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The Double Step Hybrid Linear Multistep Method for Solving Second Order Initial Value Problems

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Authors' contributions

This work was carried out in collaboration among all authors. Author YS developed the method. Author JZD analyzed the basic properties of the method and authors TYK and JS test the method on some highly stiff ordinary differential equations. All authors read and approved the final manuscript.

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Abstract

In this paper, we develop the double step hybrid linear multistep method for solving second order initial value problems via interpolation and collocation method of power series approximate solution to give a system of nonlinear equations which is solved to give a continuous hybrid linear multistep method. The continuous hybrid linear multistep method is solved for the independent solutions to give a continuous hybrid block method which is then evaluated at some selected grid points to give a discrete block method. The basic numerical properties of the hybrid block method was established and found to be zero-stable, consistent and convergent. The efficiency of the new method was conformed on some initial value problems and found to give better approximation than the existing methods.

Keywords: Double step; HLMM; IVPs; interpolation and collocation; power series.

1 Introduction

In this paper, we developed the double step hybrid linear multistep method for solving (1.1), which is implemented in block method. The method is self-starting and does not require starting values

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or predictors [1]. We consider the following equation which is a special second order initial value problem of the form

$$y^{''} = f\left(x, y, y^{'}\right), \ y(x_0) = y_0, \ y^{'}(x_0) = y_0^{'} \tag{1.1}$$

Where f is continuous within the interval of integration. It is of general knowledge that (1.1) above with such conditions imposed are known as initial value problems (IVPs).

Numerical solution of equations of this form (1.1) is a very keen area of interest for researchers in literature, ranging from first order IVPs [2-5] to higher order IVPs. The focus of this article however is on higher order IVPs (specifically, second order IVPs). Block methods for approximating the numerical solution of (1.1) have been vastly explored in literature [6]. The key advantage of the adoption of block method has been in the improved level of accuracy displayed in comparison to previously existing methods (see [7-9]).

The method of collocation and interpolation of the power series approximation to generate continuous linear multistep method has been adopted by many scholars; among others are [2-4,9-11].

The most commonly used approach for developing block methods has been the adoption of the approach seen in [12]. This is seen recent works of [2,8,13,14]. However, this article introduces another approach different from what is existing in literature on how hybrid block methods can also be developed [6]. Therefore, asides from the introduction of a new hybrid block method that performs better than previously existing methods, this article also introduces a new approach for the development of hybrid block methods.

2 Derivation of the Method

We consider a power series approximate solution of the form

$$y(x) = \sum_{j=0}^{s+r-1} a_j \left(\frac{x - x_n}{h}\right)^j \tag{2.1}$$

where $r = 2$ and $s = 8$ are the numbers of interpolation and collocation points respectively, which is considered to be a solution to (1.1). The second and third derivative of (2.1) give

$$y''(x) = \sum_{j=2}^{s+r-1} \frac{a_j \, j!}{h^2 (j-2)!} \left(\frac{x - x_n}{h}\right)^{j-2} = f\left(x, y, y'\right) \tag{2.2}$$

$$y'''(x) = \sum_{j=3}^{s+r-1} \frac{a_j \, j!}{h^3 (j-3)!} \left(\frac{x - x_n}{h}\right)^{j-3} = g\left(x, y, y'\right) \tag{2.3}$$

Substituting (2.2) and (2.3) into (1.1) gives

$$f\left(x, y, y''\right) = \sum_{j=2}^{s+r-1} \frac{a_j \, j!}{h^2 (j-2)!} \left(\frac{x - x_n}{h}\right)^{j-2} + \sum_{j=3}^{s+r-1} \frac{a_j \, j!}{h^3 (j-3)!} \left(\frac{x - x_n}{h}\right)^{j-3} \tag{2.4}$$

Collocating equation (2.3) at $x_{n+s}, \ s = 0, \frac{1}{3}, 1, \frac{5}{3}, 2$, equation (2.2) at $x_{n+s}, \ s = 0, 1, 2$ and interpolating equation (2.1) at $x_{n+r}, \ r = \frac{1}{3}, \frac{5}{3}$ which gives a system of nonlinear equation of the form

$$AX = U \tag{2.5}$$

Where

$$A = [a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9]^T$$

$$U = \left[y_{n+\frac{1}{3}}, y_{n+\frac{5}{3}}, f_n, f_{n+1}, f_{n+2}, g_n, g_{n+\frac{1}{3}}, g_{n+1}, g_{n+\frac{5}{3}}, g_{n+2} \right]^T$$

and

$$X = \begin{bmatrix} 1 & x_{n+\frac{1}{3}} & x^2_{n+\frac{1}{3}} & x^3_{n+\frac{1}{3}} & x^4_{n+\frac{1}{3}} & x^5_{n+\frac{1}{3}} & x^6_{n+\frac{1}{3}} & x^7_{n+\frac{1}{3}} & x^8_{n+\frac{1}{3}} & x^9_{n+\frac{1}{3}} \\ 1 & x_{n+\frac{5}{3}} & x^2_{n+\frac{5}{3}} & x^3_{n+\frac{5}{3}} & x^4_{n+\frac{5}{3}} & x^5_{n+\frac{5}{3}} & x^6_{n+\frac{5}{3}} & x^7_{n+\frac{5}{3}} & x^8_{n+\frac{5}{3}} & x^9_{n+\frac{5}{3}} \\ 0 & 0 & 2 & 6x & 12x^2 & 20x^3 & 30x^4 & 42x^5 & 56x^6 & 72x^7 \\ 0 & 0 & 2 & 6x_{n+1} & 12x^2_{n+1} & 20x^3_{n+1} & 30x^4_{n+1} & 42x^5_{n+1} & 56x^6_{n+1} & 72x^7_{n+1} \\ 0 & 0 & 2 & 6x_{n+2} & 12x^2_{n+2} & 20x^3_{n+2} & 30x^4_{n+2} & 42x^5_{n+2} & 56x^6_{n+2} & 72x^7_{n+2} \\ 0 & 0 & 0 & 6 & 24x & 60x^2 & 120x^3 & 210x^4 & 336x^5 & 504x^6 \\ 0 & 0 & 0 & 6 & 24x_{n+\frac{1}{3}} & 60x^2_{n+\frac{1}{3}} & 120x^3_{n+\frac{1}{3}} & 210x^4_{n+\frac{1}{3}} & 336x^5_{n+\frac{1}{3}} & 504x^6_{n+\frac{1}{3}} \\ 0 & 0 & 0 & 6 & 24x_{n+1} & 60x^2_{n+1} & 120x^3_{n+1} & 210x^4_{n+1} & 336x^5_{n+1} & 504x^6_{n+1} \\ 0 & 0 & 0 & 6 & 24x_{n+\frac{5}{3}} & 60x^2_{n+\frac{5}{3}} & 120x^3_{n+\frac{5}{3}} & 210x^4_{n+\frac{5}{3}} & 336x^5_{n+\frac{5}{3}} & 504x^6_{n+\frac{5}{3}} \\ 0 & 0 & 0 & 6 & 24x_{n+2} & 60x^2_{n+2} & 120x^3_{n+2} & 210x^4_{n+2} & 336x^5_{n+2} & 504x^6_{n+2} \end{bmatrix}$$

Solving (2.5) for a_j's using Gaussian elimination method, gives a continuous hybrid linear multistep method of the form

$$p(x) = \sum_{i=\frac{1}{3},\frac{5}{3}} \alpha_i y_{n+i} + h^2\left[\sum_{i=0,1,2} \beta_i f_{n+i}\right] + h^3\left[\sum_{i=\frac{1}{3},\frac{5}{3}} \gamma_i g_{n+i} + \sum_{i=0}^{3} \gamma_i g_{n+i}\right] \tag{2.6}$$

The coefficient of $y_{n+j},\ j = \frac{1}{3}, \frac{5}{3},\ f_{n+j},\ j = 0, 1, 2$ and $g_{n+j},\ j = 0, \frac{1}{3}, 1, \frac{5}{3}, 2$ are given by;

$$\alpha_{\frac{1}{3}} = \frac{5}{4} - \frac{3}{4}t, \quad \alpha_{\frac{5}{3}} = \frac{1}{4} + \frac{3}{4}t$$

$$\beta_0 = \frac{241891}{489888}h^2 - \frac{4493123}{2449440}th^2 + \frac{1}{2}t^2h^2 + \frac{565}{48}t^4h^2 - \frac{2271}{80}t^5h^2 + \frac{1757}{60}t^6h^2 - \frac{439}{28}t^7h^2 + \frac{957}{24}t^8h^2 - \frac{15}{32}t^9h^2$$

$$\beta_1 = \frac{239}{6804}h^2 - \frac{8}{105}th^2 - \frac{5}{3}t^4h^2 + \frac{17}{5}t^5h^2 + \frac{77}{30}t^6h^2 + \frac{6}{7}t^7h^2 - \frac{3}{28}t^8h^2$$

$$\beta_2 = -\frac{123019}{489888}h^2 + \frac{2230307}{2449440}th^2 - \frac{485}{48}t^4h^2 + \frac{1999}{80}t^5h^2 - \frac{1603}{60}t^6h^2 + \frac{415}{28}t^7h^2 - \frac{933}{224}t^8h^2 + \frac{15}{32}t^9h^2$$

$$\gamma_0 = \frac{89651}{1837080}h^3 - \frac{6438937}{36741600}th^3 + \frac{1}{6}t^3h^3 + \frac{17}{20}t^4h^3 - \frac{979}{400}t^5h^3 + \frac{66}{25}t^6h^3 - \frac{121}{84}t^7h^3 + \frac{111}{280}t^8h^3 - \frac{7}{160}t^9h^3$$

$$\gamma_{\frac{1}{3}} = \frac{5213}{2680}h^3 - \frac{725737}{907200}th^3 + \frac{27}{4}t^4h^3 - \frac{3159}{200}t^5h^3 + \frac{3213}{200}t^6h^3 - \frac{4779}{560}t^7h^3 + \frac{81}{35}t^8h^3 - \frac{81}{320}t^9h^3$$

$$\gamma_1 = \frac{44977}{20995}h^3 - \frac{5633267}{7348320}th^3 + \frac{95}{12}t^4h^3 - \frac{97}{5}t^5h^3 + \frac{491}{24}t^6h^3 - \frac{1877}{168}t^7h^3 + \frac{99}{32}t^8h^3 - \frac{11}{32}t^9h^3$$

$$\gamma_{\frac{5}{3}} = \frac{1369}{10080}h^3 - \frac{445801}{907200}th^3 + \frac{27}{5}t^4h^3 - \frac{2673}{200}t^5h^3 + \frac{1431}{100}t^6h^3 - \frac{891}{112}t^7h^3 + \frac{2511}{1120}t^8h^3 - \frac{81}{320}t^9h^3$$

$$\gamma_2 = \frac{86719}{3674160}h^3 - \frac{3149689}{36741600}th^3 + \frac{23}{24}t^4h^3 - \frac{947}{400}t^5h^3 + \frac{379}{150}t^6h^3 - \frac{587}{420}t^7h^3 + \frac{219}{560}t^8h^3 - \frac{7}{160}t^9h^3$$

Where $t = \frac{x - x_n}{h}$

Differentiating (2.6) once yields

$$p'(x) = \frac{1}{h}\sum_{j=\frac{1}{3},\frac{5}{3}} \alpha_j y_{n+j} + h\left[\sum_{j=0,1,2} \beta_j f_{n+j}\right] + h^2\left[\sum_{j=\frac{1}{3},\frac{5}{3}} \gamma_j g_{n+j} + \sum_{j=0}^{3} \gamma_j g_{n+j}\right] \quad (2.7)$$

The coefficient of first derivative f_{n+j} and g_{n+j} gives

$$\alpha'_0 = -\frac{4493123}{2449440}h^2 + th^2 + \frac{565}{12}t^3h^2 - \frac{2271}{16}t^4h^2 + \frac{1757}{10}t^5h^2 - \frac{439}{4}t^7h^2 + \frac{957}{28}t^8h^2 - \frac{135}{32}t^9h^2$$

$$\alpha'_1 = -\frac{8}{105}h^2 - \frac{20}{3}t^3h^2 + 17t^4h^2 - \frac{77}{5}t^5h^2 + 6t^6h^2 - \frac{6}{7}t^7h^2$$

$$\alpha'_2 = \frac{2230307}{2449440}th^2 - \frac{485}{12}t^3h^2 + \frac{1999}{16}t^4h^2 - \frac{1603}{10}t^5h^2 + \frac{415}{4}t^6h^2 - \frac{933}{28}t^7h^2 + \frac{135}{32}t^8h^2$$

$$\beta'_0 = -\frac{6438937}{36741600}h^3 + \frac{1}{2}t^2h^3 + \frac{17}{5}t^3h^3 - \frac{979}{80}t^4h^3 + \frac{396}{25}t^5h^3 - \frac{121}{12}t^6h^3 + \frac{111}{35}t^7h^3 - \frac{63}{160}t^8h^3$$

$$\beta'_{\frac{1}{3}} = -\frac{725737}{907200}h^3 + 27t^3h^3 - \frac{3159}{40}t^4h^3 + \frac{9639}{100}t^5h^3 - \frac{4779}{80}t^6h^3 + \frac{648}{35}t^7h^3 - \frac{720}{320}t^8h^3$$

$$\beta'_1 = -\frac{5633267}{7348320}h^3 + \frac{95}{3}t^3h^3 - 97t^4h^3 + \frac{491}{4}t^5h^3 - \frac{1877}{24}t^6h^3 + \frac{99}{4}t^7h^3 - \frac{99}{32}t^8h^3$$

$$\beta'_{\frac{5}{3}} = -\frac{445801}{907200}h^3 + \frac{108}{5}t^3h^3 - \frac{2673}{40}t^4h^3 + \frac{4293}{50}t^5h^3 - \frac{891}{16}t^6h^3 + \frac{2511}{140}t^7h^3 - \frac{720}{320}t^8h^3$$

$$\beta'_2 = -\frac{3149689}{36741600}h^3 + \frac{23}{6}t^3h^3 - \frac{947}{80}t^4h^3 + \frac{379}{25}t^5h^3 - \frac{587}{60}t^6h^3 + \frac{219}{70}t^7h^3 - \frac{63}{160}t^8h^3$$

Evaluating (2.7) at all points gives a discrete block formula of the form

$$A^{(0)}Y_m^{(i)} = \sum_{i=0}^{2} h^i e_i y_n^{(i)} + h^2 b_i f(y_n) + h^2 d_i f(Y_m) + h^3 c_i f(y_n) + h^3 r_i f(Y_m) \quad (2.8)$$

Where

$$Y_m = \left[y_{n+\frac{1}{3}}, y_{n+1}, y_{n+\frac{5}{3}}, y_{n+2} \right]^T, \quad f(y_n) = \left[f_{n-\frac{1}{3}}, f_{n-1}, f_{n-\frac{5}{3}}, f_n \right]^T, \quad f(Y_m) = \left[y_{n+\frac{1}{3}}, y_{n+1}, y_{n+\frac{5}{3}}, y_{n+2} \right]^T,$$

$$g(y_m) = \left[g_{n-\frac{1}{3}}, g_{n-1}, g_{n-\frac{5}{3}}, g_n \right]^T, \quad g(Y_m) = \left[g_{n+\frac{1}{3}}, g_{n+1}, g_{n+\frac{5}{3}}, g_{n+2} \right]^T, \quad y_n^{(i)} = \left[y_{n-\frac{1}{3}}^{(i)}, y_{n-1}^{(i)}, y_{n-\frac{5}{3}}^{(i)}, y_n^{(i)} \right]^T$$

and $A^{(0)} = 4 \times 4$ identity matrix.

When $i = 0$

$$e_0 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad e_1 = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{5}{3} \\ 0 & 0 & 0 & 2 \end{bmatrix}, \quad b_0 = \begin{bmatrix} 0 & 0 & 0 & \frac{1883725}{734832} \\ 0 & 0 & 0 & \frac{31}{24} \\ 0 & 0 & 0 & \frac{307}{105} \\ 0 & 0 & 0 & \frac{432379}{3674160} \end{bmatrix}, \quad d_0 = \begin{bmatrix} 0 & \frac{625}{6804} & 0 & -\frac{930625}{734832} \\ 0 & -\frac{1}{12} & 0 & -\frac{17}{24} \\ 0 & \frac{16}{105} & 0 & -\frac{113}{105} \\ 0 & -\frac{331}{34020} & 0 & -\frac{192511}{3674160} \end{bmatrix},$$

$$c_0 = \begin{bmatrix} 0 & 0 & 0 & \frac{1072625}{4408992} \\ 0 & 0 & 0 & \frac{2039}{16800} \\ 0 & 0 & 0 & \frac{146}{525} \\ 0 & 0 & 0 & \frac{151411}{15746400} \end{bmatrix}, \quad r_0 = \begin{bmatrix} \frac{120125}{108864} & \frac{2344375}{2204496} & \frac{10625}{15552} & \frac{75125}{629856} \\ \frac{6129}{11200} & \frac{58}{105} & \frac{4239}{11200} & \frac{1129}{16800} \\ \frac{216}{175} & \frac{116}{105} & \frac{108}{175} & \frac{52}{525} \\ \frac{14311}{388800} & \frac{455341}{11022480} & \frac{76171}{2721600} & \frac{548119}{110224800} \end{bmatrix}$$

When $i = 1$

$$e_0 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad b_0 = \begin{bmatrix} 0 & 0 & 0 & \frac{248299}{272160} \\ 0 & 0 & 0 & \frac{6907}{3360} \\ 0 & 0 & 0 & \frac{230285}{163296} \\ 0 & 0 & 0 & \frac{97}{105} \end{bmatrix}, \quad d_0 = \begin{bmatrix} 0 & -\frac{2362}{25515} & 0 & -\frac{397153}{734832} \\ 0 & \frac{8}{105} & 0 & -\frac{3903}{3360} \\ 0 & \frac{1250}{5103} & 0 & \frac{625}{54432} \\ 0 & \frac{16}{105} & 0 & \frac{97}{105} \end{bmatrix},$$

$$c_0 = \begin{bmatrix} 0 & 0 & 0 & \frac{1018291}{12247200} \\ 0 & 0 & 0 & \frac{3307}{16800} \\ 0 & 0 & 0 & \frac{66475}{489888} \\ 0 & 0 & 0 & \frac{47}{525} \end{bmatrix}, \quad r_0 = \begin{bmatrix} \frac{105187}{302400} & \frac{26875}{69984} & \frac{78563}{302400} & \frac{565459}{12247200} \\ \frac{10233}{11200} & \frac{83}{96} & \frac{6777}{11200} & \frac{601}{5600} \\ \frac{6875}{12096} & \frac{26875}{69984} & \frac{475}{12096} & -\frac{3125}{489888} \\ \frac{54}{175} & 0 & -\frac{54}{175} & -\frac{47}{525} \end{bmatrix}$$

3 Analysis of Basic Properties of the Proposed Hybrid Method

3.1 Order and error constant of the block

Applying the linear operator on

$$\nabla\{y(x);h\} = A^{[0]}Y_m^{[1]} - A^{[1]}Y_m^{[0]} - \sum_{i=0}^{k} B^i F_m^{[i]} - \sum_{i=0}^{k} D^i G_m^{[i]} \tag{3.1}$$

where $y(x)$ is an arbitrary function, continuously differentiable on an interval of integration [14,15]. The equation (3.1) written in Taylor expansion about the point x gives

$$l[y(x):h] = c_0\, y(x) + c_1 h\, y'(x) + c_2 h\, y''(x) + \cdots + c_{p+2} h\, y^{p+2}(x) + \cdots \tag{3.2}$$

we obtained the coefficients of h as $\bar{c}_0 = \bar{c}_1 = \bar{c}_2 = \bar{c}_4 = \bar{c}_5 = \bar{c}_6 = \bar{c}_7 = \bar{c}_8 = \bar{c}_9 = 0$, implying that the order $p = [8, 8, 8, 8]^T$ and the error constant is given by $\bar{c}_{10} = [5.2240 \times 10^{-8}, 7.1955 \times 10^{-7}, 1.6386 \times 10^{-6}, 2.3796 \times 10^{-6}]^T$

3.2 Consistency

The hybrid block method [16] is said to be consistent if it has an order more than or equal to one. Therefore, our method is consistent.

3.3 Zero stability of the method

Definition: A block method is said to be zero-stable if as $h \to 0$, the root, $z_i,\ i = 1(1)k$ of the first characteristic polynomial $\rho(z) = 0$ that is $\rho(z) = \det\left[\sum_{j=0}^{k} A^{(i)} z^{k-1}\right] = 0$ satisfies $|z_i| \le 1$ and for those roots with $|z_i| = 1$, multiplicity must not exceed two. The block method for k=1, with three off grid collocation point expressed in the form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{h}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & h \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{5h}{3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2h \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = z^6 (z-1)^2$$

$$\rho(z) = z^6 (z-1)^2 = 0,$$

Hence our method is zero-stable [16,17].

3.5 Regions of absolute stability (RAS)

According to [18], the stability polynomial of the method is given by,

$$\begin{aligned} t\ \bar{h}(w) = {} & \left(\left(\frac{5}{10287648}\right)w^4 - \left(\frac{5}{10287648}\right)w^3\right)h^{12} + \left(\left(\frac{36203}{1111065984}\right)w^3 + \left(\frac{505}{185177664}\right)w^4\right)h^{11} + \\ & \left(\left(\frac{2135443}{370355280}\right)w^3 + \left(\frac{10795}{740710656}\right)w^4\right)h^{10} + \left(\left(\frac{532729609}{1499939078\ \ 400}\right)w^3 + \left(\frac{333215719}{9999593860\ \ 0}\right)w^4\right)h^9 + \\ & \left(\left(\frac{4120086210\ \ 1}{9999593856\ \ 00}\right)w^3 - \left(\frac{2075003806\ \ 3}{1111065984\ \ 000}\right)w^4\right)h^8 + \left(\left(\frac{390493709}{4999796280\ \ 0}\right)w^3 - \left(\frac{102983201}{4999796280\ \ 0}\right)w^4\right)h^7 \\ & + \left(\left(\frac{1172292947\ \ 23}{2777664960\ \ 00}\right)w^3 - \left(\frac{2105701638\ \ 317}{9999593856\ \ 000}\right)w^4\right)h^6 + \left(\left(\frac{8313014266\ \ 61}{1666598976\ \ 000}\right)w^3 - \left(\frac{8805666258\ \ 1}{3333197952\ \ 00}\right)w^4\right)h^5 \\ & + \left(\left(\frac{24343}{8817984}\right)w^3 - \left(\frac{111347}{44089920}\right)w^4\right)h^4 + \left(\left(\frac{26515081}{6123600}\right)w^3 - \left(\frac{502170419}{220449600}\right)w^4\right)h^3 - \\ & \left(\left(\frac{973243}{3674160}\right)w^3 - \left(\frac{498691}{3674160}\right)w^4\right)h^2 - 2w^3 + w^4 \end{aligned}$$

The region of absolute stability of the computational method with three partitions is shown in the Fig. 1 below.

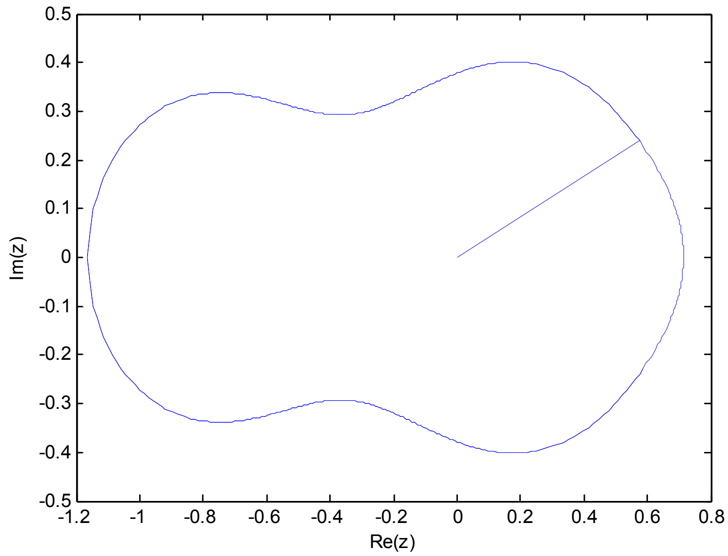

Fig. 1. Absolute stability region of our method

4 Numerical Experiments

In this section, the method derrived shall be employed in solving some problems so as to test how computationally reliable the method is.

Problem 4.1

Consider a highly stiff problem

$$f(x, y, y') = -1001y' - 1000y, \; y(0) = 1, \; y'(0) = -1$$

With exact solution, $y(x) = e^{-x}$ with $h = \frac{1}{10}$

Source: [5].

Table 1. Shown the results for problem 4.1

x-values	Exact solution	Computed solution	Error in our method	Error in [4]
0.100	0.90483741803595957316	0.90483741803595956730	5.860000E(-18)	1.054712E(-14)
0.200	0.81873075307798185867	0.81873075307798185149	7.180000E(-18)	1.776357E(-14)
0.300	0.74081822068171786607	0.74081822068171776080	1.052700E(-16)	2.342571E(-14)
0.400	0.67032004603563930074	0.67032004603563906918	2.315600E(-16)	2.797762E(-14)
0.500	0.60653065971263342360	0.60653065971263296686	4.567400E(-16)	3.130829E(-14)
0.600	0.54881163609402643263	0.54881163609402568898	7.436500E(-16)	3.397282E(-14)
0.700	0.49658530379140951470	0.49658530379140839765	1.117050E(-15)	3.563816E(-14)
0.800	0.44932896411722159143	0.44932896411722004301	1.548420E(-15)	3.674838E(-14)
0.900	0.40656965974059911188	0.40656965974059705187	2.060010E(-15)	3.730349E(-14)
1.000	0.36787944117144232160	0.36787944117143969108	2.630520E(-15)	3.741452E(-14)

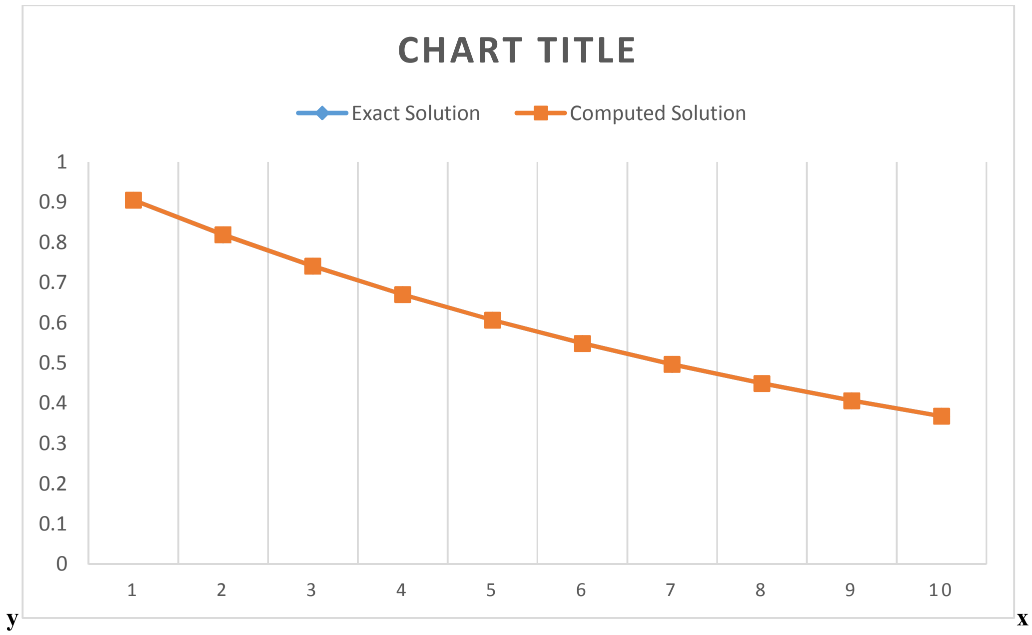

Fig. 2. Graphical solution of problem 4.1

Problem 4.2

Consider a highly stiff problem

$$f(x, y, y') = y',\ y(0) = 0,\ y'(0) = -1$$

With exact solution, $y(x) = 1 - e^{-x}$ with $h = \dfrac{1}{100}$

Source: [5,19]

Table 2. Shown the results for problem 4.2

x-values	Exact solution	Computed solution	Error in our method	Error in [19]	Error in [5]
0.01	-0.10517091807564762480	-0.10517091807564762837	5.643000E(-18)	2.858824E(-15)	5.551115E(-17)
0.02	-0.22140275816016983390	-0.22140275816016980445	2.294500E(-17)	1.439682E(-12)	1.387779E(-16)
0.03	-0.34985880757600310400	-0.34985880757600249997	6.040300E(-16)	5.591383E(-11)	3.330669E(-16)
0.04	-0.49182469764127031780	-0.49182469764126916471	1.153090E(-15)	4.796602E(-09)	4.996004E(-16)
0.05	-0.64872127070012814680	-0.64872127070012616300	1.983800E(-15)	1.003781E(-08)	7.771561E(-16)
0.06	-0.82211880039050897490	-0.82211880039050590783	3.067070E(-15)	1.590163E(-08)	1.332268E(-15)
0.07	-1.01375270747047652160	-1.01375270747047198400	4.537600E(-15)	2.870014E(-08)	1.776357E(-15)
0.08	-1.22554092849246760460	-1.22554092849246124010	6.364500E(-15)	4.284730E(-08)	2.886580E(-15)
0.09	-1.45960311115694966380	-1.45960311115694094630	8.717500E(-15)	5.857869E(-08)	3.774758E(-15)
0.10	-1.71828182845904523540	-1.71828182845903367110	1.156430E(-14)	8.449297E(-08)	5.107026E(-15)

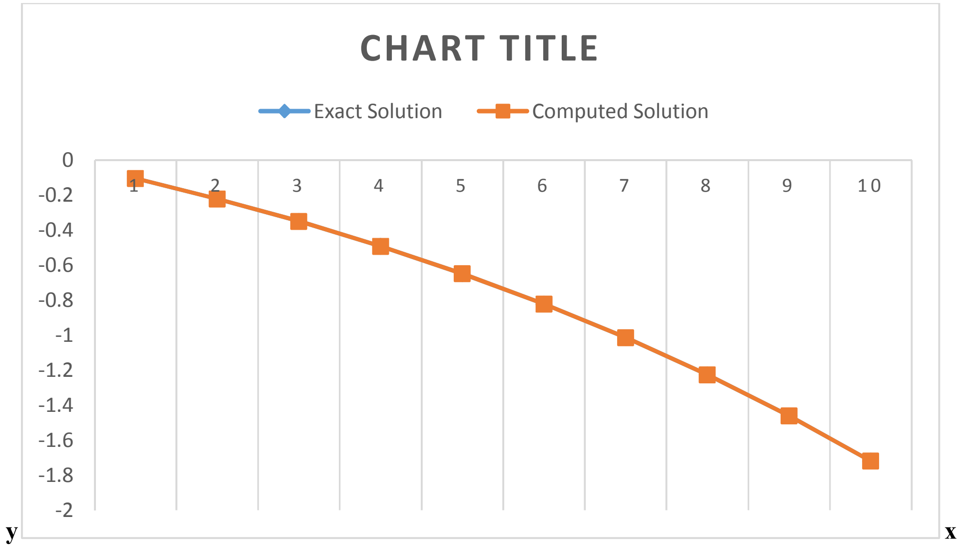

Fig. 3. Graphical solution of Problem 4.2

Problem 4.3

Consider a the stiff problem

$$f(x, y, y') = -100y', \; y(0) = 1, \; y'(0) = -10$$

With exact solution, $y(x) = e^{-10x}$ with $h = \frac{1}{100}$

Source: [5].

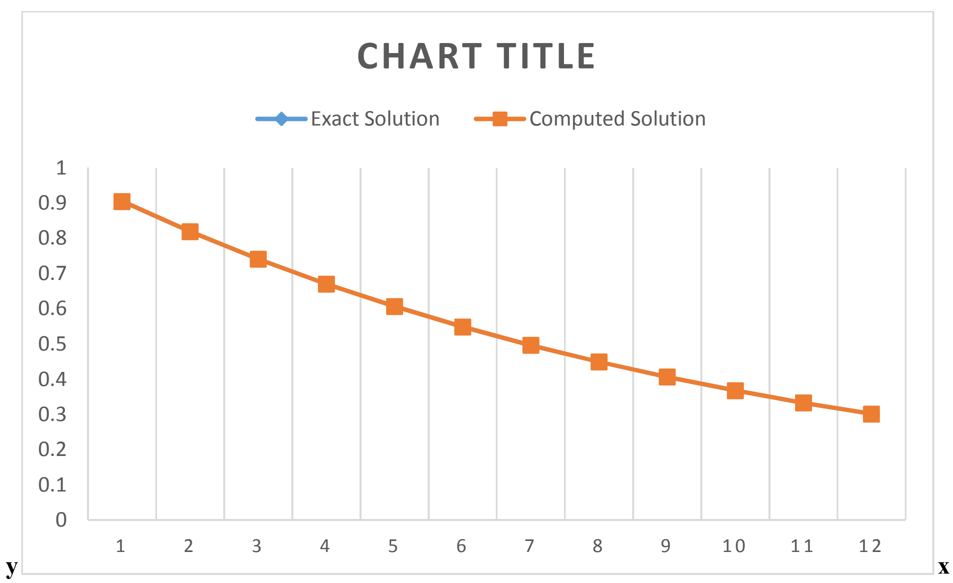

Fig. 4. Graphical solution of problem 4.3

Table 3. Shown the results for problem 4.3

x-values	Exact solution	Computed solution	Error in our method	Error in [4]
0.01	0.90483741803595957316	0.90483741803595950520	6.796000E(-17)	0.000000E(+00)
0.02	0.81873075307798185867	0.81873075307798163683	2.218400E(-16)	2.431388E(-14)
0.03	0.74081822068171786607	0.74081822068171737017	4.959000E(-16)	7.105427E(-14)
0.04	0.67032004603563930074	0.67032004603563845606	8.446800E(-16)	1.384448E(-13)
0.05	0.60653065971263342360	0.60653065971263212510	1.298500E(-15)	2.257083E(-13)
0.06	0.54881163609402643263	0.54881163609402461016	1.822470E(-15)	3.316236E(-13)
0.07	0.49658530379140951470	0.49658530379140707097	2.443730E(-15)	4.555800E(-13)
0.08	0.44932896411722159143	0.44932896411721845519	3.136240E(-15)	5.974665E(-13)
0.09	0.40656965974059911188	0.40656965974059518702	3.924860E(-15)	7.575052E(-13)
0.10	0.36787944117144232160	0.36787944117143753054	4.791060E(-15)	9.361956E(-13)
0.11	0.33287108369807955329	0.33287108369807379508	5.758210E(-15)	1.134093E(-13)
0.12	0.30119421191220209664	0.30119421191219528229	6.814350E(-15)	1.352474E(-13)

5 Conclusion

In this paper, the double step hybrid linear multistep method for solving (1.1) is derived via the interpolation and collocation approach of power series method. This hybrid block method has satisfied possessing properties that will confirm its convergence when applied to solve second order ODEs, which was found to be consistent, convergent, and zero-stable, with region of absolute stability within which is stable. It is evident from the above tables that our proposed methods are indeed accurate, and can handle stiff equations. Comparing our method with the existing methods of [5,19], the result presented in the Tables 1, 2 and 3 shows that our method performs better than the existing method of [5,19] and we further displayed the performance of our numerical solution with exact solution of each problems tested, and it was shown that the numerical solution converges towards the exact solution.

Competing Interests

Authors have declared that no competing interests exist.

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