_1 + \sqrt{\alpha_1^2 + 4\beta_1^2}}{2} \right) & \left(\frac{\alpha_1 - \sqrt{\alpha_1^2 + 4\beta_1^2}}{2} \right) \\ e^{\left(\frac{\alpha_1 + \sqrt{\alpha_1^2 + 4\beta_1^2}}{2} \right)h} & e^{\left(\frac{\alpha_1 - \sqrt{\alpha_1^2 + 4\beta_1^2}}{2} \right)h} \end{pmatrix} \begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{P_1}{\beta_1^2} \end{pmatrix}, \tag{3.30}$$

Solving for the constants in equation (3.30), we have the following:

$$A_2 = \frac{P_1}{\beta_1^2} \left(\frac{1}{\dfrac{m_{11}}{m_{21}} e^{m_{21}h} - e^{m_{11}h}} \right) \tag{3.35}$$

$$B_2 = \frac{m_{11}P_1}{m_{21}\beta_1^2} \left(\frac{1}{e^{m_{11}h} - \dfrac{m_{11}}{m_{21}} e^{m_{21}h}} \right) \tag{3.37}$$

The blood velocity profile is obtained after we've substituted the steady state velocity in equation (3.17) and pulsatile velocity profile in equation (3.19) into equation (3.10), which is:

$$w(y,t) = \left(A_1 e^{m_1 y} + B_1 e^{m_2 y} + \frac{P_0}{\beta_0^2} \right) + \left(A_2 e^{m_{11} y} + B_2 e^{m_{21} y} + \frac{P_1}{\beta_1^2} \right) e^{i\omega t} \tag{3.38}$$

5 Presentation of Results and Discussion

In this section, we coded equation (3.38) using Mathematica and simulate with the pertinent parameters and study the effect of the pertinent parameters on the velocity profile. The parameters values are obtained from Bunonyo and Amos [3], and Misra and Adhikary [24] and Eldesoky [25]. The simulated results are labeled Fig. 1 to Fig. 7, which depicts the effect of the various entering parameters on the velocity profile.
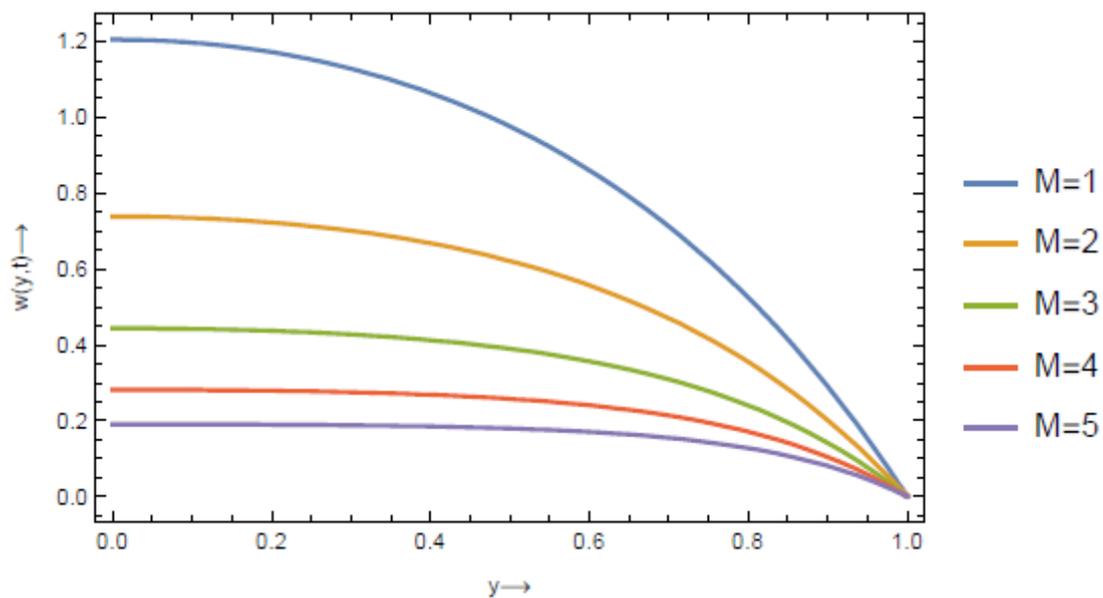

Fig. 1. Effect of Magnetic field on Velocity Profile $w(y,t)$ **, with**
$$k = 0.5, \alpha = 0.2, Re = 0.5, R_T = 0.5, \omega = 3, \varepsilon = 0.02, t = 5$$
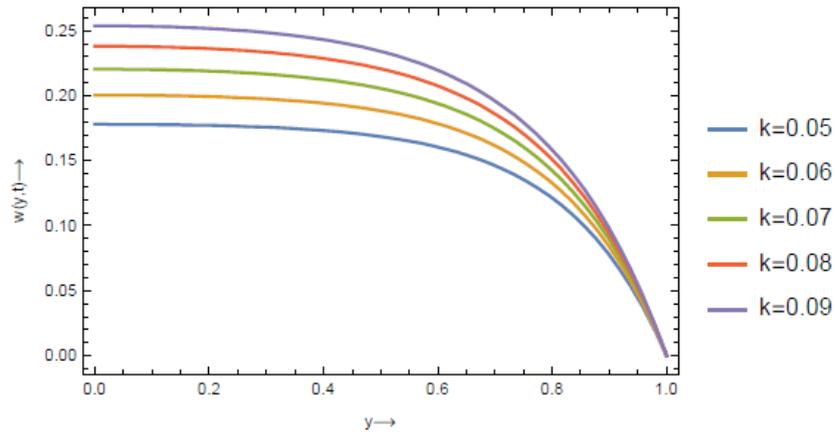
Fig. 2. Effect of permeability parameter on velocity profile $w(y,t)$, **with**
$M = 3, \alpha = 0.2, Re = 0.5, R_T = 0.5, \omega = 3, \varepsilon = 0.02, t = 5$
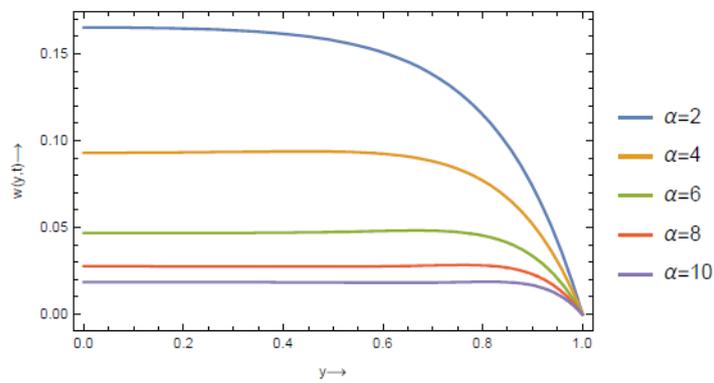

Fig. 3. Effect of womersley number on velocity profile $w(y,t)$, **with**
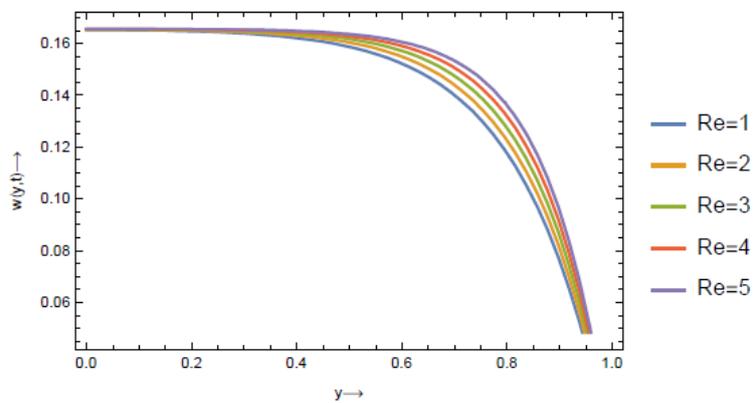$k = 0.5, M = 3, Re = 0.5, R_T = 0.5, \omega = 3, \varepsilon = 0.02, t = 5$

Fig. 4. Effect of Reynolds number on velocity profile $w(y,t)$, **with**
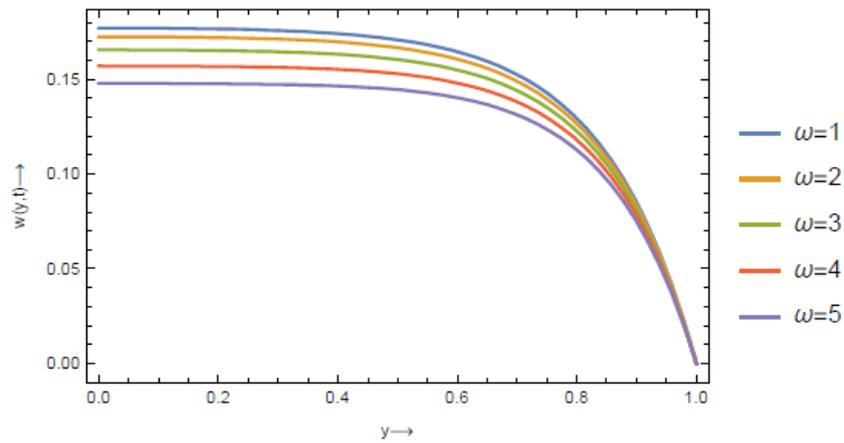$k = 0.5, \alpha = 0.2, M = 3, R_T = 0.5, \omega = 3, \varepsilon = 0.02, t = 5$

Fig. 5. Effect of oscillatory frequency on velocity profile $w(y,t)$ **, with**
$k = 0.5, \alpha = 0.2, Re = 0.5, R_T = 0.5, M = 3, \varepsilon = 0.02, t = 5$
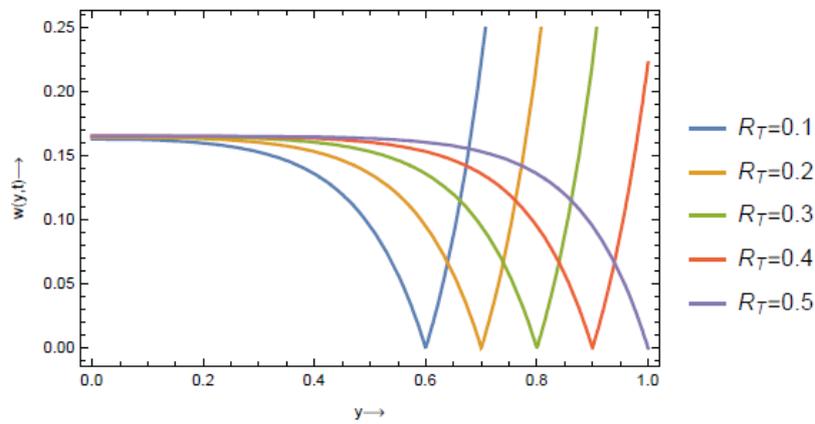

Fig. 6. Effect of oscillatory frequency on velocity profile $w(y,t)$ **, with**
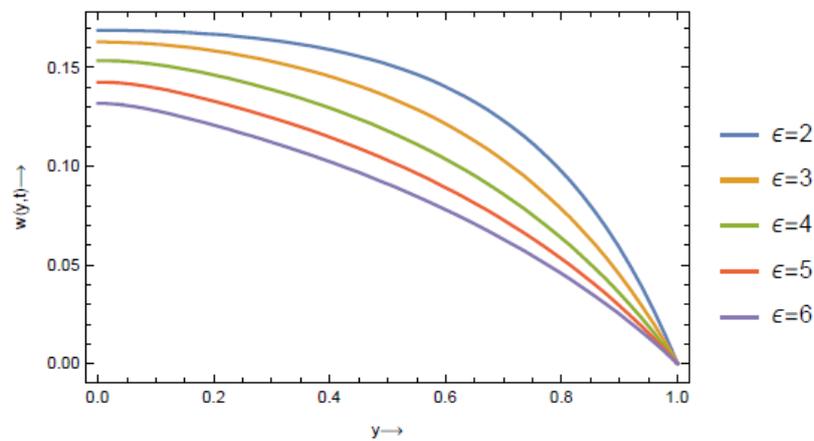$k = 0.5, \alpha = 0.2, Re = 0.5, M = 3, \varepsilon = 0.02, t = 5$

Fig. 7. Effect of oscillatory frequency on velocity profile $w(y,t)$ **, with**
$k = 0.5, \alpha = 0.2, Re = 0.5, R_T = 0.5, M = 3, t = 5$

5.1 Discussion

Fig. 1 illustrates the effect of magnetic field on the axial velocity in a tumor affected blood flow channel. The velocity of the blood decreases with the increase in magnetic field, which is in good agreement with studies carried out by Shit and Roy (2012). It is observed that the axial velocity decreases with increasing magnetic parameter M. It indicates that the blood velocity can be reduced by applying suitable magnetic field strength. However, the reduction in blood velocity can be used with surgical patients during surgery. This decrease is caused by Lorentz force due to the intersection of magnetic field with electrically conducting fluid in motion.

Fig. 2 depicts an increase in velocity profile with a maximum value at the centerline of the channel and minimum at the walls, for an increase in the value of the permeability parameter, while other involved parameters values are considered as shown in Fig. 2. This result is consistent with those obtained by Bunonyo and Amos [3].

Fig. 3 show that the parabolic axial velocity has a peak value at the centerline of the channel and minimum value at the walls. But the velocity profile decreases for an increase in the value of the womersley number, while other involving parameters values are considered as it is in Fig. 3.

Fig. 4 depicts a maximum value of the velocity at the centerline of the channel and decreases closer to the wall. It is noticed that the velocity profile increases for different values of the Reynolds number while other entering parameters values are kept as shown in Fig. 4, and the result of this investigation is consistent with those obtained by Eldesoky [25].

Fig. 5 depicts an increase in velocity profile, for an increase in the value of the oscillatory frequency parameter, while other involved parameters values are considered as shown in Fig. 5. This result is consistent with those obtained by Bunonyo and Amos [3]. Fig. 6 depicts the behavior of blood flow through the channel with tumor, it is seen that flow showed an inconsistent behavior due to an increasing dosage of the treatment. But is shows normalization as the treatment gets closer to the prescribed threshold milligram of $R_T = 0.5$, in order to effectively treat the tumor. Finally, Fig. 7 illustrates the contribution of perturbation parameters on the flow. It is seen that the flow velocity profile decreases, for the increasing values of the perturbation parameter ε, the decreases in profile is due to suction contribution.

6 Conclusion

This research was carried out to investigate the tumor growth effect and treatment on blood flow through a channel with magnetic field. The channel is subjected to time dependent suction/injection at the walls, considering blood as an incompressible electrically conducting fluid. Thus, has the potential for further exploration of the causes and development of arterial diseases like atherosclerosis and atheroma and the application of treatment. In this investigation, we conclude as follows:

1. It is seen that the effect of magnetic field on the axial velocity in a tumor affected blood flow channel. The velocity of the blood decreases with the increase in magnetic field, which is in good agreement with studies carried out by Shit and Roy [26].
2. The research showed an increase in velocity profile with a maximum value at the centerline of the channel and minimum at the walls, for an increase in the value of the permeability parameter, while other involved parameters values are considered, this result is consistent with those obtained by Bunonyo and Amos [3].
3. It is observed that the parabolic axial velocity has a peak value at the centerline of the channel and minimum value at the walls. But the velocity profile decreases for an increase in the value of the womersley number, while other involving parameters values are considered.
4. It is illustrated that there is a maximum value of the velocity at the centerline of the channel and decreases closer to the wall. It is noticed that the velocity profile increases for different values of the Reynolds number while other entering parameters values are kept, in which the result of this investigation is consistent with those obtained by Eldesoky [25].

5. The study showed that there is an increase in velocity profile, for an increase in the value of the oscillatory frequency parameter, while other involved parameters values are considered. This result is consistent with those obtained by Bunonyo and Amos [3].

However, the scope of this research only considered blood flow through micro channel with the effect of magnetic field, womersley number, permeability, oscillatory frequency, Reynolds number, without consideration of concentration of lipid and blood cells concentration through the haematocrit, and the energy generated through the body metabolism and external heat source which the future research could address.

Competing Interests

Authors have declared that no competing interests exist.

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