Asian Journal of Probability and Statistics

Volume 26, Issue 8, Page 9-27, 2024; Article no.AJPAS.117052
ISSN: 2582-0230

Efficiency of Modified Exponential Dual to Ratio-Product-Cum Type Estimator under Stratified Sampling Using Two Auxiliary Variables

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information

DOI: https://doi.org/10.9734/ajpas/2024/v26i8635

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/117052

Received: 01/05/2024
Accepted: 22/05/2024
Published: 19/07/2024

Original Research Article

Abstract

Separate and Combined dual to ratio-product-cum estimators of the population mean under stratified random sampling scheme are suggested using the ideas and the analogy of (Walsh [1], Reddy [2], Tripathi [3]) and adopted strategy initiated by (Koyuncu and Kadilar [4], Singh et al., [5], Yasmeen et al., [6], Rather and Kadilar [7]) as well as the two auxiliaries variable. Asymptotic properties of proposed estimators such as BIASs, MSEs and MMSEs up to first order of approximation by Tailor series approach were deduced and reported. Performance is evaluated and examined with others related estimators considered using empirical study utilized two natural populations and simulated data sets. The statistical package R plus is used for computations. However, Results eventually indicated the superiority of proposed estimators over existing traditional estimators mentioned in studied.

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Cite as: *Suleiman, Haruna, Jibrin Abdullahi Yafu, and Audu Ahmed. 2024. "Efficiency of Modified Exponential Dual to Ratio-Product-Cum Type Estimator under Stratified Sampling Using Two Auxiliary Variables". Asian Journal of Probability and Statistics 26 (8):9-27. https://doi.org/10.9734/ajpas/2024/v26i8635.*

Keywords: Auxiliary variable; dual to ratio-product-cum type; ratio-product-cum type; stratum; stratified random sampling; study variable; optimum values.

1 Introduction

The modified exponential dual to Ratio-Product-Cum Type (RPCT) estimator using stratified random sampling under two auxiliary variables is a statistical method for estimating population parameters. It is designed to be efficient in situations where two auxiliary variables are available and can be used to improve the accuracy of the estimation process. The efficiency of the modified exponential dual to RPCT estimator depends on several factors, including the sample size, the distribution of the population, the correlation between the auxiliary variables and the target variable, and the sampling design. In general, if these factors are favorable, the estimator can be very efficient and provide accurate estimates of the population parameters. One advantage of the modified exponential dual to RPCT estimator is that it can be used with non-normal populations and can still produce accurate estimates. This makes it a valuable tool for researchers who work with populations that do not conform to normal distributions. Overall, the efficiency of the modified exponential dual to RPCT estimator using stratified random sampling under two auxiliary variables will depend on the specific circumstances of the study. It is important to carefully consider the sampling design, the choice of auxiliary variables, and the size of the sample to ensure that the estimator is used in the most effective way possible.

Numerous effort have been carry out to obtain practical solution which gives approximation optimum stratification, namely: Equalization of $W_h\sigma_h$, Equalization of $W_h\mu_h$, Equalization of $W_h\frac{1}{2}[r(y)+f(y)]$, Equalization of cumulative(cum) of $\sqrt{f(y)}$, Equalization of Cum $\sqrt[3]{f(y)}$ and Equalization of $W_h(y_h-y_{h-1})$ all this rules was founded by scholars Dalenius and Gurney [8], Mahalanobis [9], Durbin [10], Aoyama [11], Dalenius and Hodges Jr [12], Thomsen [13] and Ekman [14] respectively. The major concerned here is the Equalization of cumulative(cum)$\sqrt{f(y)}$ which is original proposed by Dalenius and Hodges Jr[12] under assumption that the distribution is bounded and the numbers of strata is large it is a simplification of optimum stratification for Neyman allocation and founded to be more efficient if stratifying variable come in class interval or can be formed into classes. in this context reader is referred to [Page 126]Okafor [15] for more detail.

At the estimation stage, the use of the supplementary variable started with the effort of Cochran [16]. Murthy [17], Chand [18] who developed two chain-ratio estimators presented a dual estimator Bahl and Tuteja [19] were the pioneer of exponential type estimators Srivenkataramana [20] and Bandyopadhyay [21] are the one who break new ground on Dual transformation Ahmed et al., [22] developed class of ratio estimators with known function of auxiliary variable for estimating finite population variance Audu et al., [23] suggested Difference-Cum-Ratio estimators for estimating finite population coefficient of Variation in Simple Random Sampling. but there are some many practical situation when the an auxiliary information is quantitative in nature, that is auxiliary variables is available in the form of an attribute such as sex, beaut and sweat so many estimator is proposed on this unlike Naik and Gupta [24], Shabbir and Gupta [25], Solanki and Singh [26], Singh and Audu [27] also with condition when population coefficients variation C_P, kurtosis $\beta_2(\Phi)$, correlation ρ_{pb} both variation C_P kurtosis $\beta_2(\Phi)$ are known is proposed by (Abd-Elfattah et al., [28]).

Consider $U=(U_1,U_2,U_3,...,U_N)$ be finite population of size N and it divided into L homogeneous of strata size $N_h(h=1,2,...L)$. A sample n_h is drawn each stratum using simple random sampling without replacement. Let y be the study variable and x and z be the auxiliary variables

$\bar{Y}_h^s=\frac{1}{N_h}\sum_{h=1}^{L}y_{hi}$: h^{th} Population mean of the study variate y in h^{th} stratum.
$\bar{X}_h^s=\frac{1}{N_h}\sum_{h=1}^{L}x_{hi}$: h^{th} Population mean of the auxiliary variate x in h^{th} stratum.
$\bar{Z}_h^s=\frac{1}{N_h}\sum_{h=1}^{L}z_h$: h^{th} Population mean of the auxiliary variate z in h^{th} stratum.
$\bar{Y}^c=\bar{Y}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h}y_h=\sum_{h=1}^{L}W_h\bar{Y}_h$: Population mean of the study variate y.
$\bar{X}^c=\bar{X}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h}x_h=\sum_{h=1}^{L}W_h\bar{X}_h$: Population mean of the auxiliary variate x.
$\bar{Z}^c=\bar{Z}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N}z_h=\sum_{h=1}^{L}W_h\bar{Z}_h$: Population mean of the auxiliary variate z.
$\bar{y}_h^s=\frac{1}{n_h}\sum_{h=1}^{L}y_h$: h^{th} sample mean of the study variate y in h^{th} stratum.
$\bar{x}_h^s=\frac{1}{n_h}\sum_{h=1}^{L}x_h$: h^{th} sample mean of the auxiliary variate x in h^{th} stratum.

$\bar{z}_h^s = \frac{1}{n_h}\sum_{h=1}^{L} z_h$: h^{th} sample mean of the auxiliary variate z in h^{th} stratum.
$\bar{x}_*^s$ dual separate transform variable of the auxiliary variate y in h^{th} stratum.
$\bar{z}_*^s$ dual separate transform variable of the auxiliary variate y in h^{th} stratum.
$\bar{x}_*^c$ dual combined transform variable of the auxiliary variate y in h^{th} stratum.
$\bar{z}_c^*$ dual combined transform variable of the auxiliary variate y in h^{th} stratum.
$S_{yh}^2 = \sum_L^{N_h}\left(y_{hi} - \bar{Y}_h\right)^2 (N_h - 1)^{-1}$ population variance in each h^{th} stratum y
$S_{xh}^2 = \sum_L^{N_h}\left(x_{hi} - \bar{X}_h\right)^2 (N_h - 1)^{-1}$ population variance in each h^{th} stratum x
$S_{zh}^2 = \sum_L^{N_h}\left(z_{hi} - \bar{Z}_h\right)^2 (N_h - 1)^{-1}$ population variance in each h^{th} stratum z
$S_{yxh} = \sum_L^{N_h}\left(x_{hi} - \bar{X}_h\right)\left(y_{hi} - \bar{Y}_h\right)(N_h - 1)^{-1}$ population variance in each h^{th} stratum x and y.
$S_{yzh} = \sum_L^{N_h}\left(y_{hi} - \bar{Y}_h\right)\left(z_{hi} - \bar{Z}_h\right)(N_h - 1)^{-1}$ population variance in each h^{th} stratum y and z
$S_{xzh} = \sum_L^{N_h}\left(x_{hi} - \bar{X}_h\right)\left(z_{hi} - \bar{Z}_h\right)(N_h - 1)^{-1}$ population variance in each h^{th} stratum x and z

2 Pre-existing Estimators

2.1 Stratified random sampling based on ratio and product estimator

Usual separate, combined and product ratio estimators envisaged by Kadilar and Cingi [29] is $\hat{\bar{Y}}_{11}^{ST}$ and Hansen et al., [30] are $\hat{\bar{Y}}_{12}^{ST}$ $\hat{\bar{Y}}_{13}^{ST}$ all in population $\bar{Y}$ in stratified random sampling estimators are defined respectively as

$$\hat{\bar{Y}}_{11}^{ST} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h^s}{\bar{x}_h^s}\right) \tag{1}$$

$$\hat{\bar{Y}}_{12}^{ST} = \bar{y}_c \left(\frac{\sum_{h=1}^{L} W_h \bar{X}_h^c}{\sum_{h=1}^{L} W_h \bar{x}_h^c}\right) \tag{2}$$

$$\hat{\bar{Y}}_{13}^{ST} = \bar{y}_c \left(\frac{\sum_{h=1}^{L} W_h \bar{z}_h^c}{\sum_{h=1}^{L} W_h \bar{Z}_h^c}\right) \tag{3}$$

Approximated BIASs and MSEs of the estimators in 1, 2 and 3 are give as

$$Bias(\hat{\bar{Y}}_{11}^{ST}) = \sum_{h=1}^{L} W_h \lambda_h \left(R_{xh} S_{xh}^2 - S_{yxh}\right) \bar{X}_h^{-1} \tag{4}$$

$$Bias(\hat{\bar{Y}}_{12}^{ST}) = \sum_{h=1}^{L} W_h \lambda_h \left(R_{xc} S_{xh}^2 - S_{yxh}\right) \bar{X}^{-1} \tag{5}$$

$$Bias(\hat{\bar{Y}}_{13}^{ST}) = \sum_{h=1}^{L} W_h \lambda_h S_{yzh} \bar{Z}^{-1} \tag{6}$$

where $R_{xh} = \frac{\bar{Y}_h^s}{\bar{X}_h^s}$, $R_{zh} = \frac{\bar{Y}_h^s}{\bar{Z}_h^s}$ and $R_{xc} = \frac{\bar{Y}^c}{\bar{X}^c}$ $R_{zc} = \frac{\bar{Y}^c}{\bar{Z}^c}$

$$MSE(\hat{\bar{Y}}_{11}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + R_{xh}^2 S_{xh}^2 + 2R_{xh} S_{yxh}\right) \tag{7}$$

$$MSE(\hat{\bar{Y}}_{12}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + R_{xc}^2 S_{xh}^2 - 2R_{xc} S_{yxh}\right) \tag{8}$$

$$MSE(\hat{\bar{Y}}_{13}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + R_{zc}^2 S_{zh}^2 + 2R_{zc} S_{yzh}\right) \tag{9}$$

Motivated by Singh(1967), Koyuncu and Kadilar [4] had suggested ratio cum ratio , product cum product, ratio cum product and product cum ratio estimators of population mean are given in eq 10, 11, 12 and 13 in stratified random sampling $\bar{Y}$ as follow

$$\hat{\bar{Y}}_{21}^{ST} = \bar{y}_c \left(\frac{\sum_{h=1}^{L} W_h \bar{X}_h^c}{\sum_{h=1}^{L} W_h \bar{x}_h^c} \right) \left(\frac{\sum_{h=1}^{L} W_h \bar{Z}_h^c}{\sum_{h=1}^{L} W_h \bar{z}_h^c} \right) \tag{10}$$

$$\hat{\bar{Y}}_{22}^{ST} = \bar{y}_c \left(\frac{\sum_{h=1}^{L} W_h \bar{x}_h^c}{\sum_{h=1}^{L} W_h \bar{X}_h^c} \right) \left(\frac{\sum_{h=1}^{L} W_h \bar{z}_h^c}{\sum_{h=1}^{L} W_h \bar{Z}_h^c} \right) \tag{11}$$

$$\hat{\bar{Y}}_{23}^{ST} = \bar{y}_c \left(\frac{\sum_{h=1}^{L} W_h \bar{X}_h^c}{\sum_{h=1}^{L} W_h \bar{x}_h^c} \right) \left(\frac{\sum_{h=1}^{L} W_h \bar{z}_h^c}{\sum_{h=1}^{L} W_h \bar{Z}_h^c} \right) \tag{12}$$

$$\hat{\bar{Y}}_{24}^{ST} = \bar{y}_c \left(\frac{\sum_{h=1}^{L} W_h \bar{x}_h^c}{\sum_{h=1}^{L} W_h \bar{X}_h^c} \right) \left(\frac{\sum_{h=1}^{L} W_h \bar{Z}_h^c}{\sum_{h=1}^{L} W_h \bar{z}_h^c} \right) \tag{13}$$

The BIASs and MSEs of the estimators $\hat{\bar{Y}}_{21}^{ST}$, $\hat{\bar{Y}}_{22}^{ST}$, $\hat{\bar{Y}}_{23}^{ST}$ and $\hat{\bar{Y}}_{24}^{ST}$ under stratified random sampling are given below in eq 14, 15, 16, 17 and 18, 19, 20, 21 respectively

$$Bias\left(\hat{\bar{Y}}_{21}^{ST}\right) = \sum_{h=1}^{L} W_h \lambda_h \left(R_{xc}^2 S_{xh}^2 + R_{zc}^2 S_{zh}^2 + R_{xc} R_{zc} S_{xzh} - R_{xc} S_{yxh} - R_{zc} S_{yzh} \right) \bar{Y}^{-1} \tag{14}$$

$$Bias\left(\hat{\bar{Y}}_{22}^{ST}\right) = \sum_{h=1}^{L} W_h \lambda_h \left(R_{xc} R_{zc} S_{xzh} + R_{xc} S_{yxh} + R_{zc} S_{yzh} \right) \bar{Y}^{-1} \tag{15}$$

$$Bias\left(\hat{\bar{Y}}_{23}^{ST}\right) = \sum_{h=1}^{L} W_h \lambda_h \left(R_{xc}^2 S_{xh}^2 - R_{xc} R_{zc} S_{xzh} - R_{xc} S_{yxh} + R_{zc} S_{yzh} \right) \bar{Y}^{-1} \tag{16}$$

$$Bias\left(\hat{\bar{Y}}_{24}^{ST}\right) = \sum_{h=1}^{L} W_h \lambda_h \left(R_{zc}^2 S_{zh}^2 - R_{xc} R_{zc} S_{xzh} + R_{xc} S_{yxh} \right) \bar{Y}^{-1} \tag{17}$$

$$MSE(\hat{\bar{Y}}_{21}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + R_{xc}^2 S_{xh}^2 + R_{zc}^2 S_{zh}^2 + 2R_{xc} R_{zc} S_{xzh} - 2R_{xc} S_{yxh} - 2R_{zc} S_{yzh} \right) \tag{18}$$

$$MSE(\hat{\bar{Y}}_{22}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + R_{xc}^2 S_{xh}^2 + R_{zc}^2 S_{zh}^2 + 2R_{xc} R_{zc} 2S_{xzh} + 2R_{xc} S_{yxh} + 2R_{zc} S_{yzh} \right) \tag{19}$$

$$MSE(\hat{\bar{Y}}_{23}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + R_{xc}^2 S_{xh}^2 + R_{zc}^2 S_{zh}^2 - 2R_{xc} R_{zc} S_{xzh} - 2R_{xc} S_{yxh} + 2R_{zc} S_{yzh} \right) \tag{20}$$

$$MSE(\hat{\bar{Y}}_{24}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + R_{xc}^2 S_{xh}^2 + R_{zc}^2 S_{zh}^2 + 2R_{xc} R_{zc} S_{xzh} + 2R_{xc} S_{yxh} - 2R_{zc} S_{yzh} \right) \tag{21}$$

Vishwakarma et al.,[31] proposed separate estimator of population mean using multi-auxiliary variate under post stratification the proposed estimator is defined be as

$$\hat{\bar{Y}}_{31}^{ST} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\vartheta_h \frac{\bar{X}_h^s}{\bar{x}_h^s} + (1 - \vartheta_h) \frac{\bar{z}_h^s}{\bar{Z}_h^s} \right) \tag{22}$$

where ϑ of the estimator 22 is the constant parameter to minimized MSE of the estimator $\hat{\bar{Y}}_{31}^{ST}$, the equation of the BIAS, MSE and MMSE up to first term approximation are given as below.

$$Bias(\hat{\bar{Y}}_{31}^{ST}) = \sum_{h=1}^{L} W_h \lambda_h \left(\vartheta_h R_{xh}^2 S_{xh}^2 - \vartheta_h R_{xh} S_{yxh} - (1-\vartheta_h) R_{zh} S_{yzh} \right) \bar{Y}_h^{-1} \tag{23}$$

$$MSE(\hat{\bar{Y}}_{31}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + R_{xh}^2 S_{xh}^2 + (1-\vartheta_h)^2 R_{zh}^2 S_{zh}^2 - 2(1-\vartheta_h) R_{xh} R_{zh} S_{xzh} - 2\vartheta_h R_{xh} S_{yxh} + 2(1-\vartheta_h) R_{zh} S_{yzh} \right) \tag{24}$$

$$\vartheta_h min = \left(\frac{R_{zh} S_{zh}^2 - R_{xh} R_{zh} S_{xzh} + R_{xh} S_{yxh} + R_{zh} S_{yzh}}{R_{xh}^2 S_{xh}^2 + R_{zh}^2 S_{zh}^2} \right) \tag{25}$$

2.2 Stratified random sampling based on exponential ratio and product estimator

Singh et al., [32] defined exponential ratio product type estimator which is on stratified random sampling motivated from Bahl and Tuteja [19] which is on simple random sampling of the population mean $\bar{Y}$. The Ratio exponential estimator in stratified random sampling is given by

$$\hat{\bar{Y}}_{41}^{ST} = \bar{y}_c \exp \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h^c - \bar{x}_h^c \right)}{\sum_{h=1}^{L} W_h \left(\bar{X}_h^c + \bar{x}_h^c \right)} \right) \tag{26}$$

$$\hat{\bar{Y}}_{42}^{ST} = \bar{y}_c \exp \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{z}_h^c - \bar{Z}_h^c \right)}{\sum_{h=1}^{L} W_h \left(\bar{z}_h^c + \bar{Z}_h^c \right)} \right) \tag{27}$$

BIASs and MSEs of the modified ratio exponential estimator $\bar{Y}_{41}$ and product exponential estimator $\bar{Y}_{42}$ under stratified random sampling are given as

$$Bias\left(\hat{\bar{Y}}_{41}^{ST}\right) = \sum_{h=1}^{L} W_h \lambda_h \left(\frac{3}{8} R_{xc} S_{xh}^2 - \frac{1}{2} S_{yxh} \right) \bar{X}^{-1} \tag{28}$$

$$Bias\left(\hat{\bar{Y}}_{42}^{ST}\right) = \sum_{h=1}^{L} W_h \lambda_h \left(-\frac{1}{8} R_{zc} S_{zh}^2 + \frac{1}{2} S_{yzh} \right) \bar{Z}^{-1} \tag{29}$$

$$MSE(\hat{\bar{Y}}_{41}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + R_{xc}^2 \frac{S_{xh}^2}{4} - R_{xc} S_{yxh} \right) \tag{30}$$

$$MSE(\hat{\bar{Y}}_{42}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + R_{zc}^2 \frac{S_{zh}^2}{4} + R_{zc} S_{yzh} \right) \tag{31}$$

Singh et al.,[5] motivated by Singh (1965,1967) Introduced various Exponential Ratio cum Ratio, product cum product, Ratio cum product and product cum Ratio type estimators of finite population mean in stratified random sampling

$$\hat{\bar{Y}}_{51}^{ST} = \bar{y}_c exp \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h^c - \bar{x}_h^c \right)}{\sum_{h=1}^{L} W_h \left(\bar{X}_h^c + \bar{x}_h^c \right)} \right) exp \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{Z}_h^c - \bar{z}_h^c \right)}{\sum_{h=1}^{L} W_h \left(\bar{Z}_h^c + \bar{z}_h^c \right)} \right) \tag{32}$$

$$\hat{\bar{Y}}_{52}^{ST} = \bar{y}_c exp \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{x}_h^c - \bar{X}_h^c \right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_h^c + \bar{X}_h^c \right)} \right) exp \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{z}_h^c - \bar{Z}_h^c \right)}{\sum_{h=1}^{L} W_h \left(\bar{z}_h^c + \bar{Z}_h^c \right)} \right) \tag{33}$$

$$\hat{\bar{Y}}_{53}^{ST} = \bar{y}_c exp \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h^c - \bar{x}_h^c \right)}{\sum_{h=1}^{L} W_h \left(\bar{X}_h^c + \bar{x}_h^c \right)} \right) exp \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{z}_h^c - \bar{Z}_h^c \right)}{\sum_{h=1}^{L} W_h \left(\bar{z}_h^c + \bar{Z}_h^c \right)} \right) \tag{34}$$

$$\hat{\bar{Y}}_{54}^{ST} = \bar{y}_c exp \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{x}_h^c - \bar{X}_h^c \right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_h^c + \bar{X}_h^c \right)} \right) exp \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{Z}_h^c - \bar{z}_h^c \right)}{\sum_{h=1}^{L} W_h \left(\bar{Z}_h^c + \bar{z}_h^c \right)} \right) \tag{35}$$

The expressions for Biases (BIASs) and Mean Square Errors (MSEs) of the estimators in 32, 33, 34 and 35 up to second degree approximation are as follow

$$Bias\left(\hat{\bar{Y}}_{51}^{ST}\right) = \sum_{h=1}^{L} W_h\lambda_h\left(\frac{3}{8}R_{xc}^2S_{xh}^2 + \frac{3}{8}R_{zc}^2S_{zh}^2 + \frac{1}{4}R_{xc}R_{zc}S_{xzh} - \frac{1}{2}R_{xc}S_{yxh} - \frac{1}{2}R_{zc}S_{yzh}\right)\bar{Y}^{-1} \tag{36}$$

$$Bias\left(\hat{\bar{Y}}_{52}^{ST}\right) = \sum_{h=1}^{L} W_h\lambda_h\left(-\frac{1}{8}R_{xc}^2S_{xh}^2 - \frac{1}{8}R_{zc}^2S_{zh}^2 + \frac{1}{4}R_{xc}R_{zc}S_{xzh} + \frac{1}{2}R_{xc}S_{yxh} + \frac{1}{2}R_{zc}S_{yzh}\right)\bar{Y}^{-1} \tag{37}$$

$$Bias\left(\hat{\bar{Y}}_{53}^{ST}\right) = \sum_{h=1}^{L} W_h\lambda_h\left(-\frac{3}{8}R_{xc}^2S_{xh}^2 - \frac{1}{8}R_{zc}^2S_{zh}^2 - \frac{1}{4}R_{xc}R_{zc}S_{xzh} - \frac{1}{2}R_{xc}S_{yxh} + \frac{1}{2}R_{zc}S_{yzh}\right)\bar{Y}^{-1} \tag{38}$$

$$Bias\left(\hat{\bar{Y}}_{54}^{ST}\right) = \sum_{h=1}^{L} W_h\lambda_h\left(-\frac{1}{8}R_{xc}^2S_{xh}^2 - \frac{3}{8}R_{zc}^2S_{zh}^2 - \frac{1}{4}R_{xc}R_{zc}S_{xzh} + \frac{1}{2}R_{xc}S_{yxh} - \frac{1}{2}R_{zc}S_{yzh}\right)\bar{Y}^{-1} \tag{39}$$

$$MSE\left(\hat{\bar{Y}}_{51}^{ST}\right) = \sum_{h=1}^{L} W_h^2\lambda_h\left(S_{yh}^2 + \frac{1}{4}R_{xc}^2S_{xh}^2 + \frac{1}{4}R_{zc}^2S_{zh}^2 + \frac{1}{2}R_{xc}R_{zc}S_{xzh} - R_{xc}S_{yxh} - R_{zc}S_{yzh}\right) \tag{40}$$

$$MSE\left(\hat{\bar{Y}}_{52}^{ST}\right) = \sum_{h=1}^{L} W_h^2\lambda_h\left(S_{yh}^2 + \frac{1}{4}R_{xc}^2S_{xh}^2 + \frac{1}{4}R_{zc}^2S_{zh}^2 + \frac{1}{2}R_{xc}R_{zc}S_{xzh} + R_{xc}S_{yxh} + R_{zc}S_{yzh}\right) \tag{41}$$

$$MSE\left(\hat{\bar{Y}}_{53}^{ST}\right) = \sum_{h=1}^{L} W_h^2\lambda_h\left(S_{yh}^2 + \frac{1}{4}R_{xc}^2S_{xh}^2 + \frac{1}{4}R_{zc}^2S_{zh}^2 - \frac{1}{2}R_{xc}R_{zc}S_{xzh} - R_{xc}S_{yxh} + R_{zc}S_{yzh}\right) \tag{42}$$

$$MSE\left(\hat{\bar{Y}}_{54}^{ST}\right) = \sum_{h=1}^{L} W_h^2\lambda_h\left(S_{yh}^2 + \frac{1}{4}R_{xc}^2S_{xh}^2 + \frac{1}{4}R_{zc}^2S_{zh}^2 - \frac{1}{2}R_{xc}R_{zc}S_{xzh} + R_{xc}S_{yxh} - R_{zc}S_{yzh}\right) \tag{43}$$

2.3 Stratified random sampling based on dual ratio and product estimator

Kashwaha et al.,[33] obtained dual to combined ratio and product estimators $\hat{\bar{Y}}_{61}$ and $\hat{\bar{Y}}_{62}$ as

$$\hat{\bar{Y}}_{61}^{ST} = \bar{y}_c\left(\frac{\sum_{h=1}^{L} W_h\bar{x}_h^*}{\sum_{h=1}^{L} W_h\bar{X}_h}\right) \tag{44}$$

$$\hat{\bar{Y}}_{62}^{ST} = \bar{y}_c\left(\frac{\sum_{h=1}^{L} W_h\bar{Z}_h}{\sum_{h=1}^{L} W_h\bar{z}_h^*}\right) \tag{45}$$

Here are the Biases (BIASs) and Means Square Error (MSEs) of the estimators up first degree of approximations respectively

$$Bias\left(\hat{\bar{Y}}_{61}^{ST}\right) = \sum_{h=1}^{L} W_h\lambda_h\left(-g_cS_{yxh}\right)\bar{X}^{-1} \tag{46}$$

$$Bias\left(\hat{\bar{Y}}_{62}^{ST}\right) = \sum_{h=1}^{L} W_h\lambda_h\left(R_{zc}g_c^2S_{zh}^2 + g_cS_{yzh}\right)\bar{Z}^{-1} \tag{47}$$

$$MSE\left(\hat{\bar{Y}}_{61}^{ST}\right) = \sum_{h=1}^{L} W_h^2\lambda_h\left(S_{yh}^2 + R_{xc}^2g_c^2S_{xh}^2 - 2g_cR_{xc}S_{yxh}\right) \tag{48}$$

$$MSE\left(\hat{\bar{Y}}_{62}^{ST}\right) = \sum_{h=1}^{L} W_h^2\lambda_h\left(S_{yh}^2 + R_{zc}^2g_c^2S_{zh}^2 + 2g_cR_{zc}S_{yzh}\right) \tag{49}$$

Singh [34] [as cited in][pg. 740]Singh [35] introduced an improved separate ratio to dual estimator of population mean $\bar{Y}$in stratified random sampling

$$\hat{\bar{Y}}_{71}^{ST} = \sum_{h=1}^{L} W_h\bar{y}_h\left(\frac{\bar{x}_*^s}{\bar{X}_h^s}\right) \tag{50}$$

The Biases (BIAS) and Mean Square Error (MSE) according to first order of approximation are stated in equations 51 and 52.

$$Bias(\hat{\bar{Y}}_{71}^{ST}) = \sum_{h=1}^{L} W_h \lambda_h \left(-g_h S_{yxh}\right) \bar{X}_h^{-1} \tag{51}$$

$$MSE(\hat{\bar{Y}}_{71}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 - R_{xh}^2 g_h^2 S_{xh}^2 + 2R_{xh} g_h S_{yxh}\right) \tag{52}$$

2.4 Stratified random sampling based on dual exponential ratio and product estimator

Tailor et al., [36] used dual transformation to Singh et al., [32] suggested ratio and product type exponential estimator as

$$\hat{\bar{Y}}_{81}^{ST} = \bar{y}_c exp\left(\frac{\sum_{h=1}^{L} W_h(\bar{x}_*^c - \bar{X}_h^c)}{\sum_{h=1}^{L} W_h(\bar{x}_*^c + \bar{X}_h^c)}\right) \tag{53}$$

$$\hat{\bar{Y}}_{82}^{ST} = \bar{y}_c exp\left(\frac{\sum_{h=1}^{L} W_h(\bar{Z}_h^c - \bar{z}_*^c)}{\sum_{h=1}^{L} W_h(\bar{Z}_h^c + \bar{z}_*^c)}\right) \tag{54}$$

Final, the Biases (BIASs) and Mean Squared Error (MSEs) of the suggest ratio type exponential estimators $\hat{\bar{Y}}_{81}^{ST}$ and $\hat{\bar{Y}}_{82}^{ST}$ up to first order approximations are given as

$$Bias(\hat{\bar{Y}}_{81}^{ST}) = -\sum_{h=1}^{L} W_h \lambda_h g_c \left(S_{yxh} + \frac{3}{4} R_{xc} g_c S_{xh}^2\right) \bar{X}^{-1} \tag{55}$$

$$Bias(\hat{\bar{Y}}_{82}^{ST}) = \sum_{h=1}^{L} W_h \lambda_h g_c \left(S_{yzh} + \frac{5}{8} R_{zc} g_c S_{zh}^2\right) \bar{Z}^{-1} \tag{56}$$

$$MSE(\hat{\bar{Y}}_{81}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + \frac{1}{4} R_{xc}^2 g_c^2 S_{xh}^2 - R_{xc} g_c S_{yxh}\right) \tag{57}$$

$$MSE(\hat{\bar{Y}}_{82}^{ST}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + \frac{1}{4} R_{xc}^2 g_c^2 S_{zh}^2 + R_{zc} g_c S_{yzh}\right) \tag{58}$$

Rather and Kadilar [7] proposed dual to Ratio cum product type exponential estimator of finite population mean in stratified random sampling but estimator was wrongly stated by Rather and Kadilar [7] to be ”dual to product cum ratio type” and correct bias and mse is obtained in (60) and (61) as follow:

$$\hat{\bar{Y}}_{91}^{ST} = \bar{y}_c \exp\left(\frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h^c - \bar{x}_*^c\right)}{\sum_{h=1}^{L} W_h \left(\bar{X}_h^c + \bar{x}_*^c\right)}\right) exp\left(\frac{\sum_{h=1}^{L} W_h \left(\bar{z}_*^c - \bar{Z}_h^c\right)}{\sum_{h=1}^{L} W_h \left(\bar{z}_*^c + \bar{Z}_h^c\right)}\right) \tag{59}$$

To the second degree of approximation BIASs and MSEs of the suggested ratio type estimators are defined below respectively

$$Bias\left(\hat{\bar{Y}}_{91}^{ST}\right) = \sum_{h=1}^{L} W_h \lambda_h \left(\frac{3}{8} R_{xc}^2 g_c^2 S_{xh}^2 - \frac{1}{8} R_{zc}^2 g_c^2 S_{zh}^2 - \frac{1}{4} R_{xc} R_{zc} g_c^2 S_{xzh} + \frac{1}{2} R_{xc} g_c S_{yxh} - \frac{1}{2} R_{zc} g_c S_{yzh}\right) \bar{Y}^{-1} \tag{60}$$

$$MSE\left(\hat{\bar{Y}}_{91}^{ST}\right) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + \frac{1}{4} R_{xc}^2 g_c^2 S_{xh}^2 + \frac{1}{4} R_{zc}^2 g_c^2 S_{zh}^2 - \frac{1}{2} R_{xc} R_{zc} g_c^2 S_{xzh} + R_{xc} g_c S_{yxh} - R_{zc} g_c S_{yzh}\right) \tag{61}$$

3 Proposed Estimators

The ideas and the analogy of (Walsh [1], Reddy [2], Tripathi [3]) and adopted strategy initiated by (Koyuncu and Kadilar [4], Singh et al., [5], Yasmeen et al., [6], Rather and Kadilar [7]) as well as two auxiliaries variable lead to postulate the proposed estimators based on stratified random sampling such as:

Separate Estimator 62 is proposed as

$$\hat{\bar{Y}}^{*,ST}_{PCRDOSE} = \sum_{h=1}^{L} W_h \bar{y}_h exp\left(\frac{\bar{X}_h^s}{\alpha_h \bar{x}^s_{*h} + (1-\alpha_h)\bar{X}_h^s} - \frac{\alpha_h \bar{z}^s_{*h} + (1-\alpha_h)\bar{Z}_h^s}{\bar{Z}_h^s}\right) \tag{62}$$

were

$$\bar{x}^s_* = (N_h^s \bar{X}_h^s - n_h^s \bar{x}_h)/(N_h^s - n_h^s) \quad g_h = n_h^s\left(N_h^s - n_h^s\right)^{-1}$$
$$\bar{z}^s_* = (N_h^s \bar{Z}_h^s - n_h^s \bar{z}_h^s)/(N_h^s - n_h^s) \quad \lambda_h = (1-f_h)/n_h^s$$

Combined Estimator 63is proposed as

$$\hat{\bar{Y}}^{*,ST}_{PCRDOCE} = \bar{y}_c exp\left(\frac{\sum_{h=1}^{L} W_h \bar{X}_h^c}{\alpha_c \sum_{h=1}^{L} W_h \bar{x}^c_{*h} + (1-\alpha_c)\sum_{h=1}^{L} W_h \bar{X}_h^c} - \frac{\alpha_c \sum_{h=1}^{L} W_h \bar{z}^c_{*h} + (1-\alpha_c)\sum_{h=1}^{L} W_h \bar{Z}_h^c}{\sum_{h=1}^{L} W_h \bar{Z}_h^c}\right) \tag{63}$$

$$\bar{x}^c_* = (N\bar{X}^c - n\bar{x}^c)(N-n)^{-1} \quad g_c = n(N-n)^{-1}$$
$$\bar{z}^c_* = (N\bar{Z}^c - n\bar{z}^c)(N-n)^{-1} \quad \lambda_h = (1-f_h)n_h^{-1}$$

where α is a suitable chosen constant to be determined such that MSE of the proposed estimator in (62) and (63) is minimum.

4 Properties (BIAS, MSE and MMSE) of Proposed Estimators

In this section, BIASs, MSEs and MMSEs of the proposed estimator will be establish using Tailor series expansion up to second degree of expansion.

4.1 BIAS, MSE and MMSE of the proposed separate estimator

To obtain Bias and mean square error let define the followings notations:$e_{yh} = \frac{\bar{y}_h}{\bar{Y}_h} - 1$,$e_{xh} = \frac{\bar{x}_h}{\bar{X}_h} - 1$, $e_{zh} = \frac{\bar{z}_h}{\bar{Z}_h} - 1$,$e_{yc} = \frac{\bar{y}_c}{\bar{Y}} - 1$, $e_{xc} = \frac{\bar{x}_c}{\bar{X}} - 1$ and $e_{zc} = \frac{\bar{z}_c}{\bar{Z}} - 1$

$$e_{i,j,k(c)} = \sum_{h=1}^{L} W_h^{i+j+k} \frac{E[(\bar{y}_h - \bar{Y}_h)^i (\bar{x}_h - \bar{X}_h)^j (\bar{z}_h - \bar{Z}_h)^k]}{\bar{Y}^i \bar{X}^j \bar{Z}^k} \tag{64}$$

The expectation of error terms of proposed estimator (62)

$E(e_{yh}) = E(e_{xh}) = E(e_{zh}) = 0$, $E(e_{xh}^2) = \lambda_h \frac{S_{xh}^2}{\bar{X}_n^2}$, $E(e_{yh}^2) = \lambda_h \frac{S_{yh}^2}{\bar{Y}_h^2}$, $E(e_{zc}^2) = \lambda_h \frac{S_{zh}^2}{\bar{Z}^{c2}}$, $E(e_{xc}^2) = \lambda_h \frac{S_{xh}^2}{\bar{X}^{c2}}$, $E(e_{yh}e_{xh}) = \lambda_h \frac{S_{yxh}}{\bar{X}_h \bar{Y}_h}$, $E(e_{xh}e_{zh}) = \lambda_h \frac{S_{xzh}}{\bar{X}_h \bar{Z}_h}$, $E(e_{yh}e_{zh}) = \lambda_h \frac{S_{yzh}}{\bar{X}_h \bar{Z}_h}$

The expectation of error terms of proposed estimator (63)
$E(e_{yc}) = E(e_{xc}) = E(e_{zc}) = 0$, $E(e_{xc}^2) = \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S_{xh}^2}{\bar{X}^{c2}}$, $E(e_{yc}^2) = \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S_{yh}^2}{\bar{Y}^{c2}}$, $E(e_{zc}^2) = \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S_{zh}^2}{\bar{Z}^{c2}}$, $E(e_{xc}^2) = \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S_{xh}^2}{\bar{X}^{c2}}$, $E(e_{yc}e_{xc}) = \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S_{yxh}}{\bar{X}^c \bar{Y}^c}$, $E(e_{xc}e_{zc}) = \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S_{xzh}}{\bar{X}^c \bar{Z}^c}$, $E(e_{yc}e_{zc}) = \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S_{yzh}^2}{\bar{X}^c \bar{Z}^c}$

Express equation 62 in terms of error e_{xh}, e_{yh}, e_{zh} and simplify the results accordingly

$$\hat{\bar{Y}}^{*ST}_{PCRDOSE} = \sum_{h=1}^{L} W_h \bar{y}_h exp\left(\frac{\bar{X}_h^s}{\bar{X}_h^s(1+\alpha_h g_h) - \alpha_h g_h \bar{x}_h^s} - \frac{\bar{Z}_h^s(1+\alpha_h g_h) - \alpha_h g_h \bar{z}_h^s}{\bar{Z}_h^s}\right) \tag{65}$$

Now assume that $\mid e_{xh} \mid < 1$ so that $(1 - \alpha_h g_h e_{xh})^{-1}$ are expandable. Expanding the right hand side up to second degree approximation.

$$\hat{\bar{Y}}^{*ST}_{PCRDOSE} = \sum_{h=1}^{L} W_h \bar{y}_h exp\left(\frac{\bar{X}_h^s}{\bar{X}_h^s(1+\alpha_h g_h) - \alpha_h g_h(1+e_{xh})\bar{X}_h^s} - \frac{\bar{Z}_h^s(1+\alpha_h g_h) - \alpha_h g_h(1+e_{zh})\bar{Z}_h^s}{\bar{Z}_h^s}\right) \tag{66}$$

$$\hat{\bar{Y}}^{*ST}_{PCRDOSE} = \sum_{h=1}^{L} W_h \bar{y}_h exp\left[(1-\alpha_h g_h e_{xh})^{-1} - (1-\alpha_h g_h e_{zh})\right] \tag{67}$$

Apply binomial expansion to expand LHS of (67) we obtain the result of RHS as follow

$$(1-\alpha_h g_h e_{xh})^{-1} = 1 + \alpha_h g_h e_{xh} + \alpha_h^2 g_h^2 e_{zh}^2 + ... \tag{68}$$

Neglect the higher terms power of 2 or more than and add the result to R.H.S. of (67)

$$\hat{\bar{Y}}^{*ST}_{PCRDOSE} = \sum_{h=1}^{L} W_h \bar{y}_h exp\left(\alpha_h g_h e_{xh} + \alpha_h g_h e_{xh} + \alpha_h^2 g_h^2 e_{zh}^2\right) \tag{69}$$

Taking the exponential of equation (69) using Taylor's series expansion we have

$$\hat{\bar{Y}}^{*ST}_{PCRDOSE} = \sum_{h=1}^{L} W_h \bar{y}_h \left(1 + \alpha_h g_h e_{xh} + \alpha_h g_h e_{zh} + \alpha_h^2 g_h^2 e_{xh}^2 + \frac{(\alpha_h g_h e_{xh} + \alpha_h g_h e_{zh} + \alpha_h^2 g_h^2 e_{xh}^2)^2}{2}\right) \tag{70}$$

Expand the L.H.S of equation (70) and neglate the higher terms greater than 2 we obtain R.H.S of the same equation below

$$(\alpha_h g_h e_{xh} + \alpha_h g_h e_{zh} + \alpha_h^2 g_h^2 e_{xh}^2)^2 = \left(\alpha_h^2 g_h^2 e_{xh}^2 + 2\alpha_h^2 g_h^2 e_{xh} e_{zh} + \alpha_h^2 g_h^2 e_{zh}^2\right) \tag{71}$$

Substitute the R.H.S. 0f (71) into equation (70) place of L.H.S. equation (3.3.5) arrange according to magnitude power of error terms

$$\hat{\bar{Y}}^{*ST}_{PCRDOSE} = \sum_{h=1}^{L} W_h \bar{y}_h \left(1 + \alpha_h g_h e_{xh} + \alpha_h g_h e_{zh} + \alpha_h^2 g_h^2 e_{xh}^2 + \frac{\alpha_h^2 g_h^2 e_{xh}^2}{2} + \frac{\alpha_h^2 g_h^2 e_{zh}^2}{2} + \alpha_h^2 g_h^2 e_{xh} e_{zh}\right) \tag{72}$$

Subtract $\bar{Y}_h^s$ from both side of equation (72)
$Bias(\hat{\bar{Y}}^{*ST}_{PCRDOSE}) = E(\bar{Y}_h^s - \hat{\bar{Y}}^{*}_{PCRDOSE})$

$$Bias\left(\hat{\bar{Y}}^{*ST}_{PCRDOSE}\right) = \bar{Y}_h \sum_{h=1}^{L} W_h \left(e_{yh} + \alpha_h g_h e_{xh} + \alpha_h g_h e_{zh} + \frac{3\alpha_h^2 g_h^2 e_{xh}^2}{2} + \alpha_h^2 g_h^2 e_{xh} e_{zh} + \frac{\alpha_h^2 g_h^2 e_{zh}^2}{2} + \alpha_h g_h e_{xh} e_{yh} + \alpha_h g_h e_{zh} e_{yh}\right) \tag{73}$$

To obtain the Bias we take expectation errors terms of (73) and used the results of 64.

$$Bias\left(\hat{\bar{Y}}^{*ST}_{PCRDOSE}\right) = \bar{Y}_h \sum_{h=1}^{L} W_h \left(E(e_{yh}) + \alpha_h g_h E(e_{xh}) + \frac{3\alpha_h^2 g_h^2 E(e_{xh}^2)}{2} + \alpha_h g_h E(e_{zh}) + \alpha_h^2 g_h^2 E(e_{xh} e_{zh}) + \frac{\alpha_h^2 g_h^2 E(e_{zh}^2)}{2} + \alpha_h g_h E(e_{xh} e_{yh}) + \alpha_h g_h E(e_{zh} e_{yh})\right) \tag{74}$$

$$Bias\left(\hat{\bar{Y}}^{*ST}_{PCRDOSE}\right) = \bar{Y}_h^s \sum_{h=1}^{L} W_h \lambda_h \left(\frac{3}{2}\alpha_h^2 g_h^2 \frac{S_{xh}^2}{\bar{X}_h^{s2}} + \alpha_h^2 g_h^2 \frac{S_{zh}^2}{2\bar{Z}_h^{s2}} + \alpha_h^2 g_h^2 \frac{S_{xzh}}{\bar{X}_h^{s2} \bar{Z}_h^{s2}} + \alpha_h g_h \frac{S_{yxh}}{\bar{X}_h^s \bar{Y}_h^s} + \alpha_h g_h \frac{S_{yzh}}{\bar{Y}_h^s \bar{Z}_h^s}\right) \tag{75}$$

$$Bias\left(\hat{\bar{Y}}^{*ST}_{PCRDOSE}\right) = \sum_{h=1}^{L} W_h \lambda_h \left(\frac{3}{2} R_{xh}^2 \alpha_h^2 g_h^2 S_{xh}^2 + \frac{1}{2} R_{zh}^2 \alpha_h^2 g_h^2 S_{zh}^2 + R_{xh} R_{zh} \alpha_h^2 g_h^2 S_{xzh} + R_{xh} \alpha_h g_h S_{yxh} + R_{zh} \alpha_h g_h S_{yzh}\right) / \bar{Y}_h^s \tag{76}$$

Taking only the leading term of eq (73) above square the it, Expand the result up to the first order approximation to get the mean square error (MSE) of $\hat{\bar{Y}}^{*ST}_{PCRDOSE}$ which is derive as follow,

$$\hat{\hat{Y}}^{*ST}_{PCRDOSE} = \left(\bar{Y}^s_h \left(e_{yh} + \alpha_h g_h e_{xh} + \alpha_h g_h e_{zh}\right)\right)^2 \tag{77}$$

$$MSE\left(\hat{\hat{Y}}^{*ST}_{PCRDOSE}\right) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S^2_{yh} + R^2_{zh}\alpha_h^2 g_h^2 S^2_{zh} + R^2_{xh}\alpha_h^2 g_h^2 S^2_{xh} + 2R_{xh}R_{zh}\alpha_h^2 g_h^2 S_{xzh} + 2R_{xh}\alpha_h g_h S_{yxh} + 2R_{zh}\alpha_h g_h S_{yzh}\right) \tag{78}$$

To obtain minimum mean square error or test for optimality, differentiate (78) with respect to α_h, equate to zero and solve for the α_h, the optimal and minimum mean square erro is attained as in 80 and 81 respectively.

$$MSE\left(\hat{\hat{Y}}^{*ST}_{PCRDOSE}\right)min = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S^2_{yh} + R^2_{zh}\alpha^2_{min} g_h^2 S^2_{zh} + R^2_{xh}\alpha^2_{min} g_h^2 S^2_{xh} + 2R_{xh}R_{zh}\alpha^2_{min} g_h^2 S_{xzh} + 2R_{xh}\alpha_{min} g_h S_{yxh} + 2R_{zh}\alpha_{min} g_h S_{yzh}\right) \tag{79}$$

$$\alpha_{min} = \frac{-\sum_{h=1}^{L} W_h^2 \lambda_h (R_{xh}S_{yxh} + R_{zh}S_{yzh})}{g_h \sum_{h=1}^{L} W_h^2 \lambda_h (R^2_{xh}S^2_{xh} + R^2_{zh}S^2_{zh} + 2R_{xh}R_{zh}S_{xzh})} \tag{80}$$

$$MSE\left(\hat{\hat{Y}}^{*ST}_{PCRDOCE}\right)min = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S^2_{yh} - \frac{(R_{xh}S_{yxh} + R_{zh}S_{yzh})^2}{(R^2_{xh}S^2_{xh} + R^2_{zh}S^2_{zh} + 2R_{xh}R_{zh}S_{xzh})}\right) \tag{81}$$

4.2 BIAS, MSE and MMSE of the proposed combined estimator

$$\hat{\hat{Y}}^{*ST}_{PCRDOCE} = \bar{y}_{st} exp \left\{ \begin{array}{c} \frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\left[\sum_{h=1}^{L} W_h \bar{X}_h \left(\alpha_c + \alpha_c \frac{n_h}{N_h - n_h}\right) - \sum_{h=1}^{L} W_h \bar{x}_h \left(\alpha_h \frac{n_h}{N_h - n_h}\right)\right] + (1-\alpha_c)\sum_{h=1}^{L} W_h \bar{X}_h} \\ - \frac{\left[\sum_{h=1}^{L} W_h \bar{Z}_h \left(\alpha_c + \alpha_c \frac{n_h}{N_h - n_h}\right) - \sum_{h=1}^{L} W_h \bar{z}_h \left(\alpha_c \frac{n_h}{N_h - n_h}\right)\right] + (1-\alpha_h)\sum_{h=1}^{L} W_h \bar{Z}_h}{\sum_{h=1}^{L} W_h \bar{Z}_h} \end{array} \right\} \tag{82}$$

Bias of the estimator 63 can be obtain from (82) as follow

$$\hat{\hat{Y}}^{*ST}_{PCRDOCE} = \bar{Y}^c (1 + e_{yc}) exp \left[(1 - \alpha_c g_c e_{xc})^{-1} - (1 - \alpha_c g_c e_{zc})\right] \tag{83}$$

If we assume that $|e_{xc}| < 1$, the expression $(1 - \alpha_c g e_{xc})^{-1}$ inside equation 3.4.6 can be expanded to a convergent infinite series using binomial expansion. Hence

$$(1 - \alpha_c g_c e_{xc})^{-1} = 1 + \alpha_c g_c e_{xc} + \alpha_c^2 g_c^2 e^2_{zc} + ... \tag{84}$$

Deduct $\bar{Y}$ from both side of equation (85) and take expectation of the we the bias in (**??**)
$Bias(\hat{\hat{Y}}^{*ST}_{PCRDOCE}) = E(\bar{Y}^c - \hat{\hat{Y}}^{*}_{PCRDOCE})$

$$Bias\left(\hat{\hat{Y}}^{*ST}_{PCRDOCE}\right) = \bar{Y}^c - \bar{Y}^c \left(1 + e_{yc} + \alpha_c g_c e_{xc} + \alpha_c g_c e_{zc} + \alpha_c^2 g_c^2 e^2_{zc} + \alpha_c^2 g_c^2 e_{xc} e_{zc} + \frac{3}{2}\alpha_c^2 g_c^2 e^2_{xc} + \alpha_c g_c e_{xc} e_{yc} + \alpha_c g_c e_{yc} e_{zc}\right) \tag{85}$$

$$Bias\left(\hat{\hat{Y}}^{*ST}_{PCRDOCE}\right) = \sum_{h=1}^{L} W_h \lambda_h \left(\frac{3}{2} R^2_{xc}\alpha_c^2 g_c^2 S^2_{xh} + \frac{1}{2} R^2_{zc}\alpha_c^2 g_c^2 S^2_{zh} + R_{xc}R_{zc}\alpha_c^2 g_c^2 S_{xzh} + R_{xc}\alpha_c g_c S_{yxh} + R_{xc}\alpha_c g_c S_{yzh}\right) / \bar{Y}^c \tag{86}$$

Now to obtain the MSE of 63, pick the lower terms of error of (86), Square them and find their expectation; after substituting the result of expectation of (64) we obtain

$$MSE\left(\hat{\hat{Y}}^{*ST}_{PCRDOCE}\right) = \bar{Y}^{c2} \left(\sum_{h=1}^{L} W_h^2 \lambda_h \frac{S^2_{yh}}{\bar{Y}^{c2}} + 2\alpha_c g_c \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S_{yxh}}{\bar{X}^c \bar{Z}^c} + 2\alpha_c g_c \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S_{yzh}}{\bar{Y}^c \bar{Z}^c} \right.$$
$$\left. + 2\alpha_c^2 g_c^2 \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S_{xzh}}{\bar{X}^c \bar{Z}^c} + \alpha^2 g_c^2 \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S^2_{xh}}{\bar{X}^{c2}} + \alpha_c^2 g_c^2 \sum_{h=1}^{L} W_h^2 \lambda_h \frac{S^2_{zh}}{\bar{Z}^{c2}}\right) \tag{87}$$

$$MSE\left(\hat{\hat{Y}}^{*ST}_{PCRDOCE}\right) = \bar{Y}^{c2}\sum_{h=1}^{L} W_h^2\lambda_h\left(\frac{S^2_{yh}}{\bar{Y}^{c2}} + \alpha_c^2 g_c^2\frac{S^2_{xh}}{\bar{X}^{c2}} + \alpha_c^2 g_c^2\frac{S^2_{zh}}{\bar{Z}^{c2}} + 2\alpha_c^2 g_c^2\frac{S_{xzh}}{\bar{X}^c\bar{Z}^c} + 2\alpha_c g_c\frac{S_{yxh}}{\bar{X}^c\bar{Z}^c} + 2\alpha_c g_c\frac{S_{yzh}}{\bar{Y}^c\bar{Z}^c}\right) \quad (88)$$

Finally, noting that

$$MSE\left(\hat{\hat{Y}}^{*ST}_{PCRDOCE}\right) = \sum_{h=1}^{L} W_h^2\lambda_h\left(S^2_{yh} + R^2_{xc}\alpha_c^2 g_c^2 S^2_{xh} + R^2_{zc}\alpha_c^2 g_c^2 S^2_{zh} + 2R_{xc}R_{zc}\alpha_c^2 g_c^2 S_{xzh} + 2R_{xc}\alpha_c g_c S_{yxh} + 2R_{zc}\alpha_c g_c S_{yzh}\right) \quad (89)$$

From equation (89) differentiate with respect to α_c

$$\alpha_c = -\sum_{h=1}^{L} W_h^2\lambda_h\left(\frac{R_{xc}S_{yxh} + R_{zc}S_{yzh}}{R^2_{xc}S^2_{xh} + R^2_{zc}S^2_{zh} + 2R_{xc}R_{zc}S_{xzh}}\right)\frac{1}{g_c} \quad (90)$$

$$MSE\left(\hat{\hat{Y}}^{*ST}_{PCRDOCE}\right) = \sum_{h=1}^{L} W_h^2\lambda_h\left[S^2_{yh} + \alpha^2_{min}g_c^2\left(R^2_{xc}S^2_{xh} + R^2_{zc}S^2_{zh} + 2R_{xc}R_{zc}S_{xzh}\right) + 2\alpha_{min}g_c\left(R_{xc}S_{yxh} + R_{zc}S_{yzh}\right)\right] \quad (91)$$

Substituted (90) into (91) we obtain complete equation of minimum mean square error below

$$MSE\left(\hat{\hat{Y}}^{*ST}_{PCRDOCE}\right)min = \sum_{h=1}^{L} W_h^2\lambda_h\left(S^2_{yh} - \frac{(R_{xc}S_{yxh} + R_{zc}S_{yzh})^2}{(R^2_{xc}S^2_{xh} + R^2_{zc}S^2_{zh} + 2R_{xc}R_{zc}S_{xzh})}\right) \quad (92)$$

5 Empirical Study

The performance of proposed estimator is always assessed by many author using empirical study, comparing existing estimators over proposed estimators. Twenty existing estimators were compare with two proposed estimators using the following three criteria BIAS, Mean Square Error (MSE) and Percentage Relative Efficiency (PRE) by adopted real life data and simulated data sets as given in the following subsection (5.1) and (5.2).

5.1 Real life application

The data sets are considered to see the efficiency of the proposed estimators with respect to the conventional estimators review in literature, summary or descriptive statistics of the two natural population are reported. The population one was earlier presented by Tailor and Tailor [37], Tailor et al., [38], Lone et al., [39], Yadav [40] & Rather and Kadiler [7]. The population two is earlier employed by Koyuncu and Kadiler [4], Singh and Singh [41], Shahzad et al., [42], Javed and Irfan [43], Singh and Kumar [44], Shahzad [45], Kadilar and Cingi [29], Javed and Irfan [46], Yadav [47], Sardar et al., [48], Ahmad [49], Seh [50], Lone et al., [39], Siraj et al., [51], Priam [52], Zakir and Seh [53] and Malik and Singh [54]. Description of the populations are given to details in Tables 1 and 2 bellow respectively:

Table 1. population one [Source: Murthy [55, pg. 228]], Y: Output, X: Fixed Capital, Z: Numbers of Workers

		Stratum 1	Stratum 2
Str Population Size	N_h	5	5
Str Samample Size	n_h	2	3
Str Sample mean y	$\bar{Y}_h$	1925.8	3115.6
Str Sample mean x	$\bar{X}_h$	214.4	333.8
Str Sample mean z	$\bar{Z}_h$	51.8	60.6
Str Standard Dev y	S_{yh}	615.92	340.38
Str Standard Dev x	S_{xh}	74.87	66.35
Str Standard Dev z	S_{zh}	0.75	4.84
Str Standard Dev xy	S_{yxh}	39360.68	22356.5
Str Standard Dev yz	S_{yzh}	411.16	1536.24
Str Standard Dev xz	S_{xzh}	38.08	287.92

Table 2. Population two [Source: Six different district in Turkey in the years 2007 (2007)], Y: Number of teachers X: Number of students Z: Number of classes

	Stratum 1	Stratum 2	Stratum 3	Stratum 4	stratum 5	Stratum 6
N_h	127	117	103	170	205	201
n_h	31	21	29	38	22	39
$\bar{Y}_h$	703.74	413.6	573.17	424.66	267.63	393.84
$\bar{X}_h$	20804.59	9211.79	14309.3	9478.85	5569.95	12997.59
$\bar{Z}_h$	498.28	318.33	431.36	311.32	227.2	313.71
S_{yh}	883.835	644.922	1033.467	810.585	403.654	711.723
S_{xh}	30486.751	15180.769	27549.697	18218.931	8497.77	23094.141
S_{zh}	555.5816	365.4776	612.9509281	458.0282	260.8511	397.0481
S_{yxh}	25237153.52	9747942.85.5	28298397.04	14523885	3393591.75	15864573.97
S_{yzh}	480688.2	230092.8	623019.3	364943.4	101539.01	277696.1
S_{xzh}	15914648	5379190.92	16490674.5	8041254	2144057	8857729

Table 3. Statistical Analysis of different estimators based on real population one

S/No.	Estimators	Authors of Recent sep and comb Estimators	POPULATION ONE		
			BIAS	**MSE**	**PRE**
1	$\hat{\bar{Y}}_{11}^{ST}$		161.5036	12276.97	126.36
2	$\hat{\bar{Y}}_{12}^{ST}$	Kadilar and Cingi [29] & Hansen et al., [30]	13.48875	33892.38	45.772
3	$\hat{\bar{Y}}_{13}^{ST}$		180.5041	145961	10.62833
4	$\hat{\bar{Y}}_{21}^{ST}$		7.124364	12944	119.849
5	$\hat{\bar{Y}}_{22}^{ST}$	Koyuncu and Kadilar [4]	19.63428	151966	10.2083
6	$\hat{\bar{Y}}_{23}^{ST}$		-22.71543	11749.19	132.037
7	$\hat{\bar{Y}}_{24}^{ST}$		28.40568	148948	10.4152
8	$\hat{Y}_{31}^{ST}$	Vishwakarma et al., [31]	4.486646	145961	10.6283
9	$\hat{\bar{Y}}_{41}^{ST}$		-54.88745	10304	150.5552
10	$\hat{\bar{Y}}_{42}^{ST}$	Singh et al., [32]	59.6446	33085.67	46.88804
11	$\hat{\bar{Y}}_{51}^{ST}$		1.147122	10093.5	153.695
12	$\hat{\bar{Y}}_{52}^{ST}$	Singh et al., [5]	4.793144	79604.7	19.4878
13	$\hat{\bar{Y}}_{53}^{ST}$		-14.47166	10549.3	147.054
14	$\hat{\bar{Y}}_{54}^{ST}$		3.610031	77042.5	20.1359
15	$\hat{\bar{Y}}_{61}^{ST}$		-11.25815	19282.5	80.4523
16	$\hat{\bar{Y}}_{62}^{ST}$	Kashwaha et al., [33]	118.4702	34254.81	45.28771
17	$\hat{\bar{Y}}_{71}^{ST}$	Singh [34] (as cited in Singh [35])	-12.52805	44695.92	34.70836
18	$\hat{\bar{Y}}_{81}^{ST}$		26.08017	14866.7	104.349
19	$\hat{\bar{Y}}_{82}^{ST}$	Tailor et al., [36]	31.9425	33246.74	46.66087
20	$\hat{\bar{Y}}_{91}^{ST}$	Rather and Kadilar [7]	11.09063	13042	118.948
21	$\hat{\bar{Y}}_{PCRDOSE}^{*,ST}$		**-3.691608**	**7168.4**	**216.41**
22	$\hat{\bar{Y}}_{PCRDOCE}^{*,ST}$	Proposed Estimators	**-1.834172**	**7631.012**	**203.2918**

Table 4. Statistical Analysis of different estimators based on real population two

S/No.	Estimators	Authors of Recent sep and comb Estimators	POPULATION TWO BIAS	MSE	PRE
1	$\hat{\bar{Y}}^{ST}_{11}$	Koyuncu and Cingi [29] & Hansen et al., [30]	4.739442	200.937	37.3094
2	$\hat{\bar{Y}}^{ST}_{12}$		-4.929927	2324.342	3.225366
3	$\hat{\bar{Y}}^{ST}_{13}$		5.085323	9195.542	0.8153
4	$\hat{\bar{Y}}^{ST}_{21}$	Koyuncu and Kadilar [4]	2.935441	130.4	57.4914
5	$\hat{\bar{Y}}^{ST}_{22}$		8.229396	8996.7	0.83329
6	$\hat{\bar{Y}}^{ST}_{23}$		146.7123	204.29	36.6972
7	$\hat{\bar{Y}}^{ST}_{24}$		20.43594	8808.62	0.85108
8	$\hat{\bar{Y}}^{ST}_{31}$	Vishwakarma et al., [31]	0.425800	9195.54	0.81527
9	$\hat{\bar{Y}}^{ST}_{41}$	Singh et al.,[32]	1.865101	1092.6	6.8615
10	$\hat{\bar{Y}}^{ST}_{42}$		-0.561662	2276.544	3.293085
11	$\hat{\bar{Y}}^{ST}_{51}$	Singh et al., [5]	1.515125	622.236	12.0482
12	$\hat{\bar{Y}}^{ST}_{52}$		2.368766	5020.75	1.49317
13	$\hat{\bar{Y}}^{ST}_{53}$		-5.710349	670.411	11.1825
14	$\hat{\bar{Y}}^{ST}_{54}$		-0.280461	4880.86	1.53597
15	$\hat{\bar{Y}}^{ST}_{61}$	Kashwaha et al., [33]	-1.228824	1324.3	5.66099
16	$\hat{\bar{Y}}^{ST}_{62}$		43.42974	2251.897	3.329127
17	$\hat{\bar{Y}}^{ST}_{71}$	Singh [34] (as cited in Singh [35])	-1.519934	40946.5	0.18309
18	$\hat{\bar{Y}}^{ST}_{81}$	Tailor et al., [36]	4.519089	1734.79	4.32147
19	$\hat{\bar{Y}}^{ST}_{82}$		5.952849	2240.565	3.345966
20	$\hat{\bar{Y}}^{ST}_{91}$	Rather and Kadilar [7]	0.280461	1721.92	4.35378
21	$\hat{\bar{Y}}^{*,ST}_{PCRDOSE}$	Proposed Estimators	**-0.455423**	**62.391**	**120.16**
22	$\hat{\bar{Y}}^{*,ST}_{PCRDOCE}$		**-1.507569**	**103.16**	**72.675**

Tables 3 and 4 show the results of the BIASs, MSEs and PREs of the proposed and related existing estimators considered in this studied obtained from population one and two respectively. The results revealed that proposed estimators $\hat{\bar{Y}}^{*,ST}_{PCRDOSE}$ and $\hat{\bar{Y}}^{*,ST}_{PCRDOCE}$ have the BIAS, MSE and higher PRE compared to that of classical $\hat{\bar{Y}}^{ST}_{11}$, $\hat{\bar{Y}}^{ST}_{12}$, $\hat{\bar{Y}}^{ST}_{13}$, $\hat{\bar{Y}}^{ST}_{21}$, $\hat{\bar{Y}}^{ST}_{22}$, $\hat{\bar{Y}}^{ST}_{23}$, $\hat{\bar{Y}}^{ST}_{24}$, $\hat{\bar{Y}}^{ST}_{31}$, $\hat{\bar{Y}}^{ST}_{41}$, $\hat{\bar{Y}}^{ST}_{42}$, $\hat{\bar{Y}}^{ST}_{51}$, $\hat{\bar{Y}}^{ST}_{52}$, $\hat{\bar{Y}}^{ST}_{53}$, $\hat{\bar{Y}}^{ST}_{54}$, $\hat{\bar{Y}}^{ST}_{61}$, $\hat{\bar{Y}}^{ST}_{62}$, $\hat{\bar{Y}}^{ST}_{71}$, $\hat{\bar{Y}}^{ST}_{81}$, $\hat{\bar{Y}}^{ST}_{82}$, and $\hat{\bar{Y}}^{ST}_{91}$, related estimators reviews in the study

5.2 Simulated study

After real life justification of the results, stimulation studies were also consider to assess the performance of the proposed estimators over other estimators considered in the studied. Data of size 1000 units were generated for study population in stratified random. This classified into $3(h = 1,2,3)$ and the error term e_h are normally distributed with mean of zero and standard deviation of one for non-overlapping heterogeneous groups of size 500, 200 and 300 using function defined in Table 4 Samples of sizes 60, 40 and 50 were selected 10,000 times by the SRSWOR method from each stratum respectively. The precision Biases, MSEs and PREs of the considered estimators were computed using eq 5.2, 5.2 and 5.2

$$Bias(\hat{\theta}_s) = \frac{1}{10000}\sum_{s=1}^{10000}\left(\hat{\theta}_s - \bar{Y}\right), \hat{\theta}_s = \bar{y}, \hat{\bar{Y}}^{ST}_{ij}, i,j = \{(1,2,3,4,5,6,7,8,9),(1,2,3,4)\}, \hat{\bar{Y}}^{*,ST}_{PCRDOSE}$$

$$^{*,ST}_{PCRDOCE} (93)$$

$$MSE(\hat{\theta}_s) = \frac{1}{10000}\sum_{s=1}^{10000}\left(\hat{\theta}_s - \bar{Y}\right)^2, \hat{\theta}_s = \bar{y}, \hat{\bar{Y}}^{ST}_{ij}, i,j = \{(1,2,3,4,5,6,7,8,9),(1,2,3,4)\}, \hat{\bar{Y}}^{*,ST}_{PCRDOSE}$$

$$^{*,ST}_{PCRDOCE}(94)$$

$$PRE(\hat{\theta}_s) = \left(\frac{VAR(\hat{\theta}_s)}{MSE(\hat{\theta}_s}\right) \times 100, \hat{\theta}_s = \bar{y}, \hat{\hat{Y}}^{ST}_{ij}, i, j = \{(1,2,3,4,5,6,7,8,9), (1,2,3,4)\}, \hat{\hat{Y}}^{*,ST}_{PCRDOSE}$$

$$^{*,ST}_{PCRDOCE}(95)$$

Table 5. Distributions used to Simulate Population Data

Populations	Auxiliary variable(x)	Auxiliary variable(z)	Study variable(y)
I	$X_h \sim exp(0.2, 0.3, 0.1)$	$Z_h \sim exp(0.2, 0.3, 0.1)$	$Y_h = 3X_h + 6Z_h + e_h$
II	$X_h \sim poi(2, 3, 1)$	$Z_h \sim poi(2, 3, 1)$	$Y_h = 3X_h + 6Z_h + e_h$
III	$X_h \sim gamma((1.2, 1.8), (1.3, 1.8), (1.1, 1.8))$	$Z_h \sim gamma((1.2, 1.8), (1.3, 1.8), (1.1, 1.8))$	$Y_h = 60X_h + 50Z_h + e_h$
IV	$X_h \sim Chisquare(5, 3, 1)$	$Z_h \sim Chisquare(5, 3, 1)$	$Y_h = 60X_h + 50Z_h + e_h$
V	$X_h \sim Geometric(0.2, 0.3, 0.1)$	$Z_h \sim Geometric(0.2, 0.3, 0.1)$	$Y_h = 3X_h + 6Z_h + e_h$
VI	$X_h \sim Beta((1, 2), (1, 3), (1, 4))$	$Z_h \sim Beta((1, 2), (1, 3), (1, 4))$	$Y_h = 30X_h + 60Z_h + e_h$
VII	$X_h \sim Weibull((2, 1), (3, 1), (4, 1))$	$Z_h \sim Weibull((2, 1), (3, 1), (4, 1))$	$Y_h = 30X_h + 6Z_h + e_h$
VIII	$X_h \sim Log_N(0.2, 0.3, 0.1)$	$Z_h \sim Log_N(0.2, 0.3, 0.1)$	$Y_h = 3X_h + 6Z_h + e_h$

Table 6. Statistical Analysis of different estimators based on Simulated Data

	Exponential			**Poisson**			**Gamma**			**Chi-Square**		
estimator	Bias	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE
$\hat{\hat{Y}}^{ST}_{10}$	-274.3394	877.1656	100	-26.55745	72.10354	100	-102.35	1068.2	100	-52.2276	276.7463	100
$\hat{\hat{Y}}^{ST}_{11}$	-274.8128	848.3692	103.3943	-26.39243	70.27144	102.6072	101.88	1038.69	106.805	-51.9888	270.8413	102.1802
$\hat{\hat{Y}}^{ST}_{12}$	-274.3113	856.6403	102.396	-26.55264	71.10939	101.3981	102.38	1048.84	101.846	-52.182	272.8444	101.4301
$\hat{\hat{Y}}^{ST}_{13}$	-275.5203	7676.113	98.32204	-84.75121	722.9542	99.16823	-299.18	9035.3	98.6313	-151.16	2302.866	98.47763
$\hat{\hat{Y}}^{ST}_{21}$	-275.3555	858.6065	102.1615	-26.38945	71.39358	100.9944	-101.41	1052.48	101.494	-51.917	272	101.745
$\hat{\hat{Y}}^{ST}_{22}$	91.1735	938.1256	93.36779	26.39646	75.4993	95.91874	101.25	1124.83	95.0188	51.7168	289.7105	95.54438
$\hat{\hat{Y}}^{ST}_{23}$	274.3792	7549.862	99.96958	84.90047	718.123	99.93711	298.34	8924.02	99.768	150.839	2279.72	100.0869
$\hat{\hat{Y}}^{ST}_{24}$	-276.5694	7597.95	97.29975	-84.75121	726.5095	98.68294	-300.26	1115.32	95.829	-151.341	2315.529	98.47763
$\hat{\hat{Y}}^{ST}_{31}$	275.7948	7628.909	98.93374	84.86384	721.7827	99.43039	299.59	8996.21	97.2273	151.34	2294.932	99.42342
$\hat{\hat{Y}}^{ST}_{41}$	-274.1813	864.3138	101.4869	-26.57703	71.46801	100.8893	-102.49	1056.15	101.141	-52.2552	274.3447	100.8754
$\hat{\hat{Y}}^{ST}_{42}$	-274.7844	7597.95	99.33351	-84.64182	719.3267	99.66834	-298.53	8960.47	99.455	150.941	2289.123	99.61361
$\hat{\hat{Y}}^{ST}_{51}$	-274.5582	862.5657	101.6926	-26.51885	71.45644	100.9056	-102.14	1055.33	101.22	52.1814	273.3339	101.2485
$\hat{\hat{Y}}^{ST}_{52}$	92.10306	900.8361	97.2327	26.54478	73.6649	98.30731	102.08	1091.43	97.9266	52.0927	282.1184	98.11559
$\hat{\hat{Y}}^{ST}_{53}$	274.0663	7522.581	100.3321	84.59045	716.6259	100.1459	297.97	8889.46	100.746	150.787	2276.544	100.2265
$\hat{\hat{Y}}^{ST}_{54}$	-275.1619	7624.191	98.99162	-63.33711	720.4366	99.51479	-298.91	1085.57	98.4552	-150.995	2293.615	100.2206
$\hat{\hat{Y}}^{ST}_{61}$	-60.75757	9422.936	9.308834	-84.69365	904.6404	7.970409	67.613	125877	9.62774	-168.492	2842.215	9.736997
$\hat{\hat{Y}}^{ST}_{62}$	1025.168	113470.1	6.651365	316.2041	10282.95	6.972134	1085.7	167125	7.07965	537.649	56543.5	9.736997
$\hat{\hat{Y}}^{ST}_{71}$	-208.2759	2524.003	34.75295	-47.97971	230.7687	31.24494	227.3	3006.38	35.5311	-84.5942	716.7549	38.61101
$\hat{\hat{Y}}^{ST}_{81}$	-230.8916	5331.789	16.45162	-70.82577	501.6972	14.37192	149.24	6315.38	16.9143	-127.334	1621.555	38.61101
$\hat{\hat{Y}}^{ST}_{82}$	-864.5172	77899.26	9.688552	268.2003	7338.302	9.769847	928.79	89546.7	9.951 94	1778.15	22212.07	10.26594
$\hat{\hat{Y}}^{ST}_{91}$	-2970.82	962718.9	0.090983	-921.1678	88418.37	0.081904	-3169.9	1075599	0.09937	-1578.68	262861.6	0.105303
$\hat{\hat{Y}}^{*,ST}_{PCRDOSE}$	136.2099	1860.394	405.6971	44.15938	196.0911	365.9888	159.37	2546.6	349.942	-77.1917	597.1727	382.0837
$\hat{\hat{Y}}^{*,ST}_{PCRDOCE}$	92.21787	850.4137	887.5168	26.28987	69.11576	1038.361	102.14	1043.28	854.194	-51.8998	269.3592	847.0845

Table 6. Continue

	Geometric			Beta			Weibull			Log-Normal		
estimator	Bias	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE
$\hat{\bar{Y}}^{ST}_{10}$	-78.3889	634.2146	100	-40.0412	163.7676	100	105.69	1124.167	100	-32.2405	107.6702	100
$\hat{\bar{Y}}^{ST}_{11}$	-78.217	612.4341	103.5564	-39.8036	158.8724	103.0812	106.25	1129.613	99.51792	-32.1117	103.7727	103.7558
$\hat{\bar{Y}}^{ST}_{12}$	-78.6373	619.0049	102.4571	-39.9924	160.3901	102.1058	105.58	1115.367	100.7769	-32.2759	104.8022	102.7366
$\hat{\bar{Y}}^{ST}_{13}$	76.9635	5864.074	97.79753	-123.163	1523.941	99.23088	-106.09	2618.476	98.46149	92.3042	866.2929	97.49899
$\hat{\bar{Y}}^{ST}_{21}$	-77.5404	621.9547	101.9712	-39.6484	161.1702	101.6116	105.88	1131.863	99.32003	-31.721	105.0108	102.5325
$\hat{\bar{Y}}^{ST}_{22}$	76.8045	691.012	91.89332	39.8463	171.7442	95.50574	-106.09	1168.725	96.3168	31.5574	118.506	91.28535
$\hat{\bar{Y}}^{ST}_{23}$	238.944	5729.411	100.1016	122.941	1517.968	99.68614	160.22	2576.054	100.1217	-91.708	844.9595	99.99388
$\hat{\bar{Y}}^{ST}_{24}$	-241.565	5936.279	96.61323	-123.503	1531.863	98.71768	-161.02	2637.819	97.73946	-92.7778	878.284	96.16786
$\hat{\bar{Y}}^{ST}_{31}$	240.518	5864.074	98.79203	123.305	1522.474	99.3911	160.7	2591.784	99.51404	92.5037	859.755	98.27309
$\hat{\bar{Y}}^{ST}_{41}$	-78.6638	624.1599	101.6109	-40.0531	161.762	101.2399	-105.59	1117.786	100.7638	-32.3267	105.8489	101.7206
$\hat{\bar{Y}}^{ST}_{42}$	77.8446	5786.999	99.10545	-123.011	1516.557	99.714	-160.45	2594.791	99.36021	-91.9208	853.2051	98.99458
$\hat{\bar{Y}}^{ST}_{51}$	-78.2754	622.8615	101.8227	-39.92	161.8129	101.208	105.7	1122.2	100.3674	-32.1276	105.494	102.0628
$\hat{\bar{Y}}^{ST}_{52}$	77.9215	656.8484	96.67281	40.018	167.2154	98.09238	105.79	1141.313	98.63011	32.0628	112.2298	96.39028
$\hat{\bar{Y}}^{ST}_{53}$	238.711	5708.617	100.4662	122.862	1512.525	100.0449	160.18	2570.661	100.3317	91.5752	840.7621	100.4931
$\hat{\bar{Y}}^{ST}_{54}$	-239.996	5810.01	98.71292	-123.142	1518.928	99.55837	-160.57	2600.907	99.12657	-92.0916	856.9695	98.55974
$\hat{\bar{Y}}^{ST}_{61}$	-267.715	7178.6	8.834796	-137.804	1899.305	8.622503	-206.72	4278.378	26.32595	-102.612	1054.921	10.20647
$\hat{\bar{Y}}^{ST}_{62}$	918.037	103579.9	5.537012	454.646	21540.9	7.020226	1711.1	276293	0.933136	-352.9	42471.76	1.988679
$\hat{\bar{Y}}^{ST}_{71}$	-136.398	1865.123	34.0039	-68.8069	474.8188	34.49054	-65.6198	15.08474	7466.641	-53.8322	290.8466	37.01957
$\hat{\bar{Y}}^{ST}_{81}$	-200.685	4028.102	15.74475	-103.197	1064.784	15.38036	-96.154	924.8662	121.7823	-77.6898	603.7272	17.83424
$\hat{\bar{Y}}^{ST}_{82}$	-447.405	61819.45	9.277389	-386.284	15209.5	9.942604	-942.81	100630	2.562048	-288.023	8884.211	9.507056
$\hat{\bar{Y}}^{ST}_{91}$	-2658.93	785624.7	0.080827	-1318.48	182220.1	0.090015	-4405.4	1967194	0.057223	-998.371	116474.2	0.092878
$\hat{\bar{Y}}^{*,ST}_{PCRDOSE}$	119.624	1435.403	399.5553	61.6236	380.1873	398.0152	-15.726	142.9112	1804.749	49.7726	249.4568	338.6991
$\hat{\bar{Y}}^{*,ST}_{PCRDOCE}$	78.3861	614.438	933.4108	39.6993	157.6003	960.1525	-105.84	1120.291	230.2249	32.026	102.5666	823.765

Table 6 showed the results of the BIASs, MSEs and PREs of the proposed and related existing estimators considered in this study obtained from simulated data under eight different distributions respectively. The results revealed that proposed estimators (62) and (63) have the minimum BIAS, MSE and higher PRE compared to the reviewed estimators of (1), (2), (3), (10), (11), (12), (13), (22), (26), (27), (32), (33), (34), (35), (44), (45), (50), (53), (54) and (59) related estimators reviewed in the studied.

6 Results and Discussion

Twenty existing estimators in stratified sampling were review from different authors and two ratio-product-cum type estimators were proposed for separate and combined in stratified random sampling under multiple auxiliaries variable.The estimator were formulated using ideas and version of (Walsh [1], Reddy [2], Tripathi [3]) and adopted strategy initiated by (Koyuncu and Kadilar [4], Singh et al., [5], Yasmeen et al., [6], Rather and Kadilar [7]) as well as multiple auxiliaries variable. The BIAS, MSE, and PRE of the proposed estimators were derived up to fist order of approximation by Taylor series expansion. The efficiency comparison were perform empirically (real life and simulated data).

It was observed that proposed estimator are better than existing estimator with less minimum mean square error and maximum percentage relative efficiency indicate in Tables 3, 4 and 6.

7 Conclusion

The results of the empirical study revealed that proposed estimators 62 and 63 do better than other mentioned existing estimators (1), (2), (3) Proposed by (Kadilar and Cingi [29] & Hansen et al., [30]), estimators (10), (11), (12), (13) proposed by Koyuncu and Kadilar [4], estimator (22) proposed by Vishwakarma et al., [31], estimators (26), (27) proposed by Singh et al.,[32], estimators (32), (33), (34), (35) proposed by Singh et al., [5], estimators (44), (45) proposed by Kashwaha et al., [33], estimator (50) proposed by Singh[34] [as cited in][pg. 740]Singh [35], estimators (53), (54) proposed by Tailor et al.,[36] estimator (59) proposed by Rather and Kadilar [7] having less BIAS, mean square error (MSE) and the maximum Percentage Relative Efficiency (PRE). Which proved that the proposed estimator are more efficient as shown in Table 3, Table 4 and Table 6.

In general, proposed estimator (On The Efficiency of Modified Exponential Dual to Ratio-Product-Cum Type Estimator) are better than existing estimators.

Disclaimer (Artificial Intelligence)

The authors(s) of this manuscript solemnly declare that they have not utilized any forms generative AI tools, including Large Language Models (such as ChatGPT, Perplexity and COPILOT) and text-to-image generators, during writing process. This work is the result of their own original thinking, research, and writing effort, without any assistance from AI-generate content.

Acknowledgement

The authors are much grateful to the Chief Editor and anonymous reviewers for their valuable examine the manuscript and also fully supported given by Department of Statistics, Federal University Technology, Minna, Nigeria.

Competing Interests

Authors have declared that no competing interests exist.

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