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# Comparison of Proposed and Existing Fourth Order Schemes for Solving Non-linear Equations

**Khushbu Rajput[1*], Asif Ali Shaikh[1] and Sania Qureshi[1]**

*[1]Mehran University of Engineering and Technology, Jamshoro, Pakistan.*

***Authors' contributions***

*This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.*

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## Abstract

This paper, investigates the comparison of the convergence behavior of the proposed scheme and existing schemes in literature. While all schemes having fourth-order convergence and derivative-free nature. Numerical approximation demonstrates that the proposed schemes are able to attain up to better accuracy than some classical methods, while still significantly reducing the total number of iterations. This study has considered some nonlinear equations (transcendental, algebraic and exponential) along with two complex mathematical models. For better analysis graphical representation of numerical methods for finding the real root of nonlinear equations with varying parameters has been included. The proposed scheme is better in reducing error rapidly, hence converges faster as compared to the existing schemes.

*Keywords: Comparison; convergence behavior; derivative-free; number of iterations; fourth-order.*

## 1 Introduction

In mathematics and applied science, non-linear equations are the equations that have no proportional relationship between input and output. As a result, problems arising in the field of science and engineering are mostly non-linear. Analytic and numeric methods are effective ways to solve these non-linear equations. The Analytic methods delivered us an analytical or exact solution for nonlinear equations. Analytical

**Corresponding author: E-mail: rajputkhushi971@gmail.com;*

solutions of few non-linear equations are impossible. Mostly, science and engineering problems fail to bring their analytical solution of nonlinear equation then our focused on numerical methods. Most of the numerical methods, used to solve an equation, are based on iterative techniques. It is critical to ascertain convergence while developing any Numerical method. Numerical methods are approximated methods. For many years, estimating a root of non-linear equations has been an attraction to researchers. Researchers have introduced many variants of accelerated methods that proved instrumental in estimating non-linear equations. Many researchers have developed iterative methods and many modifications have been made for iterative methods (such as the Bisection, Regula-Falsi, and Newton methods), which have the same or better performance. Determining the root of a nonlinear equation is very important; researchers have developed numerical methods by involving derivatives [1]. Many algorithms have been introduced to accelerate the convergence of numerical methods without involving derivative [2,3,4,5,6,7]. By using covenant and suitable selection of parameters to reduce the number of evaluation of numerical method [2,8,9,6]. A family of a single step and multi-step method has been developed that can be used to find simple and real roots of nonlinear equations by using decomposition techniques [10,11].

# 2 Methods

## 2.1 Proposed methodology

Developing a two-step iterative method by removing the involvement of derivative function in the Newton Raphson method.

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)} \tag{1}$$

By using Forward difference method

$$f'(x_n) = \frac{f(x_n) - f(x_n - h)}{h} \tag{2}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{3}$$

By using the central difference formula

$$f'(x_n) = \frac{-f(x_n + 2h) + 8f(x_n + h) - 8f(x_n - h) + f(x_n - 2h)}{12h} \tag{4}$$

By replacing $h = x_n - x_{n-1}$ and Using (2) in (1) and (4) in (3), after simplification we get

$$y_n = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} \tag{5}$$

$$x_{n+1} = y_n - \frac{12f(y_n)(y_n - x_n)}{-f(3y_n - 2x_n) + f(2x_n - y_n) + 8(f(2y_n - x_n) - f(x_n))} \tag{6}$$

Eq: (5) and (6) is proposed scheme, Where $x_n$ and $x_{n-1}$ are the first two approximations, taken from the bracketing root location methods for non-linear equation (i.e. bisection method or Regula-falsi method).

## 2.2 Existing schemes

### 2.2.1 Consider the two-step iteration scheme

$$x_n^* = x_n - \frac{mf^2(x_n)}{f(x_n + mf(x_n)) - f(x_n)}$$

$$x_{n+1} = x_n^* - \frac{mf(x_n)f(x_n^*)}{f(x_n + mf(x_n)) - f(x_n)} \frac{f(x_n) + p(x_n)f(x_n^*)}{\left[p(x_n) - 1 - \frac{f(x_n)}{f(x_n + mf(x_n))}\right] f(x_n^*)}$$

Thus the existing scheme gives two ($m$ = ±1) one-parameter families of iteration formulae of order 4 for solving the non-linear equation $f(x) = 0$. The parameter $p(x)$ that appears in the algorithm is taken $p(x) = 1$. Calculations are performed using the problems that are solved taking the initial value $x0$ in the interval ($a$, $b$) of the root [6].

### 2.2.2 Given $x_0$ the approximate solution $x_{n+1}$ of f(x)=0 can be founded by following the iterative scheme

$$y_n = x_n - \frac{f(x_n)}{f[w_n, x_n]}$$

$$x_{n+1} = y_n - \frac{f(y_n)}{g(x_n)} \frac{f(x_n) + \beta f(y_n)}{f(x_n) + (\beta - 2) f(y_n)}$$

Where,

$$g(x_n) = f[w_n, x_n] + 2(w_n - x_n) f[w_n, x_n, y_n] - f[y_n, wl] + f[x_n, y_n]$$

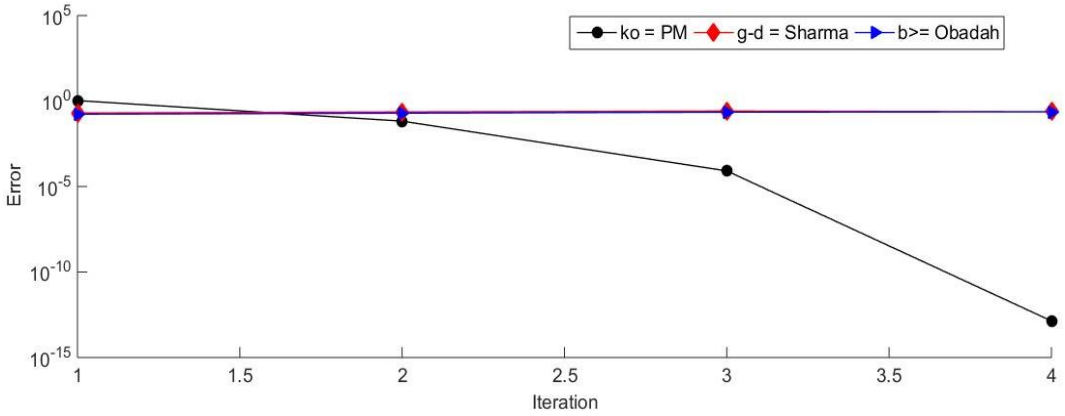
$$w_n = x_n + f(x_n), \quad f[x_n, y_n] = \frac{f(x_n) - f(y_n)}{x_n - y_n}, \quad f[x_n, y_n] = \frac{f[w_n, x_n] - f[x_n, y_n]}{w_n - y_n}$$

The method derived using the king's method with finite difference approximations. The parameter $\beta = 2$ is fixed for all numerical examples [7].

# 3 Results

**Example 1:**

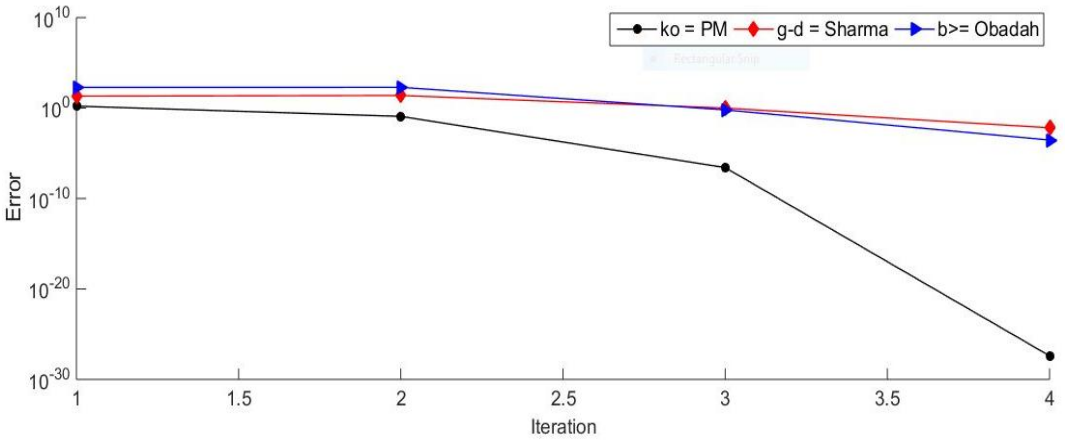
| Function | Interval | Iteration | Method | Time | Error |
|---|---|---|---|---|---|
| $x^3 + 4x^2 + 8x + 8$ | [-3,-1] | 4 | PM | 0.001339 | 1.376677e-13 |
| | | | Sharma | 0.001435 | 2.368009e-01 |
| | | | Obadah | 0.001932 | 2.346701e-01 |




In Ex: 1 proposed scheme is converging rapidly as in the graph we can see that with the increasing number of iteration on X-axis, error of the proposed scheme(PM) is decreasing upto -13 digit accuracy while other existing scheme (Sharma and Obadah) showing negligible change in accuracy and showing repetation.

**Example 2:**

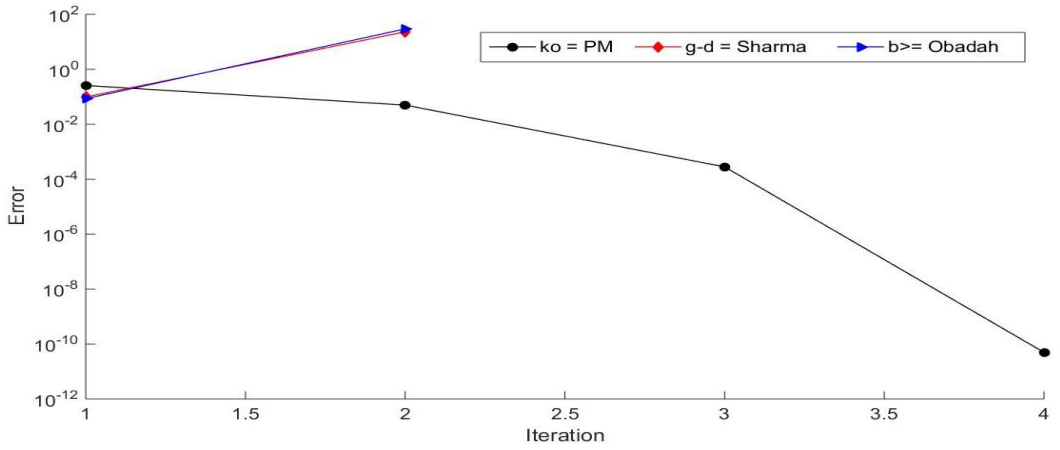
| Function | Interval | Iteration | Method | Time | Error |
|---|---|---|---|---|---|
| $\sin x - \frac{1}{2}x$ | [-1,1.5] | 4 | PM | 0.000926 | 4.038968e-28 |
| | | | Sharma | 0.001954 | 6.557702e-03 |
| | | | Obadah | 0.001405 | 2.893045e-04 |




In Ex: 2 with the increasing number of iteration existing schemes (Sharma and Obadah) are converging slowly while the proposed scheme (PM) reducing errors frequently.

**Example 3:**

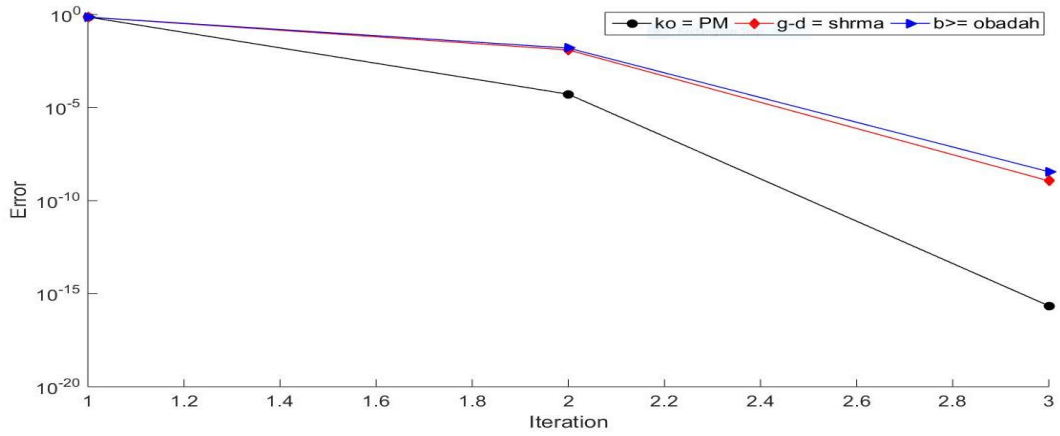
| Function | Interval | Iteration | Method | Time | Error |
|---|---|---|---|---|---|
| $xe^{x^2} - \sin^2 x + 3\cos x + 5$ | [-2,-1] | 4 | PM | 0.001822 | 4.964540e-11 |
| | | | Sharma | 0.001617 | NaN |
| | | | Obadah | 0.001708 | NaN |




In Ex: 3 through graph we can observe the proposed scheme (PM) converging while after 2 iterations the existing scheme (Sharma and Obadah) giving undefine values (NaN).

**Example 4:**

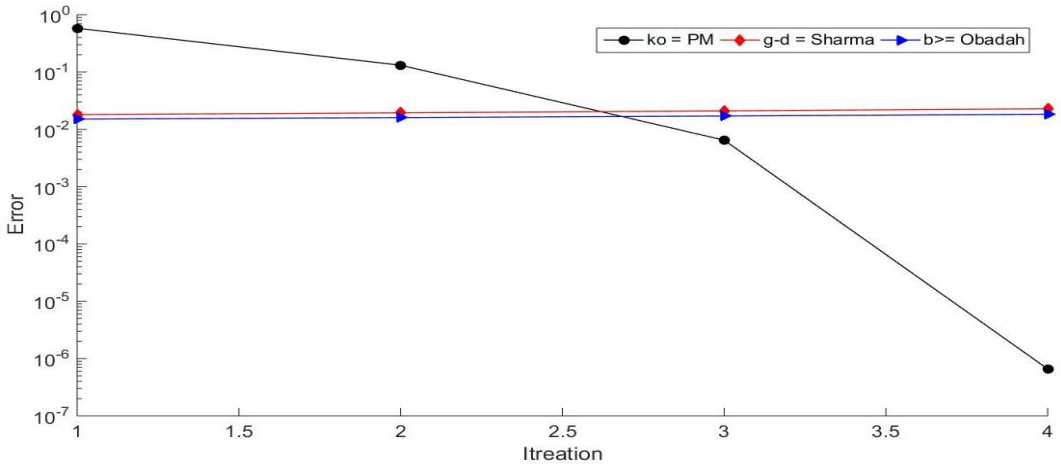
| Function | Interval | Iteration | Method | Time | Error |
|---|---|---|---|---|---|
| $x^2 - e^x - 3x + 2$ | [0,1] | 3 | PM | 0.014250 | 2.220446e-16 |
| | | | Sharma | 0.005937 | 1.152646e-09 |
| | | | Obadah | 0.007175 | 3.578957e-09 |




In Ex:4 at three iterations the proposed scheme(PM) is showing accuracy up to 16 decimals places while the existing schemes (Sharma and Obadah) having accuracy up to 9 decimal places.

**Example 5:**

| Function | Interval | Iterations | Method | Time | Error |
|---|---|---|---|---|---|
| $x^5 - 8x^4 + 44x^3 - 91x^2 + 85x - 26$ | [0,1] | 4 | PM | 0.000868 | 6.589412e-07 |
| | | | Sharma | 0.001522 | 2.285747e-02 |
| | | | Obadah | 0.001330 | 1.832951e-02 |




In Ex:5 the proposed scheme (PM) reducing error rapidly at each iteration, while the existing schemes (Sharma and Obadah) having no change in error accuracy at each iteration.

## 4 Conclusion

This study has developed a multi-step derivative-free scheme by using central difference formula for estimating root of nonlinear equations. The proposed scheme having fourth-order convergence. Different types of transcendental and algebraic problems solved which show that the proposed scheme is more accurate than the existing schemes presented in [6,7]. From the tables it observed, the proposed scheme is better in terms of CPU time, number of iterations and reducing error more frequently than the existing schemes. In the given graphs presented in example 1 and 5 existing schemes showing repetition and in Example 3 existing schemes diverge while the proposed scheme reducing error at each iteration. All the calculations and graphs carried out using MATLAB software.

## Competing Interests

Authors have declared that no competing interests exist.

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