



Effect of Quantum Correction on Jeans Instability of Magnetized Radiative Plasma

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Authors' contributions

This work was carried out in collaboration between all authors. Author AKP designed the study, performed the Mathematical analysis, and wrote the first draft of the manuscript. Author VS discussed and analyzed the results obtained in the manuscript. The all work is done under the guidance and supervision of author RKP. Author HJ managed the literature searches... All authors read and approved the final manuscript.

Original Research Article

Received 9th May 2013
Accepted 15th November 2013
Published 14th January 2014

ABSTRACT

The Jeans instability of infinite homogeneous plasma has been investigated in the presence of magnetic field considering the effect of radiative heat-loss function and quantum correction. In the present approach, it is initiated that the criterion of Jeans instability is modified due to radiative heat-loss function and quantum effect in the longitudinal mode of propagation, while in transverse mode, it is affected by the presence of magnetic field. The resulting curves are obtained, illustrating that the temperature dependent heat-loss function and quantum correction have a stabilizing influence on the growth rate of instability. However, density dependent heat-loss function has a destabilizing influence on the growth rate of the instability. This analytic approach first discusses the quantum plasma with radiative heat-loss function and a strong emphasis is put on the choice of appropriate change in the instability criterion.

Keywords: *Radiative heat-loss functions; jeans instability; quantum correction; thermal conductivity; magnetic field.*

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1. INTRODUCTION

There has been a great deal of interest in studying various collective processes in gaseous plasma, which are ubiquitous in space, including diffuse and dense interstellar media, star envelopes, accretion disks, circumstellar shells, chromosphere, dark interiors and the out flow of red giant star. Thus, in to the crucial phenomena of the interstellar medium (ISM), many body gravitating system play an essential role. The gravitational instability is of the fundamental concept of modern astrophysical plasma and it is connected with the fragmentation of interstellar matter in regard to star formations. James Jeans [1] first studied this instability problem and shows that an infinite homogeneous, self-gravitating fluid is unstable for all wave number which is less then critical Jeans wave number. Chandrashekhar [2] has given the comprehensive account of the effect of a magnetic field and rotation separately and simultaneously on the gravitational instability of an infinite homogeneous medium and observed that the Jeans criterion remains unaffected in each case. In recent years, numerous researchers {Pensia *et al.* [3], Dangarh et al. [4], Ali and Shukla [5] and Shaikh et al. [6] have been carrying out investigations on various salient features of Jeans instability of infinite homogeneous gaseous plasmas contaminated by the various parameters encountered very often in space and laboratory plasmas. Various researchers have attempted to include the problem taking different assumptions and parameters under consideration in their studies.

Field [7] has pointed out the importance of thermal effect in the process of star formation and suggested that the observed filamentary condensations in nebulae may be due to thermal effects. The problem of thermal instability in the fragmentation of a gravitating fluid has been investigated by Aggarwal and Talwar [8,9]. Bora and Talwar [10] have investigated the thermal instability in resistive plasma with radiative effects. Talwar and Bora [11] have discussed the ISM model consisting of stars and optically thin radiative plasma. In this connection, many authors have discussed the thermal instability of homogeneous plasma considering the effects of various parameters {e.g. Vyas and Chhajlani [12], Chhajlani and Parihar [13].

As a reasonably simple approximation, the radiative heat-loss mechanism plays an important role in the star formation and molecular cloud condensation process in connection with thermal instability. In the study of ISM structure, we find that the heat-loss process is the major cause for the condensation of large astrophysical compact objects. The radiative heat-loss functions have decay effects of heat in an embedded system with respect to local temperature and density. These functions are similar to those of the cooling functions considered earlier by Wolfire et al. [14] and Shadmehri and Dib [15]. Recently, Prajapati et al. [16] have discussed the effect of arbitrary radiative heat-loss function and Hall current on the self-gravitational instability of homogeneous, viscous, rotating plasma incorporating the effects of finite electrical resistivity, finite electron inertia and thermal conductivity. Patidar et al. [17] have studied the problem of radiative instability of homogeneous rotating partially ionized plasma incorporating viscosity, porosity and electron inertia in the presence magnetic field. It may therefore, be of importance in the dynamics of ISM field and is the object of the present paper to study the effects of quantum correction on the Jeans instability of magnetized radiative plasma.

In the recent study of astrophysics we find that quantum correction play a comprehensive role to discuss self-gravitational instability of gaseous plasma. The quantum plasmas was first investigated by Pines [18,19], and the kinetic model of the quantum electro-dynamical properties of non-thermal plasmas has been discussed by Bezzerrides and DuBios [20].

Masood, et al. [21] have extended the above treatment by considering multi-component quantum dusty plasma, which incorporates the quantum Bohm potential and statistical terms of electrons and ions. Salimullah et al, [22] have investigated the Jeans instability in homogeneous cold quantum dusty plasma in the presence of a magnetic field. The self-gravitational stability of a streaming nonuniform quantum dusty magnetoplasma have been discussed by Bashir et al. [23]. These studies suggest that the contributions of the quantum correction in the ISM lead to many important phenomena, which are applicable in different astrophysical processes.

From the above discussion, it is obvious that the inclusion of quantum correction in the Jeans instability problem, together with radiative heat-loss functions, is of interest because of its relevance to certain astrophysical contexts. Therefore, in the present paper we have carried out an analysis of the Jeans gravitational instability of homogeneous magnetized gaseous plasma incorporating the effects of quantum correction, radiative heat-loss functions and thermal conductivity. We have discussed the implications of the quantum correction and radiative heat-loss functions on the stability of the considered configuration.

2. EQUATIONS OF THE PROBLEM

We consider an infinite extended homogeneous, high density self-gravitating plasma containing electrons and singly charged ions including, radiative heat-loss functions and thermal conductivity. It is assumed that the above medium is permeated with a weak uniform magnetic field $\vec{B} (0, 0, B)$ along z- direction. We introduced the quantum effects through the Bohm potential term in the momentum transfer equation. Due to the consideration that the magnetic field is not strong, the quantum spin effect is not taken in the analysis [25,27]. Following Haas [24], basic set equation using the QMHD model is given as:

The momentum transfer equation for magnetized quantum plasma is

$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla}p + \rho\vec{\nabla}U + \frac{1}{4\pi}(\vec{\nabla} \times \vec{B}) \times \vec{B} + \frac{\hbar^2 \rho}{2m_e m_i} \vec{\nabla} \left(\frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right). \quad (1)$$

The equation of continuity is given by

$$\frac{d\rho}{dt} + \rho\vec{\nabla} \cdot \vec{v} = 0. \quad (2)$$

Poisson's equation for the gravitational potential

$$\nabla^2 U = -4\pi G\rho. \quad (3)$$

The idealized Ohm's law

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}). \quad (4)$$

$$\vec{\nabla} \cdot \vec{B} = 0. \tag{5}$$

The heat equation for perfect gas

$$\frac{1}{(\gamma-1)} \frac{dp}{dt} - \frac{\gamma}{(\gamma-1)} \frac{p}{\rho} \frac{d\rho}{dt} + \rho L - \vec{\nabla} \cdot (\lambda \vec{\nabla} T) = 0. \tag{6}$$

The equation of state

$$p - \rho RT = 0. \tag{7}$$

The parameters $G, \lambda, p, T, U, R, \gamma, \rho, \mathcal{L}, \hbar$, denote the gravitational constant, thermal conductivity, thermal pressure, temperature, gravitational potential, gas constant, adiabatic index, density of ionized component, radiative heat-loss function, Planck’s constant divided by 2π respectively. m_e and m_i are the electron and ion mass, respectively.

The pressure term in Eq. (1) contains both the Fermion pressure P_f and the thermal pressure

P_t . For low temperature plasma, the Fermi pressure $p_F = \left(\frac{4\pi^2 \hbar^2}{5m} \right) \left(\frac{3}{8\pi} \right)^{\frac{2}{3}} n^{\frac{5}{3}}$ is significant and

its contribution cannot be neglected. But for high temperature plasma thermal pressure dominates thus Fermi pressure can be neglected and the total pressure is simply taken as thermal pressure [27,28]. In the present analysis, we consider the case of high temperature radiative and thermally conducting plasmas, so the effect of Fermi pressure is not included in the study.

2.1 Linearized Perturbation Equations and Dispersion Relation

In the linearization, every space and time dependent quantities p, ρ, v, \vec{B} and U is supposed to have the following form

$$p = p_0 + \delta p, \quad \rho = \rho_0 + \delta \rho, \quad U = U_0 + \delta U, \quad \vec{B} = \vec{B}_0 + \delta \vec{B}, \quad \vec{v} = \vec{v}_0 + \delta \vec{v}. \tag{8}$$

The terms with subscript ‘0’ denote the unperturbed value while $\delta \vec{v} (v_x, v_y, v_z), \delta p, \delta \rho, \delta \vec{B} (B_x, B_y, B_z), \delta U$, denote the perturbation in fluid velocity, fluid pressure, fluid density, magnetic field and gravitational potential respectively.

The first order linearized perturbation equations obtained from (1) – (7), using equation (8), are

$$\rho \frac{\partial \delta \vec{v}}{\partial t} = -\vec{\nabla} \delta p + \rho \vec{\nabla} \delta U + \frac{1}{4\pi} (\vec{\nabla} \times \delta \vec{B}) \times \vec{B} + \frac{\hbar^2}{4m_e m_i} \vec{\nabla} (\nabla^2 \delta \rho) \tag{9}$$

$$\frac{\partial \delta \rho}{\partial t} + \rho \vec{\nabla} \cdot \delta \vec{v} = 0. \tag{10}$$

$$\nabla^2 \delta U = -4\pi G \delta \rho. \tag{11}$$

$$\frac{\partial \delta \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}). \tag{12}$$

$$\vec{\nabla} \cdot \delta \vec{B} = 0. \tag{13}$$

$$\frac{1}{(\gamma-1)} \frac{\partial \delta p}{\partial t} - \frac{\gamma}{(\gamma-1)} \frac{p}{\rho} \frac{\partial \delta \rho}{\partial t} + \rho (\mathcal{L}_\rho \delta \rho + \mathcal{L}_T \delta T) - \lambda \nabla^2 \delta T = 0. \tag{14}$$

$$\frac{\delta p}{p} = \frac{\delta T}{T} + \frac{\delta \rho}{\rho}. \tag{15}$$

In equation (14) the perturbation in temperature and radiative heat-loss function are given as δT , and $\delta \mathcal{L}$ respectively. $\mathcal{L}_{\rho,T}$ are the partial derivatives of the density dependent $(\partial \mathcal{L} / \partial \rho)_T$ and temperature dependent $(\partial \mathcal{L} / \partial T)_\rho$, heat-loss functions respectively.

We seek solutions of the above equation i.e., (9)-(15) whose dependence on space coordinates x, z -axis and time t is given by

$$\exp \{i(k_x x + k_z z + \omega t)\}. \tag{16}$$

where ω is the frequency of harmonic disturbances, k_x and k_z are wave numbers in x and z direction, respectively, where $k_x^2 + k_z^2 = k^2$ combining equation (14) and (15), we obtain the expression for δp as

$$\delta p = \left(\frac{\alpha + \sigma C^2}{\sigma + \beta} \right) \delta \rho, \tag{17}$$

where $\sigma = i\omega$, $C = \sqrt{\gamma p / \rho}$, is the adiabatic velocity of sound in the medium. The parameter α and β are given by

$$\alpha = (\gamma - 1) \left(\mathcal{L}_T T - \mathcal{L}_\rho \rho + \frac{\lambda k^2 T}{\rho} \right), \quad \beta = (\gamma - 1) \left(\frac{\mathcal{L}_T T \rho}{p} + \frac{\lambda k^2 T}{p} \right). \tag{18}$$

$V = (B / \sqrt{4\pi\rho})$, is the Alfvén velocity, $C^2 = \gamma C'^2$ where C and C' are the adiabatic and isothermal velocities of sound. We obtain the following matrix relation

$$X_{ij} Y_j = 0 \quad i, j = 1, 2, 3, 4 \tag{19}$$

where Y_j is a single column matrix with elements (v_x, v_y, v_z, s) and X_{ij} is fourth order matrix whose elements are

$$X_{11} = \left[\sigma + \frac{k^2 V^2}{\sigma} \right], \quad X_{12} = 0, \quad X_{13} = 0, \quad X_{14} = \frac{ik_x}{k^2} \left[\Omega_T^2 + \frac{\hbar^2 k^4}{4m_e m_i} \right], \quad X_{21} = 0, \quad X_{22} = \left[\sigma + \frac{k_z^2 V^2}{\sigma} \right]$$

$$X_{23} = 0, \quad X_{24} = 0, \quad X_{31} = 0, \quad X_{32} = 0, \quad X_{33} = \sigma, \quad X_{34} = \frac{ik_z}{k^2} \left[\Omega_T^2 + \frac{\hbar^2 k^4}{4m_e m_i} \right],$$

$$X_{41} = \frac{ik_x k^2 V^2}{\sigma}, \quad X_{42} = 0, \quad X_{43} = 0, \quad X_{44} = - \left[\sigma^2 + \Omega_T^2 + \frac{\hbar^2 k^4}{4m_e m_i} \right],$$

$$\Omega_i^2 = (k^2 \alpha - 4\pi G \rho \beta), \Omega_j^2 = (k^2 C^2 - 4\pi G \rho), \Omega_T^2 = \left(\frac{\sigma \Omega_j^2 + \Omega_i^2}{\sigma + \beta} \right).$$

Equation (19) has a nontrivial solution of the determinant of the matrix should vanish is to give the following dispersion relation.

$$\begin{aligned} & -\sigma \left[\left(\sigma + \frac{k^2 V^2}{\sigma} \right) \left(\sigma + \frac{k_z^2 V^2}{\sigma} \right) \left(\sigma^2 + \Omega_T^2 + \frac{\hbar^2 k^4}{4m_e m_i} \right) \right] \\ & + \left[\frac{k_x^2}{k^2} \left(\Omega_T^2 + \frac{\hbar^2 k^4}{4m_e m_i} \right) k^2 V^2 \left(\sigma + \frac{k_z^2 V^2}{\sigma} \right) \right] = 0 \end{aligned} \tag{20}$$

This dispersion relation shows the combined influence of quantum correction parameter, radiative heat-loss function and thermal conductivity on self-gravitating magnetized plasma. If as a first approximation, we neglect the thermal and radiative effects, then this dispersion relation reduces to Ren et al. [25] excluding resistivity in that case, and Prajapati and Chhajlani [26] in the absence of Hall current, viscosity and permeability. The most thoroughly investigated we shall examine the cases; the dispersion relation is modified by the presence of thermal conductivity and radiative heat-loss function in our case. Also in the absence of quantum correction, this dispersion relation coincides with Bora and Talwar [10] excluding the Hall current, resistivity and electron inertia effects in their case.

The effect mentioned above leads to a curious phenomenon. With these corrections, we find that the dispersion relation (20) is modified due the combined effects of thermal conductivity and radiative heat-loss functions. However, there is no doubt on the path of this goal that this dispersion relation will be able to predict the complete information about the waves and instabilities of the radiative quantum plasma considered and it will be able to explain the physical situations, like the instability of the galaxy and the problems related with star formation etc. After this lengthy dispersion relation, we precede to the brief study the effects of each parameter we have now reduced the dispersion relation (20) for the following special cases of wave propagation:

1. Longitudinal propagation $k_z = k$.
2. Transverse propagation $k_x = k$.

3. DISCUSSION

3.1 Longitudinal Propagation

For propagating along z-axis we have $k_x = 0$, and $k_z = k$. On substituting this, in equation (20), we get the following dispersion relation

$$\sigma \left(\sigma + \frac{k^2 V^2}{\sigma} \right)^2 \left(\sigma^2 + \Omega_T^2 + \frac{\hbar^2 k^4}{4m_e m_i} \right) = 0. \tag{21}$$

It is observed, from (21), that magnetic field and radiative heat-loss functions both have a separate independent mode. In the absence of radiative heat-loss function and thermal conductivity, this dispersion relation is similar to that of Prajapati, et al. [16], in the longitudinal mode of propagation, excluding Hall current, viscosity and permeability. In

addition, the present dispersion relation coincides with the dispersion relation obtained by Dangarh et al. [4], when the effect of quantum correction has been neglected. Dispersion relation (21) is the product of three independent factors; each describes different mode of wave propagation incorporating different parameters as discussed below. The first factor of (21) is $\sigma = 0$ and represents a natural stability of the system.

The second factor of the dispersion relation (21), equating to zero gives

$$\sigma^2 + k^2 V^2 = 0 \tag{22}$$

This gives the waves due to the magnetic field, which travels with Alfvén velocity and the third mode of propagation is given as

$$\sigma^3 + \sigma^2 \beta + \sigma \left(\Omega_j^2 + \frac{\hbar^2 k^4}{4m_e m_i} \right) + \Omega_j^2 + \left(\frac{\hbar^2 k^4}{4m_e m_i} \right) \beta = 0 \tag{23}$$

This dispersion relation represents a self-gravitating mode modified due to the presence of thermal conductivity, radiative heat-loss functions and quantum correction and not affected by magnetic field. It is clear from equation (23) that when constant term is less than zero (i.e. negative), then equation (23) will admit of at least one root of σ is positive (Appendix). Hence, the system is unstable. Thus for the cases of equation (23) the condition of instability is

$$k^2 \left(\mathcal{L}_T T - \mathcal{L}_\rho \rho + \frac{\lambda k^2 T}{\rho} \right) + \left(\frac{\mathcal{L}_T T \rho}{p} + \frac{\lambda k^2 T}{p} \right) \left[\frac{\hbar^2 k^4}{4m_e m_i} - 4\pi G \rho \right] < 0 \tag{24}$$

The above equation (24) represents the quantum corrected condition of radiative instability. Even the most rigorous attempt that this is the new condition of Jeans instability found in the present work. It is clear from the condition of instability (24) that the critical Jeans wave number is modified by the thermal conductivity, radiative heat-loss functions and the quantum correction parameter.

For thermally non-conducting, non-radiating, and neglecting the effect of quantum correction, we have $\alpha = \beta = Q = 0$, the dispersion relation (23) reduces to

$$\sigma^2 + (k^2 C^2 - 4\pi G \rho) = 0 \tag{25}$$

It is clear from (25) that, when $(k^2 C^2 - 4\pi G \rho) < 0$, the product of the roots of (25) and value of σ is positive, hence the system is unstable. Thus in the cases of (25), the condition of instability is

$$\Omega_j^2 = (k^2 C^2 - 4\pi G \rho) < 0$$

$$k < k_j = \left(\frac{4\pi G \rho}{C^2} \right)^{1/2} \tag{26}$$

Where k_j is the Jeans wave number. Equation (26) is an original Jeans expression for instability. The Jeans length is given as

$$\lambda_j = C \left(\frac{\pi}{G\rho} \right)^{1/2}. \tag{27}$$

The fluid is unstable for all Jeans length $\lambda > \lambda_j$, of Jeans wave number $k < k_j$. If we neglect the effect of quantum correction $Q = 0$, the dispersion relation (23) reduces to

$$\sigma^3 + \sigma^2 \beta + \sigma \Omega_j^2 + \Omega_i^2 = 0. \tag{28}$$

Consequently, means from a condition of instability (28) that the Jeans criterion of instability is modified due to inclusion of thermal conductivity and radiative heat-loss function. This condition of instability is similar to the condition of radiative instability earlier obtained by Prajapati et al. [16]. On comparing both equations (23) and (28), we found that the condition of radiative instability is modified by the effect of quantum correction.

The molecular clouds of the interstellar medium (ISM) is often unstable because of the instability of the gaseous plasma; thus, we study the effects of thermal conductivity, purely density-dependent heat-loss functions, purely temperature-dependent heat-loss functions and quantum correction parameter on the growth rate of instability by choosing the arbitrary values of these parameters in the present problem. We write the dispersion relation (23) in the non-dimensional form in terms of self-gravitation as

$$\sigma^{*3} + \sigma^{*2} \beta^* + \sigma^* (\Omega_j^{*2} + Q^* k^{*2}) + \Omega_i^{*2} + Q^* k^{*2} \beta^* = 0. \tag{29}$$

Where the various non-dimensional parameters are defined as

$$\sigma^* = \frac{\sigma}{\sqrt{4\pi G\rho}}, \quad k^* = \frac{kC}{\sqrt{4\pi G\rho}}, \quad Q^* = \frac{\hbar^2 k_j^2}{4m_e m_i}, \quad \lambda^* = \frac{(\gamma - 1) \Gamma \lambda \sqrt{4\pi G\rho}}{pC^2}, \quad \mathcal{L}_\rho^* = \frac{(\gamma - 1) \rho \mathcal{L}_\rho}{C^2 \sqrt{4\pi G\rho}},$$

$$\mathcal{L}_T^* = \frac{(\gamma - 1) \rho T \mathcal{L}_T}{\rho \sqrt{4\pi G\rho}}, \quad \alpha^* = \frac{1}{\gamma} (\mathcal{L}_T^* + \lambda^* k^{*2}) - \mathcal{L}_\rho^*, \quad \beta^* = (\mathcal{L}_T^* + \lambda^* k^{*2}), \quad \Omega_j^{*2} = (k^{*2} - 1),$$

$$\Omega_i^{*2} = (\alpha^* k^{*2} - \beta^*).$$

The growth rate is plotted against the non-dimensional wave number k^* with variation in the normalized the effect of density dependent radiative heat-loss function $\mathcal{L}_\rho^* = 0.0, 0.5, 1.0, 1.5$, the values of \mathcal{L}_T^* , Q^* and λ^* are taken 1.5 of each.

We pinpoint that the growth rate increases rapidly as the wave number increases. The growth rate decreases further with increases wave number. Hence, the density dependent heat-loss function has a destabilizing influence on the growth rate of instability.

The growth rate is plotted against the non-dimensional wave number k^* with variation in the normalized the effect of temperature dependent radiative heat-loss function $\mathcal{L}_T^* = 0.0, 0.5, 1.0, 1.5$, the values of \mathcal{L}_ρ^* , Q^* , and λ^* are taken 1.5 of each.

We find that the temperature dependent heat-loss function has a reverse effect on the growth rate compared to that of the density dependent heat-loss function parameter, in other words, due to an increase in the temperature dependent heat-loss function, the growth rate of the instability decreases. Hence, a temperature dependent heat-loss function can be made stable with increasing temperature dependent heat-loss function.

Growth rate is plotted against the non-dimensional wave number k^* with variation in the normalized the effect of quantum correction $Q^* = 0.0, 0.5, 1.0, 1.5$, the values of \mathcal{L}_T^* , \mathcal{L}_ρ^* , and λ^* are taken 1.5 of each

We find that the quantum plasma has a same effect on the growth rate compared to that of the temperature dependent heat-loss function parameter, in other words, due to an increase in the quantum plasma, the growth rate of the instability decreases.

In order to discuss the Jeans criterion of instability of the system for longitudinal propagation, it is unaffected by the magnetic field but modified by the effect of quantum correction for self-gravitational mode. From the Figure it is concluded that the equation (23); the growth rate of instability and from (24); the condition of radiative instability, we successfully detected that it is affected by quantum correction.

In the ISM, there are several cooling and heating mechanisms like free-free and free-bounding transitions, heat conduction, shock wave heating and cosmic ray heating, which depend, in a complicated way, on the local pressure and temperature. For radiative cooling, the heat-loss function depends on density and temperature but there is no single definite heat-loss function holds for an ISM, one can consider only non-dimensional parameters and discuss numerically the influence of their nonzero values on the ranges and growth rate of instability.

Numerical computation of roots of the dispersion relation (29) was done for a set of values of the parameters involved. The results are shown in Figs. 1-3. To put the assumptions graphically, we find that from Fig. 1 that the density-dependent heat-loss has a destabilizing influence on the growth rate of instability, which is reduced by quantum correction. From Fig. 2, we find that temperature-dependent heat-loss function has a stabilizing influence on the growth rate of instability, which is further increased by quantum correction.

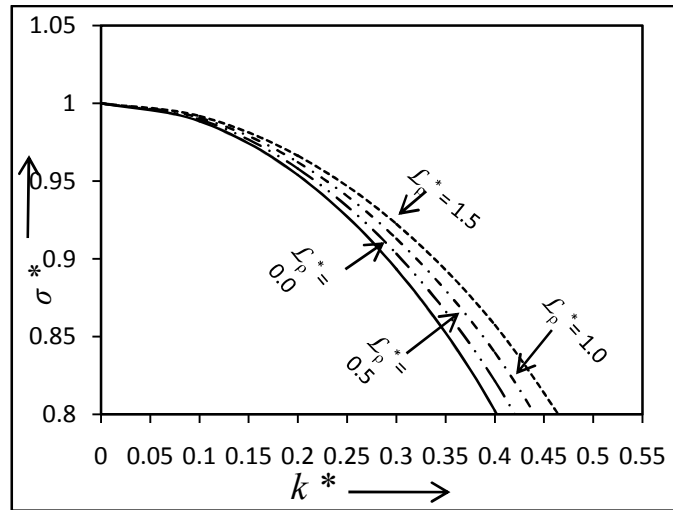


Fig. 1. Effect of density dependent radiative heat-loss function

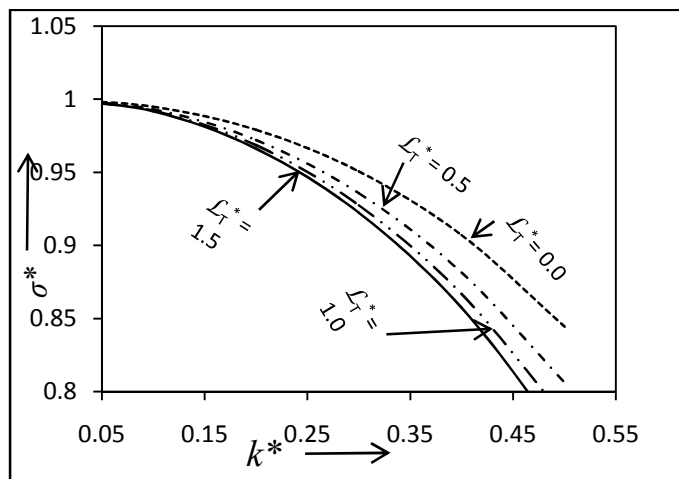


Fig. 2. Effect of temperature dependent radiative heat-loss function

Whereas from Fig. 3, we conclude that growth rate decreases with increasing quantum correction. Thus, the effect of quantum correction is stabilizing and which is reduced by density dependent heat-loss function and increased by temperature dependent heat-loss function. One can be found that thermal conductivity has a destabilizing influence. Owing to inclusion of thermal conductivity, the isothermal sound velocity is replaced by the adiabatic velocity of sound. We also obtain a non-gravitating Alfvén mode, which is not affected by quantum correction.

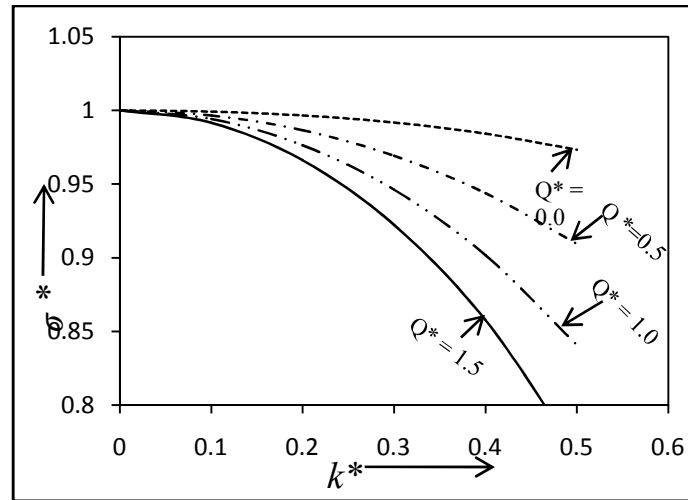


Fig. 3. Effect of quantum correction

3.2 Transverse Propagation

For propagating along x-axis we have $k_z = 0$, and $k_x = k$. On substituting this equation (20), we get the following dispersion relation

$$\sigma^3 \left[\sigma^2 + \left(\Omega_T^2 + \frac{\hbar^2 k^4}{4m_e m_i} \right) + k^2 V^2 \right] = 0 \quad (30)$$

The dispersion relation (30) shows the combined influence of thermal conductivity arbitrary radiative heat-loss functions, magnetic field and quantum correction parameter on the self-gravitational instability of the homogeneous fluid plasma. Thus equation (30) represents a self-gravitating Alfvén mode modified by quantum correction conductivity and arbitrary radiative heat-loss functions and it is the product of two independent factors. These factors show the mode of propagating incorporating different parameters. The first mode of propagating is spurious stable mode and the second mode of transverse mode of propagation is given as

$$\sigma^3 + \sigma^2 \beta + \sigma \left(\Omega_j^2 + \frac{\hbar^2 k^4}{4m_e m_i} + k^2 V^2 \right) + \left\{ \Omega_i^2 + k^2 V^2 \beta + \left(\frac{\hbar^2 k^4}{4m_e m_i} \right) \beta \right\} = 0 \quad (31)$$

The above equation shows the radiative mode modified by the inclusion of quantum effect. The condition of instability is obtained from the constant term of (33) which is given as

$$\left\{ \Omega_i^2 + k^2 V^2 \beta + \left(\frac{\hbar^2 k^4}{4m_e m_i} \right) \beta \right\} > 0 \quad (32)$$

The condition (32) represents condition of radiative instability and it is modified due to magnetic field, and quantum correction. It is clear that magnetic field and quantum correction

parameter decrease the value of critical wave number. Hence, magnetic field and quantum correction have a stabilizing influence on the configuration.

On comparing equation (24) and (32), we found that the condition of radiative instability is modified due to the consideration of quantum effect in both longitudinal and transverse direction of propagation, while the magnetic field affects the radiative instability condition only in transverse direction.

In the transverse propagation, we obtain two separate modes of propagation, one of them is a spurious stable mode which is unaffected by all parameters involved in the systems. The second mode is self-gravitating Alfvén mode influenced by quantum correction, thermal conductivity and radiative heat-loss function. It is clear that the condition of radiative instability is dependent on quantum correction and the magnetic field. From the dispersion relation (31) we find that the coefficient of σ in dispersion relation has quantum correction, thermal conductivity, magnetic field and radiative heat-loss functions, thus the growth rate is affected by the presence of these parameters.

From the preceding discussion, it is apparent that for the wave propagation, parallel to the direction magnetic field, we get three independent modes but in the direction perpendicular to the magnetic field we get a spurious stable mode and Alfvén waves due to neutral interaction of sonic and magnetic waves under the influence of thermal conductivity and radiative heat-loss functions. For Alfvén mode the condition of instability is given by equations (32) in which along with the sonic speed, modified Alfvén velocity due to magnetic field, thermal conductivity, radiative heat-loss functions and quantum correction are also introduced.

Thus, it is well to review that magnetic field, quantum correction, thermal conductivity and radiative heat-loss functions affect the Jeans criterion in perpendicular direction to the magnetic field but in parallel direction to the magnetic field, the medium behaves as if it is non-magnetized for instability considerations.

4. CONCLUSION

The result of this work is that, with a suitable interpretation of the parameters involved, that quantum correction has a predominantly stabilizing influence on the growth rate of the system. In this colloquium, we obtain a general dispersion relation, which is modified due to the presence of these parameters. This dispersion relation is reduced for longitudinal and transverse modes of propagation. In the case of longitudinal propagation, the gravitating mode modified due to presence of thermal conductivity, radiative heat-loss function and quantum correction.

From the graphical packages mentioned above, we perceive that the density dependent heat-loss functions have a destabilizing role on the growth rate of the system. Quantum correction and temperature dependent heat-loss functions, shows the stabilizing effect on the growth rate of instability. It is easy to extend a result that the quantum correction and temperature dependent heat-loss function stabilizes the magnetized gaseous plasma system.

In the transverse mode of propagation, we find a gravitating thermal mode influenced by thermal conductivity and radiative heat-loss functions, magnetic field and quantum correction. For the aforementioned reason, there is no doubt that instability criterion is

adapted due to the presence of a magnetic field, thermal conductivity, quantum correction and radiative heat-loss functions.

The results of the present analysis may be useful to understand the problem of wave propagation and Jeans instability in self-gravitating dense strongly correlated systems (astrophysical plasma, inertial confinement plasma, laser produced plasma, semiconductor plasma). The results of the present study are applicable in the understanding of the formation of white dwarf star and neutron star.

Dust impurities exist in the quantum plasma, forming a quantum dusty plasma thus the present work can be further extended, in dusty plasma environment, considering the other non-ideal effects viz. spin magnetization, Hall current, Rotation, finite ion Larmor radius corrections which play significant role in the protoplanet formation.

ACKNOWLEDGEMENT

The authors would like to thank the referees for their useful suggestions and comments that improved the original manuscript.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Jeans JH. The Stability of Spherical Nebula. *Phil. Trans. Roy. Soc. London* 1902:A199;1-53
2. Chandrashekhar S. *Hydrodynamics and hydromagnetic stability*. Oxford: Clarendon Press; 1961.
3. Pensia RK, Prajapat V, Kumar V, Patidar AK. Effect of electron inertia and permeability on Jeans instability of viscous uniformly magnetized gaseous plasma in the presence of suspended particles. *Adv. Studies Theor. Phys.* 2011;5(15):743-54.
4. Dangarh BK, Pensia RK, Shrivastava V, Prajapat V. Analysis of Jeans instability of partially ionized molecular cloud under influence of radiative effect and electron inertia. *Adv. Studies Theor. Phys.* 2011;5(16):755–64.
5. Ali S, Shukla PK. Jeans instability in a plasma with positive-negative charged and neutral dust components. *Phys. Scr.* 2006;73:359.
6. Shaikh S, Khan A, Bhatia PK. Jeans gravitational instability of a thermally conducting plasma. *Physics Lett. A.* 2008;372:1451.
7. Field GB. Thermal Instability. *Astrophys. J.* 1965;142:531.
8. Aggarwal M, Talwar SP. Magneto-thermal instability in a rotating gravitating Fluid. *Mon. Not. Roy. Astr. Soc.* 1969;146:235.
9. Aggrawal M, Talwar SP. Equilibrium of rotating degenerate fluid cylinders. *Pub. Astron Soc. Japan.* 1969;21:176.
10. Bora MP, Talwar SP. Magneto-thermal instability with generalized ohm's law. *Phys. Fluids B.* 1993;5(3):950.
11. Talwar SP, Bora MP. Thermal instability in a star gas system. *J. Plasma Phys.* 1995;54(2):157-172.

12. Vyas MK, Chhajlani RK. Gravitational instability of thermally-conducting plasma flowing through a porous medium in the presence of suspended particles. *Astrophys. Space Sci.* 1988;149:323.
13. Chhajlani RK, Parihar AK. Stability of self-gravitating magnetized Hall plasma with electrical and thermal conductivity through porous medium. *Contrib. Plasma Phys.* 1993;33(3):227-234.
14. Wolfire MG, Hollenbach D, McKee CF, Tilens AGGM, and Baker ELO. The neutral atomic phases of the interstellar medium. *Astrophys. J.* 1995;443:152-168.
15. Shadmehari M, Dib S. Magneto-thermal condensation modes including the effects of charged dust particles. *Mon. Not. Roy. Astron. Soc.* 2009;395:985-990.
16. Prajapati RP, Pensia RK, Kaothekar S, Chhajlani RK. Self-gravitational instability of rotating viscous Hall plasma with arbitrary radiative heat-loss functions and electron inertia. *Astrophys. Space Sci.* 2010;327:139-154.
17. Patidar AK, Pensia RK, Shrivastava V. Effect of electron inertia on radiative instability of rotating two-component gaseous plasma. *Can. J. Phys.* 2012;90:1209.
18. Pines D, Classical and quantum plasmas, *J. Nucl. Energy: Part C: Plasma Phys.* 1961;2:5.
19. Pines D. Elementary excitations in solid. New York: Westview Press; 1999.
20. Bezzerides B, DuBios DF. Quantum electrodynamics of nonthermal relativistic plasmas: kinetic theory. *Ann. Phys.* 1972;70:10-66.
21. Masood W, Sallimullah M, Shah HA. A Quantum hydrodynamic model for multicomponent quantum magneto-plasma with Jeans term. *Phys. Lett. A* 2008;372:6757-6760.
22. Sallimullah M, Jamil M, Shah HA, Murtaza G. Jeans instability in a quantum dusty magneto-plasma. *Phys. Plasmas.* 2009;16:014502.
23. Bashir MF, Jamil M, Murtaza G, Salimullah M, Shah HA. Stability analysis of self-gravitational electrostatic drift waves for a streaming non uniform quantum dusty magneto plasma *Phys. Plasmas.* 2012;19:043701. Available on: <http://dx.doi.org/10.1063/1.3698165>.
24. Haas FA Magneto-hydrodynamic model for Quantum plasma, *Phys. Plasmas* 2005;12:062117.
25. Ren H, Wu Z, Cao J, Chu PK. Jeans instability in quantum magneto plasma with resistive effects. *Phys. Plasmas.* 2009;16:072101.
26. Prajapati RP, Chhajlani RK. Effect of Hall current on the Jeans instability of magnetized viscous quantum plasma. *Phys. Scr.* 2010;82:055003.
27. Lundin J, Marklund M, Brodin G. Modified Jeans instability criteria for magnetized systems. *Phys. Plasmas* . 2008;15:072116 doi:10.1063/1.2956641.
28. Ren H, Wu Z, Cao J, Chu PK. Electrostatic drift modes in quantum dusty plasmas with Jeans terms. *Phys. Plasmas.* 2009;16;103705.

APPENDIX

We applied the method, used by several authors [4,12,16,17] for determining whether a linear system is stable or not by examining the sign of the roots of the characteristic equation of the system. If the dispersion relations obtained from the linear - perturbation analysis is a polynomial of the form

$$f(w) = a_0w^n + a_1w^{n-1} + a_2w^{n-2} + \dots + a_n = 0$$

If the sign of $a_n = -1$,

So that the sign of $[f(0)] = -1$,

And because of $\lim_{w \rightarrow +\infty} f(w) = +\infty$

Thus at least one positive real root exist which renders the system unstable.

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