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Forecasting Oil Consumption with Novel Fractional Grey Prediction Model Based on Simpson Formula

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

With the development of human society, the evolving transition of energy will become a common challenge that mankind has to face together. In this context, it is crucial to make scientific and reasonable predictions about energy consumption. This paper presents a novel fractional grey prediction model FGM(1,1,k^2) based on the classical fractional grey system theory. In order to improve the prediction accuracy of the FGM(1,1,k^2) model, we further analyze the model error and propose improved grey model called as SFGM with optimization of background value. The numerical cases point out that SFGM(1,1,k^2) significantly outperforms other existing fractional grey models. Finally, the proposed SFGM(1,1,k^2) is applied to the forecasting of oil consumption, the predicted results would provide a reference for making energy policy in new situations.

Keywords: Energy economic; fractional grey system; SFGM model; Simpson formula.

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1 Introduction

Energy has always been a key issue concerning the future of human destiny. And oil determines the future development of mankind, which as the backbone of the energy. Recently, The British Petroleum (BP) released *BP energy outlook 2019-Global* with the theme of "evolving transition" in Beijing. The outlook shows that the rise of energy demand and environmental pollution force mankind to face the common situation of evolving transition. Furthermore, the energy consumption in total world will continue to rise by about a third, and China and other countries in the Asia Pacific region account for two-thirds of that in a certain period of future. Therefore, it is necessary to make a scientific and reasonable forecast of energy consumption, which will help mankind to overcome this challenge in the new situation.

In system theory, the grey system is a known system, which in an intermediate between the black system and the white system. In 1982, the concept and content about grey system theory was first proposed by Professor Deng [1], which has demonstrated excellent ability in solving uncertain problems with little information and small samples. In the past three decades, more and more researchers devoted themselves to the study of the grey system theory with its rapid development. Therefore, it also has been successfully applied in many fields including social [2-4], economic [5-6], energy [7-10], industrial [11-13], and agricultural management studies [14]. One of the most brilliant achievements of grey system theory is the grey prediction model, which represented by GM(1,1) model with one order accumulation generation and single variable of which whitening equation is $dx^{(1)}\left(t\right)/dt+az^{(1)}\left(t\right)=b$.

Although GM(1,1) model exhibits a good mathematical theoretical foundation in solving practical problems, it does not always provide a satisfactory result in the face of complex and variable raw data sequences. To tackle the challenge, many scholar have studied the preprocessing, accumulation generating, background value, initial value and the residual error correlation of GM(1,1) prediction model aim to improve its prediction performance. And the research on the background value of gray model is always a hot topic. Tan et al. [15-16] took the lead in optimizing the background value of the GM(1,1) model from it's geometric meaning, and provided a new optimization idea. Based on direct modeling, Wang et al. [17] presented a method of stepwise optimization, and the results show that the background value can be effectively improved. Luo et al. [18] used the homogeneous exponential function to simulation the sequence generated by the initial accumulation of the original sequence, thus giving a novel improved means of background value. Many researchers use interpolation and numerical integral theory to optimize the background of the gray model [19-22]. Thereafter, some researches extended the idea of optimizing the background value to the extended model of multivariable and non-equidistant gray theory and achieved good results [23-26].

On the other hand, there are also many researchers study the performances of the models from the perspective of accumulation generation of the raw data, which includes generalized accumulating generation, reciprocal accumulating generation, and fractional accumulating generation. Wu et al. [27] extend the traditional GM(1,1) to the fractional GM(1,1) (FGM(1,1) by short) which whitening equation is $dx^{(r)}\left(t\right)/dt+az^{(r)}\left(t\right)=b$ firstly, this approach provides a new modelling idea and has been widely recognized by the academia. Later, Wu et al. [28,29,30] applied the model flexibly in various fields with satisfactory results. Xiao et al. [31] studied the modelling mechanism and extension structure of GM(1,1) model which fractional accumulated generation operator was considered as a generalized accumulated generation operator. Furthermore, Mao et al. [32] presented a novel FGM(q,1) model then regarded this fractional accumulated generation as a more general form of data transformation. Wu et al. [33] recently proposed a novel fractional accumulation grey prediction model (FGM(1,1,k)), and the results of theoretical analysis and practical application show the model is reliable.

In this paper, we first presented a novel fractional grey prediction model (FGM(1,1,k^2)) which bring in 2-order time power terms to improve the prediction accuracy. With mathematical analysis, we proposed the SFGM(1,1,k^2) model by using the Simpson numerical integral formula to further optimize the background value of the FGM(1,1,k^2), then validated the SFGM(1,1,k^2) model by numerical cases. In application study, the proposed SFGM(1,1,k^2) model is established to analyze the oil consumption in China, total Asia Pacific and the total world, and make the forecast to the future consumption behaviors in the next three years from 2019 to 2021. Three alternative fractional grey models include the classical FGM(1,1) and the FGM(1,1,k), as well as the FGM(1,1,k^2) were compared. These results indicate our proposed model has advantage over other existing models.

2 Fractional Grey Model

2.1 Fractional accumulated generating operation

Accumulated generating operation (AGO) plays a crucial part in grey system theory, who is employed to reduce or even eliminate the randomness of the original data sequence and improve the use efficiency of grey system information. It also laid an important foundation for modeling grey differential equations.

In general, for an original data sequence $X^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),...,x^{(0)}(n\text{-}1),x^{(0)}(n)\right)$, We define it's r order accumulated generation operator as follow.

Definition 1. Set $X^{(r)}\left(r\in\Re\right)$ is the r order accumulated generation operator of sequence $X^{(0)}$, in which $x^{(r)}(k)=\sum_{i=1}^{k}x^{(r-1)}(i), k=1,2,...,n\text{-}1,n.$

We can obtained the expression $X^{(r)}=X^{(0)}A^r$ from the matrix operation theory, in which A^r is a r-AGO matrix . A^r is described as follow:

$$A^r=\begin{bmatrix} \binom{r}{0} & \binom{r}{1} & \binom{r}{2} & \cdots & \binom{r}{n-1} \\ 0 & \binom{r}{0} & \binom{r}{1} & \cdots & \binom{r}{n-2} \\ 0 & 0 & \binom{r}{0} & \cdots & \binom{r}{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \binom{r}{0} \end{bmatrix}_{n\times n} \tag{1}$$

where

$$\binom{r}{n}=\begin{cases} 1 & n=0 \\ \dfrac{r(1+r)\cdots(n-1+r)}{n!} & n\in N^{+} \end{cases} \tag{2}$$

Definition 2. Let $x^{(r-1)}(k)=x^{(r)}(k)-x^{(r)}(k-1), k=1,2,...,n$ be the r order inverse accumulated generated operator (r-$IAGO$). We can obtained the expression $X^{(0)}=X^{(r)}D^r$ according to the matrix operation theory, where D^r denotes the r-$IAGO$ matrix, and

$$D^r = \begin{bmatrix} \binom{-r}{0} & \binom{-r}{1} & \binom{-r}{2} & \cdots & \binom{-r}{n-1} \\ 0 & \binom{-r}{0} & \binom{-r}{1} & \cdots & \binom{-r}{n-2} \\ 0 & 0 & \binom{-r}{0} & \cdots & \binom{-r}{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \binom{-r}{0} \end{bmatrix}_{n\times n} \tag{3}$$

where $\binom{-r}{n} = \begin{cases} 1 & n=0 \\ \dfrac{(-r)(-r+1)\cdots(-r+n-1)}{n!} & n\in N^+ \end{cases}$.

2.2 FGM(1,1) model

As a remarkable branch of grey system theory, fractional grey prediction model makes up for the deficiency of classical grey prediction model to some extent, and further extends the grey prediction model.

Definition 3. Let $X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n-1), x^{(0)}(n)\right)$ be the original data sequence, $X^{(r)} = \left(x^{(r)}(1), x^{(r)}(2), \cdots, x^{(r)}(n)\right) = X^{(0)}A^r$, in which $X^{(r)}$ is the *r* order accumulated generating operator of $X^{(0)}$, and $Z^{(r)} = \left(z^{(r)}(2), z^{(r)}(3), \cdots, z^{(r)}(n\text{-}1), z^{(r)}(n)\right)$ be the background value sequence of $X^{(r)}$, in which $z^{(r)}(k) = 1/2\left(x^{(r)}(k) + x^{(r)}(k-1)\right), k = 2,3,\cdots,n$. We define

$$x^{(r-1)}(k) + az^{(r)}(k) = b \tag{4}$$

as the *r* order fractional GM(1,1) model called as FGM(1,1) by short, where b and a are the grey actuating quantity and development coefficient of the model respectively. And the parameters b and a can be solved by the least square rule. The parameter can be obtained as follow:

$$\hat{e} = [a,b]^T = \left(P^T P\right)^{-1} P^T Y, \tag{5}$$

where

$$Y = \begin{bmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{bmatrix}, P = \begin{bmatrix} -z^{(r)}(2) & 1 \\ -z^{(r)}(3) & 1 \\ \vdots & \vdots \\ -z^{(r)}(n) & 1 \end{bmatrix}. \tag{6}$$

Definition 4. Let the parameters a and b be the same as those in previous definition. We have the whitening equation of the *r* order accumulated grey FGM(1,1) model:

$$\frac{dx^{(r)}(t)}{dt}+ax^{(r)}(t)=b \tag{7}$$

Let $\hat{x}^{(0)}(1)=x^{(0)}(1)$. Then the solution for the whitening equation of the FGM(1,1) model can be obtained by

$$\hat{x}^{(r)}(k)=\left(x^{(0)}(1)-\frac{b}{a}\right)e^{-a(k-1)}+\frac{b}{a},k=1,2,\cdots,n,\cdots \tag{8}$$

The restored values of the sequence $X^{(0)}$ can be expressed by

$$\hat{X}^{(0)}=\hat{X}^{(r)}D^{r} \tag{9}$$

where $\hat{X}^{(0)}=\left(\hat{x}^{(0)}(1),\hat{x}^{(0)}(2),\cdots,\hat{x}^{(0)}(n-1),\hat{x}^{(0)}(n)\right)$, $\hat{X}^{(r)}=\left(\hat{x}^{(r)}(1),\hat{x}^{(r)}(2),\cdots,\hat{x}^{(r)}(n)\right)$.

3 FGM(1,1,k^2) Model

The traditional grey system model regards the grey action b as a constant. However, with the development and change of time and space, the grey action b is not a fixed constant. Therefore, many scholars have carried out a series of improvements and optimization on the grey actuating quantity. In literature [34], GM(1,1) model was first introduced into the linear development space by substituting bk for b to improve the grey action, and the literature [35] was optimized by introducing the first-order time power b_1k+b_2. In this paper, the higher second-order time power model with $b_2k^2+b_1k+b_0$ is extend to the fractional grey system and the following model definition is obtained.

3.1 Fractional grey FGM(1,1,k^2) model

Definition 5. Let these sequences $X^{(r)},X^{(r-1)},Z^{(r)}$ have the same definition as those of FGM(1,1) model in previous section, we have the equation

$$x^{(r-1)}(k)+az^{(r)}(k)=b_2k^2+b_1k+b_0 \tag{10}$$

is called the *r* order fractional grey FGM(1,1,k^2) model. And a is named as development coefficient, b_0, b_1 and b_2 are called as grey actuating quantity.

Definition 6. Assume the sequences and parameters be the same definitions as those in Definition 5. We obtain

$$\frac{dx^{(r)}(t)}{dt}+ax^{(r)}(t)=b_2t^2+b_1t+b_0 \tag{11}$$

is named as the whitening equation of the *r* order non-homogeneous FGM(1,1,k^2) model.

The FGM(1,1,k^2) model can be reduced to a series of representative grey system models, includes FGM(1,1,k), GM(1,1,k,c), GM(1,1,k),GM(1,1) and so on, where the parameters are a,b_1,b_2 specified suitable values.

When $b_2 = 0$, the FGM(1,1,k^2) model is reduce to FGM(1,1,k) model [33].

When $b_2 = 0, b_1 = 0$, the FGM(1,1,k^2) model is reduce to FGM(1,1) model [27].

When $r = 1, b_2 = 0$, the FGM(1,1,k^2) model is reduce to GM(1,1,k,c) model [35].

When $r = 1, b_2 = 0, b_0 = 0$, the FGM(1,1,k^2) model is reduce to GM(1,1,k) model [34].

When $r = 1, b_2 = 0, b_1 = 0$, the FGM(1,1,k^2) model is reduce to GM(1,1) model [1].

3.2 Determination of FGM(1,1,k^2) model parameters

Theorem 1. Assume the sequences $X^{(r)}, X^{(r-1)}, Z^{(r)}$ be the same as those in FGM(1,1,k^2) model, let $\hat{u} = \left[a, b_2, b_1, b_0\right]^T$ be a parameter set to be determined and

$$Y = \begin{bmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(r)}(2) & 2^2 & 2 & 1 \\ -z^{(r)}(3) & 3^2 & 3 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -z^{(r)}(n) & n^2 & n & 1 \end{bmatrix}. \tag{12}$$

According to the least square rule, we have

$$\hat{u} = (a, b_2, b_1, b_0)^T = \left(B^T B\right)^{-1} B^T Y. \tag{13}$$

Proof. Combing the above parameters and previous subsection, we have $Y = B\hat{u}$ and substitute $-az^{(r)}(k) + b_2 k^2 + b_1 k + b_0$ for $x^{(r-1)}(k), k = 2, 3, \ldots, n$. Let error sequence is satisfied $\varepsilon = Y - B\hat{u}$, $S = \varepsilon^T \varepsilon = [Y - B\hat{u}]^T [Y - B\hat{u}] = \sum_{i=2}^{n} [x^{(r-1)}(k) + az^{(r)}(k) - b_2 k^2 - b_1 k - b_0]^2$. then these parameters a, b_2, b_1, b_0 which make S minimum should satisfy

$$\begin{cases} \dfrac{\partial S}{\partial a} = 2\sum_{i=2}^{n} [x^{(r-1)}(k) + az^{(r)}(k) - b_2 k^2 - b_1 k - b_0] z^{(r)}(k) = 0 \\ \dfrac{\partial S}{\partial b_2} = -2\sum_{i=2}^{n} [x^{(r-1)}(k) + az^{(r)}(k) - b_2 k^2 - b_1 k - b_0] k^2 = 0 \\ \dfrac{\partial S}{\partial b_1} = -2\sum_{i=2}^{n} [x^{(r-1)}(k) + az^{(r)}(k) - b_2 k^2 - b_1 k - b_0] k = 0 \\ \dfrac{\partial S}{\partial b_0} = -2\sum_{i=2}^{n} [x^{(r-1)}(k) + az^{(r)}(k) - b_2 k^2 - b_1 k - b_0] = 0 \end{cases}. \tag{14}$$

Analyze the left-hand side of the equation, we obtain

$$\begin{cases} a\sum_{i=2}^{n}\left(z^{(r)}(k)\right)^2 - b_2\sum_{i=2}^{n}k^2z^{(r)}(k) - b_1\sum_{i=2}^{n}kz^{(r)}(k) - b_0\sum_{i=2}^{n}z^{(r)}(k) = -\sum_{i=2}^{n}x^{(r-1)}(k)z^{(r)}(k) \\ -a\sum_{i=2}^{n}k^2z^{(r)}(k) + b_2\sum_{i=2}^{n}k^4 + b_1\sum_{i=2}^{n}k^3 + b_0\sum_{i=2}^{n}k^2 = \sum_{i=2}^{n}x^{(r-1)}(k)k^2 \\ -a\sum_{i=2}^{n}kz^{(r)}(k) + b_2\sum_{i=2}^{n}k^3 + b_1\sum_{i=2}^{n}k^2 + b_0\sum_{i=2}^{n}k = \sum_{i=2}^{n}x^{(r-1)}(k)k \\ -a\sum_{i=2}^{n}z^{(r)}(k) + b_2\sum_{i=2}^{n}k^2 + b_1\sum_{i=2}^{n}k + b_0\sum_{i=2}^{n}1 = \sum_{i=2}^{n}x^{(r-1)}(k) \end{cases}. \quad (15)$$

Reorder the above equations, we have

$$M\cdot\left[a,b_2,b_1,b_0\right]^T = N, \quad (16)$$

where

$$M = \begin{bmatrix} \sum_{i=2}^{n}\left(z^{(r)}(k)\right)^2 & -\sum_{i=2}^{n}z^{(r)}(k)k^2 & -\sum_{i=2}^{n}z^{(r)}(k)k & -\sum_{i=2}^{n}z^{(r)}(k) \\ -\sum_{i=2}^{n}z^{(r)}(k)k^2 & \sum_{i=2}^{n}k^4 & \sum_{i=2}^{n}k^3 & \sum_{i=2}^{n}k^2 \\ -\sum_{i=2}^{n}z^{(r)}(k)k & \sum_{i=2}^{n}k^3 & \sum_{i=2}^{n}k^2 & \sum_{i=2}^{n}k \\ -\sum_{i=2}^{n}z^{(r)}(k) & \sum_{i=2}^{n}k^2 & \sum_{i=2}^{n}k^1 & \sum_{i=2}^{n}k^0 \end{bmatrix}, \quad (17)$$

$$N = \begin{bmatrix} -\sum_{i=2}^{n}x^{(r-1)}(k)z^{(r)}(k) \\ \sum_{i=2}^{n}x^{(r-1)}(k)k^2 \\ \sum_{i=2}^{n}x^{(r-1)}(k)k \\ \sum_{i=2}^{n}x^{(r-1)}(k) \end{bmatrix}, \quad (18)$$

According to the known condition, we have

$$B^TB = \begin{bmatrix} \sum_{i=2}^{n}\left(z^{(r)}(k)\right)^2 & -\sum_{i=2}^{n}z^{(r)}(k)k^2 & -\sum_{i=2}^{n}z^{(r)}(k)k & -\sum_{i=2}^{n}z^{(r)}(k) \\ -\sum_{i=2}^{n}z^{(r)}(k)k^2 & \sum_{i=2}^{n}k^4 & \sum_{i=2}^{n}k^3 & \sum_{i=2}^{n}k^2 \\ -\sum_{i=2}^{n}z^{(r)}(k)k & \sum_{i=2}^{n}k^3 & \sum_{i=2}^{n}k^2 & \sum_{i=2}^{n}k \\ -\sum_{i=2}^{n}z^{(r)}(k) & \sum_{i=2}^{n}k^2 & \sum_{i=2}^{n}k & \sum_{i=2}^{n}1 \end{bmatrix} = M, \quad (19)$$

$$B^TY = \begin{bmatrix} -\sum_{i=2}^{n}x^{(r-1)}(k)z^{(r)}(k) \\ \sum_{i=2}^{n}x^{(r-1)}(k)k^2 \\ \sum_{i=2}^{n}x^{(r-1)}(k)k \\ \sum_{i=2}^{n}x^{(r-1)}(k) \end{bmatrix} = N, \quad (20)$$

Substituting both $M = B^TB$ and $N = B^TY$ into equation (15), we have

$$B^T B\cdot(a,b_2,b_1,b_0)^T = B^T Y, \tag{21}$$

therefore,

$$\hat{u} = (a,b_2,b_1,b_0)^T = \left(B^T B\right)^{-1} B^T Y, \tag{22}$$

3.3 The time response sequence of FGM(1,1,k^2) model

Theorem 2. Set Y and B have the same definition as Theorem 1, and parameter set $\hat{u} = (a,b_2,b_1,b_0)^T = \left(B^T B\right)^{-1} B^T Y$, then the continuous time response function of the whitening equation $dx^{(r)}(t)/dt + ax^{(r)}(t) = b_2t^2 + b_1t + b_0$ of the r order FGM(1,1,k^2) model is

$x^{(r)}(t) = e^{-at}(\int (b_2t^2 + b_1t + b_0)e^{at}dt + C)$, in which C is a constant which can be determined based on initial condition.

Proof. It can be proofed by the constant variation method, omitted here.

Theorem 3. Let B, Y and the parameter set $\hat{u} = (a,b_2,b_1,b_0)^T = \left(B^T B\right)^{-1} B^T Y$ be same as those defined in Theorem 1. For the r order fractional FGM(1,1,k^2) prediction model, the form of its time response series $\hat{X}^{(r)}$ is

$$\hat{x}^{(r)}(k) = \left(x^{(0)}(1) - \frac{2b_2}{a^3} + \frac{2b_2+b_1}{a^2} + \frac{b_2+b_1+b_0}{a}\right)e^{-a(k-1)} + b_2\left(\frac{k^2}{a} - \frac{2k}{a^2} + \frac{2}{a^3}\right) + b_1\left(\frac{k}{a} - \frac{1}{a^2}\right) + \frac{b_0}{a}$$

$k = 2,3,\ldots,n-1,n.$ Thereafter, we have the restored values sequence $\hat{X}^{(0)}$ according to $\hat{X}^{(0)} = \hat{X}^{(r)}D^r$,where $\hat{X}^{(0)} = \left(\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \cdots, \hat{x}^{(0)}(n)\right)$ and $\hat{X}^{(r)} = \left(\hat{x}^{(r)}(1), \hat{x}^{(r)}(2), \cdots, \hat{x}^{(r)}(n)\right)$

Proof. This proof can be got by discretizing the time response function in Theorem 2. Omitted here.

4 SFGM(1,1,k^2) Model Based on Simpson

4.1 Error analysis of the model

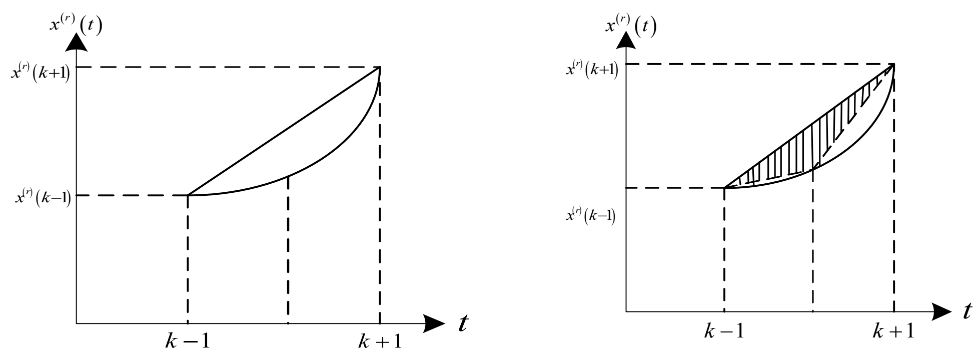
According to the time response function of FGM(1,1,k^2) we noticed the prediction performance about FGM(1,1,k^2) model depends on the parameters a and b_2,b_1,b_0. However, the parameters a,b_2,b_1,b_0 are mainly determined by the background value $z_1^{(r)}(k)$ .The integration of $dx^{(r)}(t)/dt + ax^{(r)}(t) = b_2t^2 + b_1t + b_0$ in $[k-1,k]$ can be introduced:

$$I = \int_{k-1}^{k} dx^{(r)}(\mathrm{t}) + a\int_{k-1}^{k} x^{(r)}(t)dt = \int_{k-1}^{k}\left(b_2t^2 + b_1t + b_0\right)dt, k = 2,3,\ldots,n-1,n, \tag{23}$$

Equation (14) is combined with $x^{(r-1)}(k) + az^{(r)}(k) = b_2k^2 + b_1k + b_0$ we get:

$$z^{(r)}(k)=\int_{k-1}^{k}x^{(r)}\left(t\right)dt\,, \tag{24}$$

Based on the previous knowledge, we know that the classical fractional grey system theory can be explained by the trapezoidal integral formula when calculating the background value of FGM(1,1,k^2), then its geometric significance is shown in Fig. 1(a). The trapezoid area between the curve $x^{(r)}\left(t\right)$ and the horizontal axis t on the interval $[k-1,k+1]$ is the background value of FGM (1,1,k^2) model. Obviously, when the data sequence has a large change, the background value calculated by the trapezoidal formula will have a large calculation error, which will cause the prediction effect of the model to change. The Simpson formula approximates the definite integral of the integrand curve by using a parabola to improve the error of the background value. The corresponding background value can be calculated by Fig. 1(b). The shaded area in Fig. 1(b) is the calculation error reduced by the improved Simpson formula.



(a) **(b)**

Fig. 1. The background value based on (a) trapezoidal formula and (b) Simpson formula

4.2 SFGM(1,1,k^2) model based on simpson

Combined with the analysis of the model error in the above subsection, we noticed using the trapezoidal integral formula to calculate the background value will bring a large error, reducing the prediction performance of the model. Therefore, in our research, the Simpson numerical integral formula is employed to optimize the background value aim to promote the prediction performance of the model, so as to obtain a novel fractional grey system model named as the FGM(1,1,k^2) model based on Simpson(SFGM(1,1,k^2) by short).

Considering the integral of equation (11) over the interval of $[k-1,k+1]$,

$$I=\int_{k-1}^{k+1}dx^{(r)}\left(t\right)+a\int_{k-1}^{k+1}x^{(r)}\left(t\right)dt=\int_{k-1}^{k+1}\left(b_2t^2+b_1t+b_0\right)dt\,, \tag{25}$$

the above equation can be further expressed as:

$$I=x^{(r)}\left(k+1\right)-x^{(r)}\left(k-1\right)+a\int_{k-1}^{k+1}x^{(r)}\left(t\right)dt=2\left(b_2k^2+b_1k+b_0\right)+\frac{2b_2}{3}\,, \tag{26}$$

Simpson formula is used to approximate calculate the score of $\int_{k-1}^{k+1}x^{(r)}(t)dt$ over the interval $[k-1,k+1]$, and the following equation can be obtained:

$$I = \int_{k-1}^{k+1} x^{(r)}(t)dt = \frac{1}{3}\left[x^{(r)}(k-1)+4x^{(r)}(k)+x^{(r)}(k+1)\right], \tag{27}$$

then, equation (26) can be expressed as:

$$\begin{aligned} I &= x^{(r)}(k+1)-x^{(r)}(k-1)+\int_{k-1}^{k+1} x^{(r)}(t)dt \\ &= x^{(r)}(k+1)-x^{(r)}(k-1)+\frac{a}{3}\left[x^{(r)}(k-1)+4x^{(r)}(k)+x^{(r)}(k+1)\right], \\ &= 2\left(b_2k^2+b_1k+b_0\right)+\frac{2b_2}{3} \end{aligned} \tag{28}$$

By contrast with equation (26), we obtain:

$$z^{(r)}(k)=\frac{1}{6}\left[x^{(r)}(k-1)+4x^{(r)}(k)+x^{(r)}(k+1)\right], \tag{29}$$

Further simplify equation (28):

$$\begin{aligned} \frac{a}{6}\left[x^{(r)}(k-1)+4x^{(r)}(k)+x^{(r)}(k+1)\right]+\frac{(x^{(r)}(k+1)-x^{(r)}(k-1))}{2} &= b_2k^2+b_1k+b_0+\frac{b_2}{3} \\ &= b_2\left(k^2+\frac{1}{3}\right)+b_1k+b_0 \\ k &= 2,3,...,n\text{-}1 \end{aligned} \tag{30}$$

Let $\hat{e}=\left[a,b_2,b_1,b_0\right]^T$ be the parameter set. According to the principle of least square method, the parameter set shall meet the following conditions:

$$\hat{e}=\left(P^TP\right)^{-1}P^TY, \tag{31}$$

where

$$P=\begin{pmatrix} -\frac{\left(x^{(r)}(1)+4x^{(r)}(2)+x^{(r)}(3)\right)}{6} & 2^2+\frac{1}{3} & 2 & 1 \\ -\frac{\left(x^{(r)}(2)+4x^{(r)}(3)+x^{(r)}(4)\right)}{6} & 3^2+\frac{1}{3} & 3 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -\frac{\left(x^{(r)}(n-2)+4x^{(r)}(n-1)+x^{(r)}(n)\right)}{6} & n^2+\frac{1}{3} & n & 1 \end{pmatrix}, \tag{32}$$

$$Y=\begin{pmatrix} \frac{x^{(r)}(3)-x^{(r)}(1)}{2} \\ \frac{x^{(r)}(4)-x^{(r)}(2)}{2} \\ \vdots \\ \frac{x^{(r)}(n)-x^{(r)}(n-2)}{2} \end{pmatrix}, \tag{33}$$

Hence, the time response sequence of SFGM(1,1,k^2) model improved based on Simpson formula is:

$$\hat{x}^{(r)}(k)=\left(x^{(0)}(1)-\frac{2b_2}{a^3}+\frac{2b_2+b_1}{a^2}+\frac{b_2+b_1+b_0}{a}\right)e^{-a(k-1)}+b_2\left(\frac{k^2}{a}-\frac{2k}{a^2}+\frac{2}{a^3}\right)+b_1\left(\frac{k}{a}-\frac{1}{a^2}\right)+\frac{b_0}{a} \tag{34}$$

After reduction, the predicted value of $X^{(0)}$ is

$$\hat{X}^{(0)}=\hat{X}^{(r)}D^r, \tag{35}$$

where $\hat{X}^{(0)}=\left(\hat{x}^{(0)}(1),\hat{x}^{(0)}(2),\cdots,\hat{x}^{(0)}(n\text{-}1),\hat{x}^{(0)}(n)\right)$, $\hat{X}^{(r)}=\left(\hat{x}^{(r)}(1),\hat{x}^{(r)}(2),\cdots,\hat{x}^{(r)}(n\text{-}1),\hat{x}^{(r)}(n)\right)$

4.3 Determination of the order of the model

When modelling and solving the fractional order grey prediction model, the selection of model order *r* will directly affect the solution result of model parameters, and then affect the prediction effect of the model. Therefore, the selection of the order *r* of fractional grey system model is very important. In this paper, we established an optimization problem with *SMAPE* as the objective function to minimize the modelling data, and the order *r* of the model is solved by using MATLAB optimization algorithm toolbox.

$$\min_r SMAPE=\frac{1}{n-1}\sum_{i=2}^{n}\left|\frac{\hat{x}^{(0)}(k)-x^{(0)}(k)}{x^{(0)}(k)}\right|$$

$$s.t.\begin{cases}P=\begin{pmatrix}-\dfrac{\left(x^{(r)}(1)+4x^{(r)}(2)+x^{(r)}(3)\right)}{6} & 2^2+\dfrac{1}{3} & 2 & 1\\ -\dfrac{\left(x^{(r)}(2)+4x^{(r)}(3)+x^{(r)}(4)\right)}{6} & 3^2+\dfrac{1}{3} & 3 & 1\\ \vdots & \vdots & \vdots & \vdots\\ -\dfrac{\left(x^{(r)}(n-2)+4x^{(r)}(n-1)+x^{(r)}(n)\right)}{6} & n^2+\dfrac{1}{3} & n & 1\end{pmatrix}\\[2ex] Y=\begin{pmatrix}\dfrac{x^{(r)}(3)-x^{(r)}(1)}{2}\\ \dfrac{x^{(r)}(4)-x^{(r)}(2)}{2}\\ \vdots\\ \dfrac{x^{(r)}(n)-x^{(r)}(n-2)}{2}\end{pmatrix}\\ \hat{e}=\left[a,b_2,b_1,b_0\right]^T=\left(P^TP\right)^{-1}P^TY\\ \hat{x}^{(r)}(k)=\left(x^{(0)}(1)-\dfrac{2b_2}{a^3}+\dfrac{2b_2+b_1}{a^2}+\dfrac{b_2+b_1+b_0}{a}\right)e^{-a(k-1)}+b_2\left(\dfrac{k^2}{a}-\dfrac{2k}{a^2}+\dfrac{2}{a^3}\right)+b_1\left(\dfrac{k}{a}-\dfrac{1}{a^2}\right)+\dfrac{b_0}{a}\\ \hat{x}^{(0)}(k)=\sum_{i=0}^{k-1}\hat{x}^{(r)}(i+1)\begin{pmatrix}-r\\ k-i-1\end{pmatrix}\end{cases}$$

4.4 Modelling procedure

In this subsection, we summarize the modelling procedure of the proposed fractional SFGM(1,1,k^2) model into following five steps according to the previous derivation.

Step 1: Generate r order accumulated generated operation(r-AGO) sequence $X^{(r)}$ according to the original data sequence $X^{(0)}$. The r-AGO sequence $X^{(r)}$ is calculated by $X^{(r)} = X^{(0)}A^r$.

Step 2: Generate the background value sequence $Z^{(r)}$ calculated by consecutive three neighbors of $X^{(r)}$ with the formula $z^{(r)}(k) = 1/6\left(x^{(r)}(k-1) + 4x^{(r)}(k) + x^{(r)}(k+1)\right), k = 2,3,\ldots,n-1$,which improved based on the Simpson formula.

Step 3: Estimate the parameters set $\hat{u} = \left[a, b_2, b_1, b_0\right]^T$ based on Theorem 1.

Step 4: Obtain the time response sequence $\hat{X}^{(r)}$ according to Theorem 2 and Theorem 3.

Step 5: Obtain the stored values $\hat{X}^{(0)}$ based on time response sequence $\hat{X}^{(r)}$ and the inverse accumulated generated operation.

5 Validation

In this section, we first give the metrics for evaluating the models to prepare for the subsequent validation and application. The definition and calculation formula for these metrics is shown in subsection 5.1.

Then, we demonstrate the performances of the SFGM(1,1,k^2) model with two numerical cases, all the detailed modelling process is also shown in the 2nd and 3rd subsection. And the modelling results are used for comparative studies with the FGM(1,1) prediction model and the FGM(1,1,k) prediction model, as well as the FGM(1,1,k^2) prediction model.

5.1 Metrics for evaluating of the models

The model error is an important metrics to test the practicability and effectiveness of a prediction model. In this paper, the absolute percentage error$\left(APE(k)\right)$ is taken as the basis metrics to examine the practical modelling effect and prediction performance of the model. The magnitude of its value represents the degree of relative deviation between the true value of the kth data and the predicted value. The calculation formula is as follows:

$$APE(k) = \left|\frac{\hat{x}^{(0)}(k) - x^{(0)}(k)}{x^{(0)}(k)}\right| \times 100 \tag{36}$$

where $APE(k)$ represents the absolute simulation percentage error when $k = 2,3,\ldots,n\text{-}1,n$, n represents the number of data used for simulation modelling, and $APE(k)$ represents the absolute predicted percentage error when $k = n+1, n+2, \ldots N\text{-}1, N$, N represents the number of data used for prediction.

Furthermore, we take the mean absolute percentage error $(MAPE)$ as the evaluation standard and optimization target of the model, and the value of $MAPE$ describes the deviation degree between the real value and the predicted value. The calculation formula is as follows:

$$MAPE = \frac{1}{n}\sum_{k=1}^{n} APE(k) \tag{37}$$

Which means:

$$MAPE = \frac{1}{n}\sum_{k=1}^{n}\left|\frac{\hat{x}^{(0)}(k)-x^{(0)}(k)}{x^{(0)}(k)}\right|\times 100 \tag{38}$$

where n stands for the number of data sets to be tested. Considering the initial condition ($x^{(0)}(1)=\hat{x}^{(0)}(1)$)has no influence on the model evaluation when solving the fractional grey models, the metrics of the following three evaluation models are given:

The Simulated Mean Absolute Percent Error (SMAPE):

$$SMAPE = \frac{1}{n-1}\sum_{k=2}^{n}\left|\frac{\hat{x}^{(0)}(k)-x^{(0)}(k)}{x^{(0)}(k)}\right|\times 100 \tag{39}$$

The Predicted Mean Absolute Percent Error (PMAPE):

$$PMAPE = \frac{1}{N-n}\sum_{k=n+1}^{N}\left|\frac{\hat{x}^{(0)}(k)-x^{(0)}(k)}{x^{(0)}(k)}\right|\times 100 \tag{40}$$

The Total Mean Absolute Percent Error (TMAPE):

$$TMAPE = \frac{1}{N-1}\sum_{k=2}^{N}\left|\frac{\hat{x}^{(0)}(k)-x^{(0)}(k)}{x^{(0)}(k)}\right|\times 100 \tag{41}$$

5.2 Numerical case 1

In this subsection we study the national consumer price index (CPI for short) to validate the performance of the proposed model. The raw data of the national CPI from 1999 to 2006 are used to establish models and are tabulated in Table 1. The raw data from 2007 to 2011 are compared with the FGM(1,1), the FGM(1,1,k) ,the FGM(1,1,k^2), as well as the SFGM(1,1,k^2). All the raw data are collected from the China Statistical Yearbook from 1999 to 2011.

Table 1. 1999-2006 the national consumer price index (1978=100)

Year	1999	2000	2001	2002	2003	2004	2005	2006
National CPI	432.2	434	437	433.5	438.7	455.8	464	471

According to the data in the Table 1, we establish the fractional grey model. Meanwhile, different fractional grey models are solved according to the principle of minimum error, and different time response sequences are obtained as shown in Table 2.

Through the corresponding time response sequence, the modelling value of the national CPI are calculated and express in Table 3 and the prediction results of a several fractional models are shown in Fig. 2.

The results show SFGM(1,1,k^2) has more accurate prediction effect than other fractional grey models. Further, the national CPI was extrapolated according to different time response equations. We obtain the predicted results of national CPI from 2007 to 2010 of different fractional grey models and compared them with the raw data as shown in the following Table 4. And we draw the error comparison figure in order to more intuitively show the forecast effect of different fractional grey models.

Table 2. The time response sequences of several fractional grey models

Models	Optimal order *r*	The time response sequences of models
FGM(1,1)	1.08	$\hat{x}^{(1.08)}(k)=13876.60e^{0.0333(k-1)}$-13444.40
FGM(1,1,k)	-0.18	$\hat{x}^{(1.08)}(k)=142.63e^{-0.7215(k-1)}+0.70k+288.87$
FGM(1,1,k^2)	-0.26	$\hat{x}^{(-0.26)}(k)=675.73e^{-0.7958(k-1)}-0.46k^2+4.42k+230.79$
SFGM(1,1,k^2)	-0.74	$\hat{x}^{(-0.74)}(k)=484.74e^{-1.6377(k-1)}-0.57k^2+4.01k+45.01$

Table 3. The validation results of several fractional grey prediction models

Year	Raw data	FGM(1,1)	FGM (1,1,k)	FGM(1,1,k^2)	SFGM (1,1, k^2)
1999	432.2	432.2	432.2	432.2	432.2
2000	434	435.4957	436.2881	436.2115	443.1033
2001	437	432.4847	434.3451	429.9394	432.6854
2002	433.5	436.1011	436.9377	430.7491	431.8095
2003	438.7	442.8962	443.1912	437.145	438.1955
2004	455.8	451.6742	451.533	446.5541	448.6563
2005	464	461.8845	460.8685	457.2707	461.3822
2006	471	473.234	470.5581	468.3512	475.4192

Table 4. The prediction results of several fractional grey prediction models

Year	Raw data	FGM(1,1)	FGM (1,1,k)	FGM(1,1,k^2)	SFGM (1,1, k^2)
2007	493.6	485.5531	480.2599	479.3193	490.245
2008	522.7	498.7381	489.8058	489.9555	505.556
2009	519	512.7245	499.1232	500.173	521.1655
2010	536.1	527.4718	508.1897	509.9513	536.9533
PMAPE		2.2582	4.5845	4.4156	**1.1340**
TMAPE		1.2548	2.0923	2.2562	**1.0177**

As can be seen from the data in the Table 4, the $PMAPE$ of FGM(1,1) is 2.2582, that of FGM(1,1,k) is 4.5845, that of FGM(1,1,k^2) is 4.4156 and that of SFGM(1,1,k^2) is 1.1340, respectively. And the $TMAPE$ of FGM(1,1) is 1.2548, that of FGM(1,1,k) is 2.0923, that of FGM(1,1,k^2) is 2.2562 and that of SFGM(1,1,k^2) is 1.0177, respectively.

The Fig. 3 shows more intuitively that the total mean absolute error and the prediction mean absolute error of the proposed model are less than others, it also imply that the SFGM(1,1,k^2) model performance more accurate prediction effect so that validate the reliability of the proposed model.

5.3 Numerical case 2

In the same way, this subsection we study the urban consumer price index(CPI for short) from 1999 to 2011, the raw data from 1999 to 2006 are used to establish models and are tabulated in Table 5, the left raw data from 2007 to 2011 are compared with the proposed model and other fractional grey models. All the raw data are collected from the China Statistical Yearbook from 1999 to 2011.

According to the data in the Table 5, we establish the different fractional grey models, and the optimal order *r* and the time response sequence of different fractional models are calculated as Table 6.

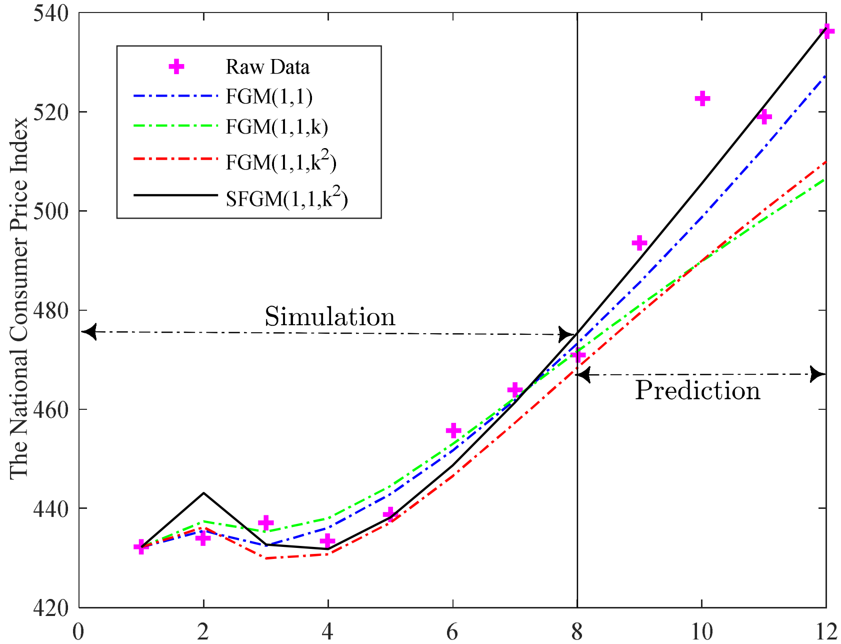


Fig. 2. The forecast results of different fractional models on the national CPI

Table 5. The urban consumer price index from 1999-2006 (1978=100)

Year	1999	2000	2001	2002	2003	2004	2005	2006
Urban CPI	472.8	476.6	479.9	475.1	479.4	495.2	503.1	510.6

Table 6. The time response sequences of different fractional grey models

Models	Optimal order *r*	The time response sequences of models
FGM(1,1)	1.07	$\hat{x}^{(1.07)}(k)=17564.24e^{0.0287(k-1)}\text{-}17091.44$
FGM(1,1,k)	-0.17	$\hat{x}^{(-0.17)}(k)=110.48e^{-0.7215(k-1)}+0.70k+321.01$
FGM(1,1,k^2)	-0.22	$\hat{x}^{(-0.22)}(k)=723.14e^{-0.7958(k-1)}\text{-}0.23k^2+1.86k+285.54$
SFGM(1,1,k^2)	-0.44	$\hat{x}^{(-0.44)}(k)=596.41e^{-0.9785(k-1)}\text{-}0.77k^2+5.33k+152.02$

Through the corresponding time response sequence, the modelling value of the urban CPI are calculated and express in Table 7, and the prediction results of a series of fractional models are shown in Fig 4.

It can be seen from the Fig 4, the prediction effect of the SFGM(1,1,k^2) prediction model is better than others. Similarly, the following Table 8 shows the prediction effect of different fractional grey models on urban CPI, and draws the error comparison chart under different prediction models.

From the Table 4 we know that the $PMAPE$ of FGM(1,1) is 2.4414, that of FGM(1,1,k) is 3.5714, that of FGM(1,1,k^2) is 2.7555 and that of SFGM(1,1,k^2) is 1.6939, respectively. And the $TMAPE$ of FGM(1,1) is 1.4301, that of FGM(1,1,k) is 1.9411, that of FGM(1,1,k^2) is 1.5858 and that of SFGM(1,1,k^2) is 1.3965, respectively.

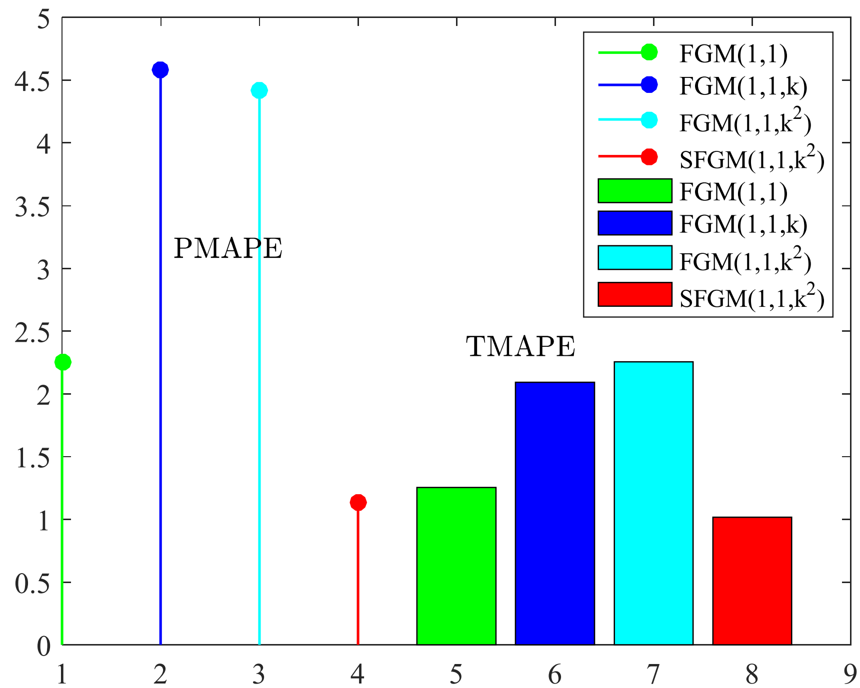


Fig. 3. The error comparison of different fractional grey models

Table 7. The validation results of several fractional grey prediction models

Year	Raw data	FGM(1,1)	FGM (1,1,k)	FGM(1,1,k^2)	SFGM (1,1, k^2)
1999	472.8	472.8	472.8	472.8	472.8
2000	476.6	477.948	479.1914	480.2695	483.4546
2001	479.9	474.7504	476.8427	476.1433	472.1995
2002	475.1	477.8659	478.5743	477.7076	470.9612
2003	479.4	484.0047	484.0684	484.1334	478.6147
2004	495.2	492.0113	491.8591	493.2598	491.4074
2005	503.1	501.3478	500.8247	503.5691	506.7579
2006	510.6	511.7245	510.2697	514.1907	523.2107

Table 8. The prediction results of several fractional models

Year	Raw data	FGM(1,1)	FGM (1,1,k)	FGM(1,1,k^2)	SFGM (1,1, k^2)
2007	533.6	522.9709	519.8012	524.6735	539.9992
2008	563.5	534.9806	529.2132	534.8057	556.7329
2009	558.4	547.685	538.4077	544.5038	573.2186
2010	576.3	561.0389	547.3466	553.7492	589.3654
PMAPE		2.4414	3.5714	2.7555	**1.6939**
TMAPE		1.4301	1.9411	1.5858	**1.3965**

Combining the results in the Table 4 with the Fig. 5, we can see that the proposed SFGM(1,1,k^2)model shows better prediction effect and validate that the proposed model is effective and reliable.

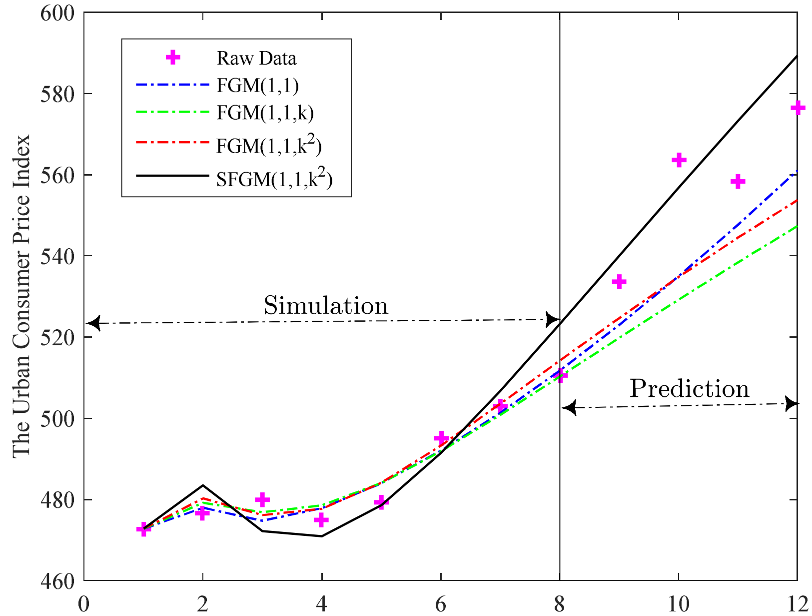


Fig. 4. The forecast results of different fractional models on the urban CPI

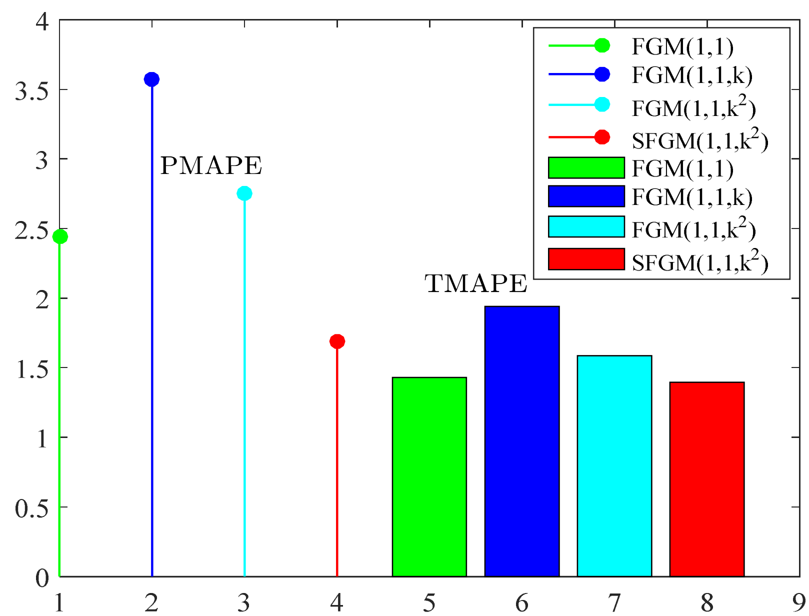


Fig. 5. The error comparison of different fractional grey models

6 Application

According to our knowledge, oil is the most important lifeblood of modern industry, which is called "the blood of industry". As an important non-renewable energy and strategic resources, the use and consumption of oil will directly affect the survival and development of human beings and play an inestimable role in national security and economic and social development. In order to more effectively demonstrate the effectiveness and practicability of the model, we studied the oil consumption in China, Asia Pacific and the total world as cases for application analysis.

In this paper, the raw data of the oil consumption from 2007 to 2018 are collected from the BP Statistical Review of World Energy 2019. We divided the data in each case into two groups, and established different fractional grey models based on the data of eight years from 2007 to 2014. The remaining data from 2015 to 2018 were used to test and contrast the prediction results of different prediction models. Finally, we gave the prediction data of different fractional-order prediction models for the next three years from 2019 to 2021. All these results are tabulated in Table 9, Table 10 and Table 11. Meanwhile, we show the prediction errors of different models in the table, and further intuitively show them in Fig. 6, Fig. 7 and Fig. 8.

6.1 Case 1: Forecasting oil consumption in China

This section mainly studied the oil consumption in China from 2007 to 2018. We used the oil consumption data of China from 2007 to 2014 to establish different fractional grey models, and solved the optimal orders of FGM(1,1,k), FGM(1,1,k^2) and SFGM(1,1,k^2) models: 0.28, 0.24, -0.40 and -0.42 respectively. The $PMAPE$ of FGM(1,1) is 5.4592, that of FGM(1,1,k) is 5.0050, that of FGM(1,1,k^2) is 1.9703 and that of SFGM(1,1,k^2) is 1.0126,respectively. And the $TMAPE$ of FGM(1,1) is 2.7163, that of FGM(1,1,k) is 2.5278, that of FGM(1,1,k^2) is 1.7233 and that of SFGM(1,1,k^2) is 1.2528 respectively.

6.2 Case 2: Forecasting oil consumption in total Asia Pacific

In this subsection, let the oil consumption in total Asia Pacific from 2007 to 2018 as the research object, through modelling and solving different fractional grey prediction models, and obtained the optimal order numbers of the FGM(1,1,k), FGM(1,1,k^2) and SFGM(1,1,k^2) models: 0.16, 0.17, -0.34 and -0.3, respectively. The $PMAPE$ of FGM(1,1) is 5.1004, that of FGM(1,1,k) is 5.3274, that of FGM(1,1,k^2) is 2.3014 and that of SFGM(1,1,k^2) is 1.3775,respectively. And the $TMAPE$ of FGM(1,1) is 2.2026, that of FGM(1,1,k) is 2.2943, that of FGM(1,1,k^2) is 1.7141 and that of SFGM(1,1,k^2) is 1.4238 respectively.

6.3 Case 3: Forecasting oil consumption in total world

Similarly, in this section we conduct a predictive analysis of global oil consumption. The global oil consumption from 2007 to 2018 is still the research object. By solving the established fractional order gray prediction model, the optimal orders corresponding to FGM(1,1,k), FGM(1,1,k^2) and SFGM(1,1,k^2) are respectively 0.06, 0.06, -0.18 and -0.12. The $PMAPE$ of FGM(1,1) is 2.2717, that of FGM(1,1,k) is 2.2826, that of FGM(1,1,k^2) is 2.8054 and that of SFGM(1,1,k^2) is 0.5179,respectively. And the $TMAPE$ of FGM(1,1) is 1.1121, that of FGM(1,1,k) is 1.1327, that of FGM(1,1,k^2) is 1.7670 and that of SFGM(1,1,k^2) is 0.7099 respectively.

7 Discussion

In comparison with the classical FGM(1,1) model, the FGM(1,1,k) model and the FGM(1,1,k^2) model, the SFGM(1,1,k^2) model proposed in this paper can make more accurate prediction results, which effectively describe the evolution trend of future oil consumption. These results indicate that in the next three years(2019-2021), the oil consumption in China would reach 13639.02, 14040.8 and 14427.59 barrels respectively, the oil consumption in total Asia Pacific would reach 36890.37, 37673.93, 38421.93 barrels respectively, and the oil consumption total world would reach 100198.2, 101091.5, 101930.8 barrels respectively.

Table 9. Prediction results of oil consumption in China

Year	Raw data	FGM(1,1)	*APE*	FGM(1,1,k)	*APE*	FGM(1,1,k^2)	*APE*	SFGM(1,1,k^2)	*APE*
		***r*=0.28**		***r*=0.24**		***r*=-0.40**		***r*=-0.42**	
2007	7784	7784	0.0000	7784	0.0000	7784	0.0000	7784	0.0000
2008	7914	7842.661	0.9031	7870.637	0.5496	8106.47	2.4303	7852.435	0.7796
2009	8295	8512.597	2.6226	8505.416	2.5361	8527.245	2.7992	8338.071	0.5187
2010	9446	9189.714	2.7084	9159.29	3.0305	9084.623	3.8211	8974.323	4.9888
2011	9808	9802.42	0.0555	9762.035	0.4673	9668.507	1.4209	9615.588	1.9605
2012	10242	10341.07	0.9647	10301.72	0.5805	10234.61	0.0747	10225.22	0.1664
2013	10750	10809.11	0.5540	10779.73	0.2808	10770.9	0.1986	10798.06	0.4513
2014	11239	11213.04	0.2341	11200.99	0.3414	11276.49	0.3303	11336.58	0.8650
2015	11986	11559.74	3.5552	11571.2	3.4595	11753.76	1.9365	11844.64	1.1782
2016	12304	11855.65	3.6416	11895.91	3.3145	12205.71	0.7965	12325.94	0.1807
2017	12840	12106.6	5.7143	12180.22	0.5141	12635.21	1.5976	12783.7	0.4412
2018	13525	12317.78	8.9256	12428.77	8.1051	13044.76	3.5506	13220.63	2.2503
2019		12493.77		12645.66		13436.52		13639.02	
2020		12638.62		12834.56		13812.31		14040.8	
2021		12755.89		12998.7		14173.71		14427.59	
SMAPE			1.1480		**1.1123**		1.5822		1.3900
PMAPE			5.4592		5.0050		1.9703		**1.0126**
TMAPE			2.7163		2.5278		1.7233		**1.2528**

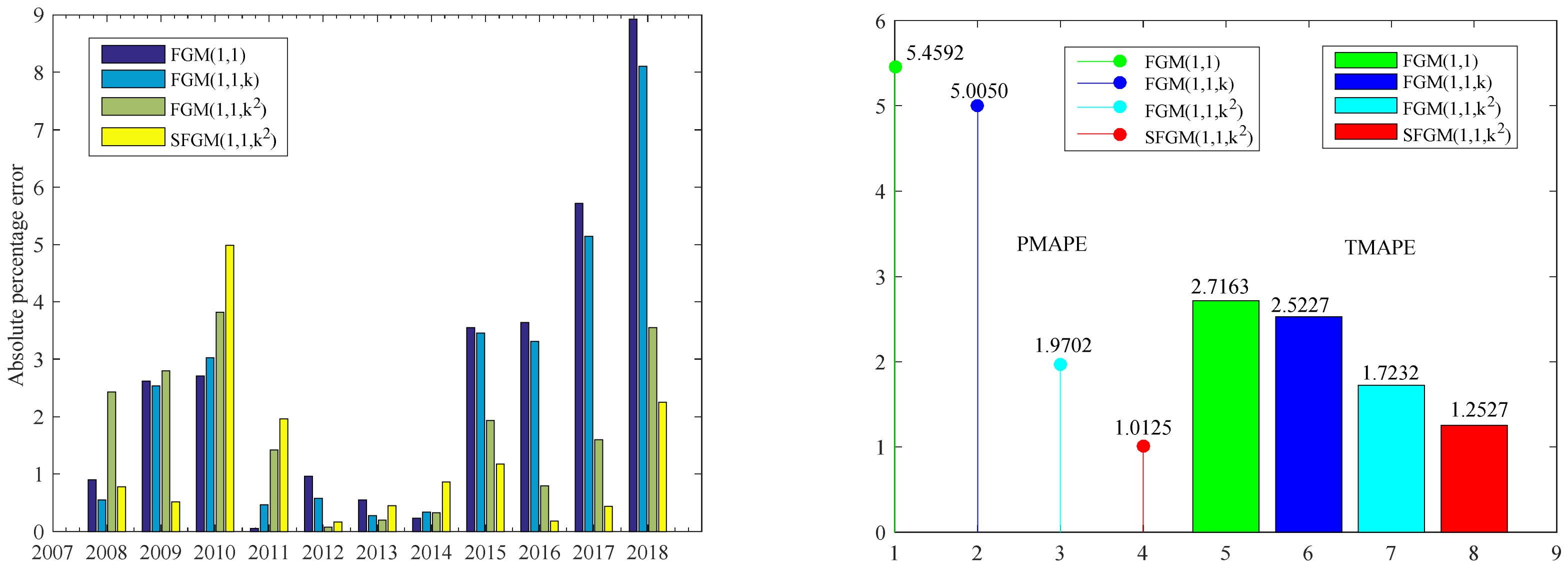


Fig. 6. Error comparison of oil consumption in China

Table 10. Prediction results of oil consumption in Asia Pacific

Year	Raw data	FGM(1,1)	*APE*	FGM(1,1,k)	*APE*	FGM(1,1,k^2)	*APE*	SFGM(1,1,k^2)	*APE*
		***r*=0.16**		***r*=0.17**		***r*=-0.34**		***r*=-0.3**	
2007	26078	26078	0.0000	26078	0.0000	26078	0.0000	26078	0.0000
2008	25940	25727.48	0.8185	25704.64	0.9067	26203.88	1.0180	25594.81	1.3300
2009	26351	26765.96	1.5728	26794.57	1.6814	26543	0.7267	26167.32	0.6989
2010	28043	27918.86	0.4420	27993.03	0.1775	27500.32	1.9345	27348.4	2.4763
2011	28942	28981.4	0.1346	29084.75	0.4917	28710.31	0.8020	28661.87	0.9694
2012	30094	29908.74	0.6158	30022.5	0.2378	29971.88	0.4060	29940.53	0.5102
2013	30759	30697.37	0.1994	30802.34	0.1418	31204.37	1.4489	31140.99	1.2428
2014	31343	31356.59	0.0440	31433.84	0.2905	32381.26	3.3132	32259.39	2.9244
2015	32551	31899.55	2.0018	31930.72	1.9060	33497.32	2.9067	33302.68	2.3087
2016	33743	32339.92	4.1594	32307.42	4.2558	34554.98	2.4049	34279.72	1.5892
2017	34835	32690.73	6.1557	32577.81	6.4799	35559.02	2.0781	35198.78	1.0440
2018	35863	32963.86	8.0846	32754.69	8.6679	36514.55	1.8160	36067	0.5681
2019		33169.91		32849.64		37426.39		36890.37	
2020		33318.28		32873.02		38298.83		37673.93	
2021		33417.21		32834.03		39135.61		38421.93	
SMAPE			**0.5467**		0.5611		1.3785		1.4503
PMAPE			5.1004		5.3274		2.3014		**1.3775**
TMAPE			2.2026		2.2943		1.7141		**1.4238**

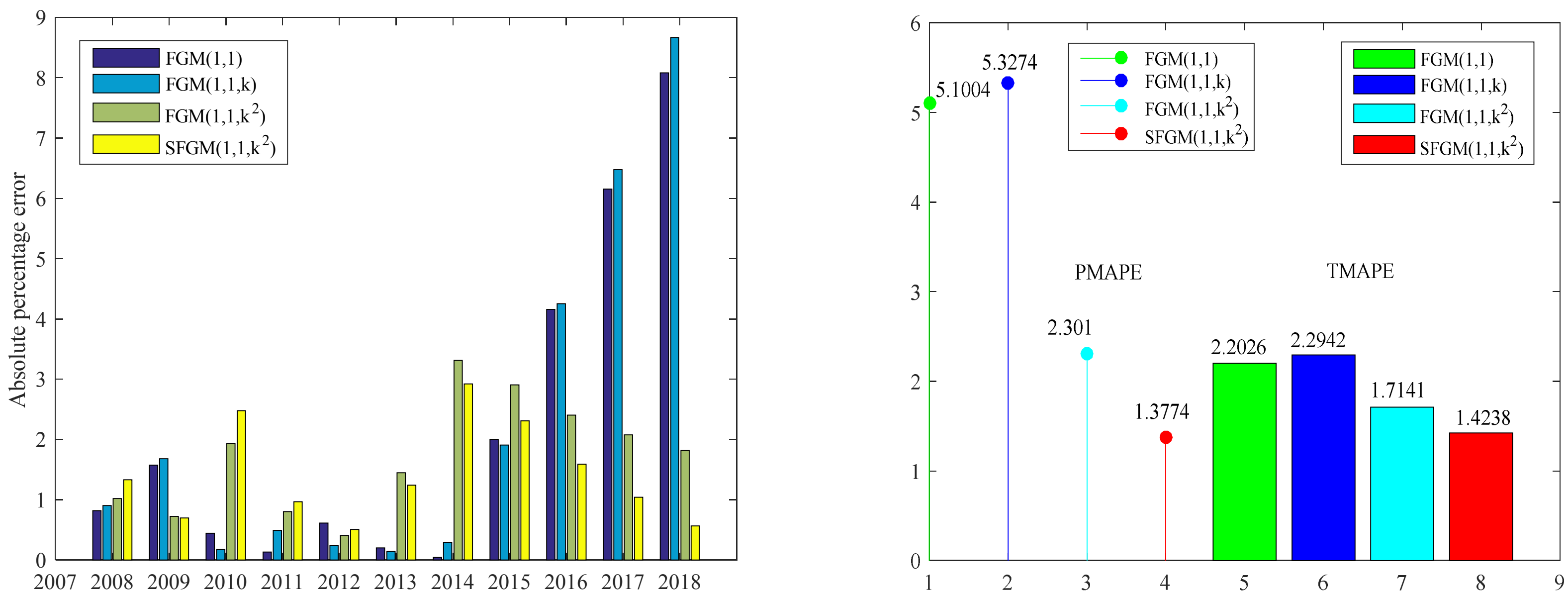


Fig 7. Error comparison of oil consumption in Asia Pacific

Table 11. Prediction results of oil consumption in total world

year	raw data	FGM(1,1)	*APE*	FGM(1,1,k)	*APE*	FGM(1,1,k^2)	*APE*	SFGM(1,1,k^2)	*APE*
		r=0.06		*r*=0.06		*r*=-0.18		*r*=-0.12	
2007	87191	87191	0.0000	87191	0.0000	87191	0.0000	87191	0.0000
2008	86619	86001.13	0.7132	85992.41	0.7233	86815.39	0.2268	85845.58	0.8928
2009	85780	86922.85	1.3319	86897.36	1.3023	86681.82	0.1051	86365.55	0.6823
2010	88730	88249.25	0.5413	88204.65	0.5916	87904.11	0.9303	87939.06	0.8910
2011	89763	89621.26	0.1574	89559.41	0.2264	89785.81	0.0258	89755.26	0.0081
2012	90724	90923.72	0.2196	90849.67	0.1380	91869.19	1.2618	91509.59	0.8654
2013	92276	92115.92	0.1730	92037.05	0.2585	93939.55	1.8032	93118.85	0.9138
2014	93194	93185.78	0.0091	93111.21	0.0892	95912.26	2.9164	94576.94	1.4836
2015	95048	94133.63	0.9615	94073.67	1.0246	97762.39	2.8563	95899.91	0.8968
2016	96737	94965.37	1.8317	94931.13	1.8671	99489.93	2.8454	97107.35	0.3825
2017	98406	95689.34	2.7602	95692.38	2.7571	101104	2.7422	98216.9	0.1917
2018	99843	96314.76	3.5336	96366.85	3.4815	102616.2	2.7776	99243.23	0.6006
2019		96850.9		96963.8		104037.7		100198.2	
2020		97306.71		97492.02		105378.8		101091.5	
2021		97690.6		97959.62		106648.4		101930.8	
SMAPE			**0.4494**		0.4756		1.1736		0.8196
PMAPE			2.2717		2.2826		2.8054		**0.5179**
TMAPE			1.1121		1.1327		1.7670		**0.7099**

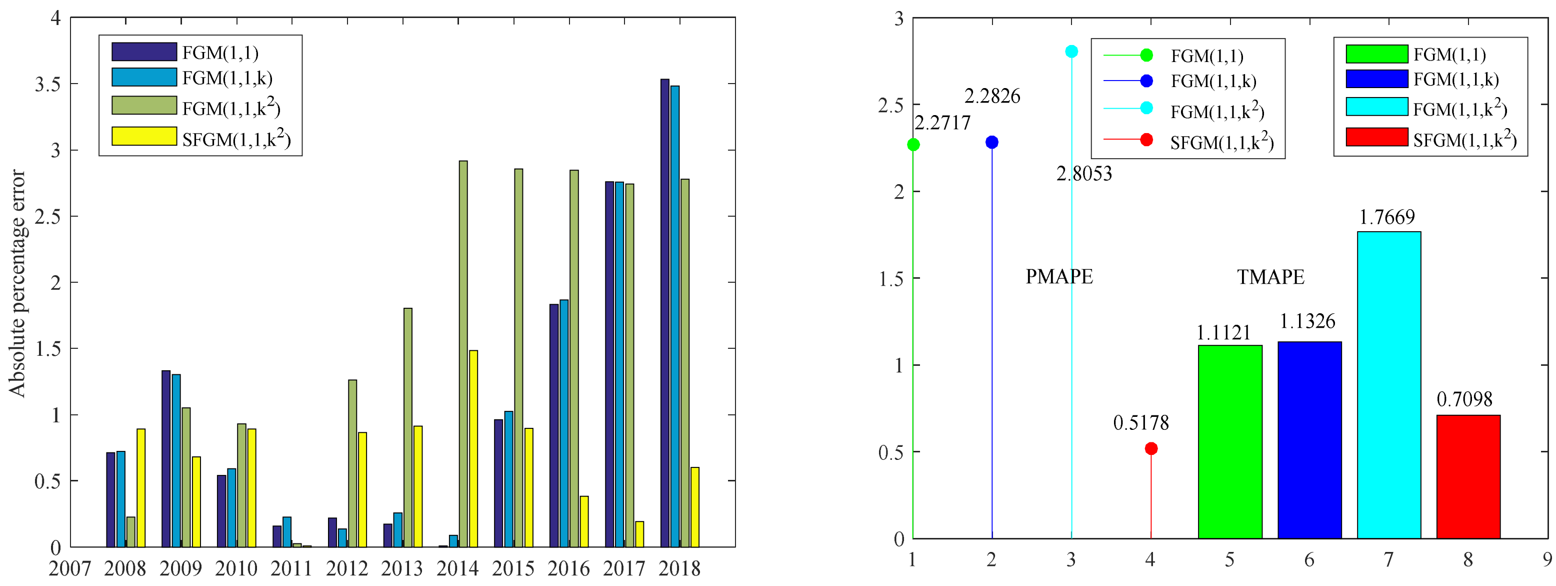


Fig 8. Error comparison of oil consumption in total world

From the forecast results, we can clearly understand the behavior of oil consumption in China, total Asia Pacific and the total world will gradually increase in the next three years. The average annual growth rate of the oil consumption in total world reached 0.9027%, and the average annual growth rate of the oil consumption in total Asia Pacific reached 2.7164%, which is significantly higher than the average annual growth rate of the total world. It is estimated that the oil consumption in Asia Pacific will account for 36.8173%, 37.2672% and 37.6941% of the oil consumption in total world in the next three years. And the average annual growth rate of the oil consumption in China in the next three years will reach 3.0431%, which is also higher than the average annual growth rate of the oil consumption in Asia Pacific and total world. The forecast results of the model predict that the proportion of the oil consumption in China will account for 13.6120%, 13.8892% and 14.1543% of the oil consumption in total world.

In terms of the energy source, the demand for oil will continue to rise in future, which is dominated by China and other Asia Pacific countries. These prediction results coincide with the outlook made by the British Petroleum (BP) this year, which again verifies the accuracy and reliability of the model. At the same time, these forecast results will provide important enlightenment for many countries to face the coming gradual transition of energy.

8 Conclusion

In this paper, a novel fractional grey system prediction model named as the SFGM$(1,1,k^2)$ model is proposed by considering the error from the background value of the model. With mathematical analysis, it is shown that the SFGM$(1,1,k^2)$ based on the Simpson formula can effectively reduce the error. The numerical cases in Section 5 validate the validity of the SFGM$(1,1,k^2)$ model.

In the end, The proposed model is employed to forecast the behavior of oil consumption in China, total Asia Pacific and total world, the results show that SFGM$(1,1,k^2)$ presents high accuracy. These prediction results are also confirmed by *BP energy outlook 2019-Global*, it also imply that the SFGM$(1,1,k^2)$ model proposed in this paper is reliable and will contribute to the development of human energy.

Competing Interests

Authors have declared that no competing interests exist.

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