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Exact Solutions of the Generalized KP-BBM Equation by the G'/G-expansion Method and the First Integral Method

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

In this paper, the generalized KP-BBM equation is considered. The G'/G-expansion method and the first integral method are applied to integrate the equation. By means of the two methods, the rational solutions, the periodic solutions and the hyperbolic function solutions are thus obtained under some parametric conditions.

Keywords: Generalized KP-BBM equation; G'/G-expansion method; First integral method.

1 Introduction

Recently, many researchers have studied the following nonlinear Kadomtsov-Petviashvili-Benjiamin-Bona-Mahony (KP-BBM) equation

$$[u_t + u_x - a(u^2)_x - bu_{xxt}]_x + ku_{yy} = 0, \quad (1)$$

which is a combination of the KP equation and the BBM equation and was deduced when Wazwaz[1] studied the BBM equation in the sense of the KP equation. The KP equation was introduced in order to discuss the stability of

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tense waves to perpendicular horizontal perturbations[2]. The BBM equation has been proposed as a model for propagation of long waves where nonlinear dispersion is incorporated[3]. Up to now, researchers have succeeded in applying several methods to study the KP-BBM equation and getting some results. Wazwaz[1] obtained some periodic solutions and solitons solutions by using the sin-cosine method and the tanh method. In addition, Wzawaz[4] used the extended tanh method to obtain some exact solutions. Abdou et al.[5] got some periodic wave solutions, solitary wave solutions and triangular wave solutions by using the extended mapping method with symbolic computation. Song et al.[6] employed the bifurcation method of dynamical systems to investigate bifurcation of solitary waves.

In this paper, we consider the following generalized KP-BBM equation

$$[(u^n)_t + (u^n)_x - a(u^{m+1})_x - b(u^n)_{xxt}]_x + k(u^n)_{yy} = 0, \tag{2}$$

where a, b, k are constants, n, m are positive integers and $m, n \geq 1$. Specially, when $m = n = 1$, (2) becomes (1). Tang et al.[7] studied travelling wave solutions of (2) with parametric conditions of $n > m \geq 1$ by bifurcation theory of dynamical systems. The goal of this paper is to obtain the rational solutions, the periodic solutions and the hyperbolic function solutions of system (2) by applying the G'/G-expansion method and the first integral method. The G'/G-expansion method was first presented by Wang [8] which can be used to deal with all types of nonlinear evolution equations. From then on, the G'/G-expansion method has been widely used, for example, Ozkan Guner et al.[9] used the method to obtain exact soliton solutions of nonlinear fractional density-dependent fractional differential equation with quadratic nonlinearity and nonlinear fractional approximate long water wave equation. The first integral method was first proposed by Feng[10] for obtaining the exact solutions of Burgers-KdV equation which is based on the ring theory of commutative algebra. It has been applied to many nonlinear evolution equations, for example, M. Eslami et al. [11] considered the resonant nonlinear Schrödinger's equation with time-dependent coefficients by employing the first integral method and obtained the exact solutions of the equation.

2 The G'/G-expansion method to the generalized KP-BBM equation

Let us assume that the solutions of (2) take the form

$$u(x, y, t) = u(\xi), \quad \xi = k_1 x + l_1 y + \lambda_1 t, \tag{3}$$

where k_1, l_1, λ_1 are constants. Using the transformation (3), (2) becomes

$$(k_1\lambda_1 + k_1^2 + kl_1^2)(u^n)'' - ak_1^2(u^{m+1})'' - b\lambda_1 k_1^3 (u^n)^{(4)} = 0.$$

Integrating the above equation twice and letting the first integral constant be zero, hence, we have the following ODE

$$(k_1\lambda_1 + k_1^2 + kl_1^2)u^n - ak_1^2 u^{m+1} - b\lambda_1 k_1^3 (u^n)^{''} = g, \tag{4}$$

where g is an integral constant and "′" is the derivative with respect to ξ. We assume that (4) has solutions as the following form [12, 13]

$$u(\xi) = D\left(\frac{G'}{G}\right)^N,$$

where D is a non-zero constant which will be determined later. N is determined by balancing the linear term of the highest order derivatives with the highest order nonlinear term of (4) and G satisfies a second order constant coefficient ODE

$$G^{''}(\xi) + \lambda G^{'}(\xi) + \mu G(\xi) = 0, \tag{5}$$

where λ, μ are constants that need to be determined later. Considering the relationship between $m+1$ and n in (4), there are the following two cases.

2.1 $m+1>n$

Balancing $(u^n)''$ with u^{m+1} of (4), we have $nN+2=N(m+1)$, that is $N=2/(m-n+1)$. Therefore, the solutions can be written as

$$u(\xi)=D\left(\frac{G'}{G}\right)^{\frac{2}{(m-n+1)}}. \tag{6}$$

Then, we obtain

$$\begin{aligned}
u^n &= D^n\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}}, \qquad u^{m+1}=D^{m+1}\left(\frac{G'}{G}\right)^{\frac{2(m+1)}{m-n+1}}\\
(u^n)'' &= \frac{2n}{m-n+1}D^n\Bigg[(\frac{2n}{m-n+1}+1)\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}+2}+(\frac{4n}{m-n+1}+1)\lambda\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}+1}\\
&\quad+\frac{2n}{m-n+1}(2\mu+\lambda^2)\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}}+(\frac{4n}{m-n+1}-1)\lambda\mu\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}-1}\\
&\quad+(\frac{2n}{m-n+1}-1)\mu^2\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}-2}\Bigg].
\end{aligned}$$

Substituting the above formulas into (4) and collecting all terms with the same order of G'/G together, we can convert the left-hand side of (4) into a polynomial in G'/G. Then, setting each coefficient of each polynomial to zero, we derive a set of algebraic equation for λ,μ and D.

$\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}+2}$ coeff:

$$-b\lambda_1k_1^3\frac{2n}{m-n+1}(\frac{2n}{m-n+1}+1)D^n-ak_1^2D^{m+1}=0, \tag{7}$$

$\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}+1}$ coeff:

$$-b\lambda_1k_1^3\frac{2n}{m-n+1}(\frac{4n}{m-n+1}+1)\lambda D^n=0, \tag{8}$$

$\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}}$ coeff:

$$-b\lambda_1k_1^3(\frac{2n}{m-n+1})^2(2\mu+\lambda^2)D^n+(\lambda_1k_1+k_1^2+kl_1^2)D^n=0. \tag{9}$$

According to the situations that whether $\frac{2n}{m-n+1}-1$ and $\frac{2n}{m-n+1}-2$ are equal to zero, we need to consider the cases as follows.

Case 1. $\frac{2n}{m-n+1}-1\neq 0$ and $\frac{2n}{m-n+1}-2\neq 0$ (i.e. $n\neq\frac{m+1}{3}$ and $n\neq\frac{m+1}{2}$)

$\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}-1}$ coeff:

$$-b\lambda_1k_1^3\frac{2n}{m-n+1}(\frac{4n}{m-n+1}-1)\lambda\mu D^n=0, \tag{10}$$

$\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}-2}$ coeff:

$$-b\lambda_1k_1^3\frac{2n}{m-n+1}(\frac{2n}{m-n+1}-1)\mu^2D^n=0, \tag{11}$$

$\left(\frac{G'}{G}\right)^0$ coeff:

$$g = 0. \tag{12}$$

Solving the set of (7)-(12) , we have

$$\lambda = \mu = 0, \quad g = 0, \quad \lambda_1 k_1 + k_1^2 + k l_1^2 = 0, \quad D = \left(\frac{-b\lambda_1 k_1^3 \frac{2n}{m-n+1}(\frac{2n}{m-n+1}+1)}{a k_1^2}\right)^{\frac{1}{m-n+1}}. \tag{13}$$

Substituting (13) into (5) and (6), we can obtain the the rational solutions

$$u(x,y,t) = \left(\frac{-b\lambda_1 k_1^3 \frac{2n}{m-n+1}(\frac{2n}{m-n+1}+1)}{a k_1^2}\right)^{\frac{1}{m-n+1}} \left(\frac{C_1}{C_1(k_1 x + l_1 y + \lambda_1 t) + C_2}\right)^{\frac{2}{m-n+1}}, \tag{14}$$

where C_1, C_2 are arbitrary constants. The solution $u(x,y,t)$ is presented in Fig. 11 at the end of this paper.

Case 2. $\frac{2n}{m-n+1} - 1 = 0$ (i.e. $n = \frac{m+1}{3}$)
$\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}-1}$ coeff:

$$-b\lambda_1 k_1^3 \frac{2n}{m-n+1}(\frac{4n}{m-n+1} - 1)\lambda\mu D^n - g = 0, \tag{15}$$

$\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}-2}$ coeff:

$$-b\lambda_1 k_1^3 \frac{2n}{m-n+1}(\frac{2n}{m-n+1} - 1)\mu^2 D^n = 0. \tag{16}$$

Solving (7)-(9), (15)-(16) and combining $n = \frac{m+1}{3}$, we can derive

$$\lambda = 0, \quad g = 0, \quad \mu = \frac{\lambda_1 k_1 + k_1^2 + k l_1^2}{2b\lambda_1 k_1^3}, \quad D = \left(\frac{-2b\lambda_1 k_1^3}{a k_1^2}\right)^{\frac{1}{2n}}. \tag{17}$$

Substituting (17) into (5) and (6), then, (5) thus becomes

$$G'' + \left(\frac{\lambda_1 k_1 + k_1^2 + k l_1^2}{2b\lambda_1 k_1^3}\right) G = 0.$$

Considering the relationship between $\frac{\lambda_1 k_1 + k_1^2 + k l_1^2}{2b\lambda_1 k_1^3}$ and zero, we have the following results in the end.
I. $\frac{\lambda_1 k_1 + k_1^2 + k l_1^2}{2b\lambda_1 k_1^3} < 0$
We obtain the hyperbolic function solutions

$$u(x,y,t) = \left(\frac{k_1\lambda_1 + k_1^2 + k l_1^2}{a k_1^2}\right)^{\frac{1}{2n}}$$
$$\left(\frac{C_3 \sinh\left(\frac{k_1\lambda_1 + k_1^2 + k l_1^2}{-2b\lambda_1 k_1^3}\right)^{1/2}(k_1 x + l_1 y + \lambda_1 t) + C_4 \cosh\left(\frac{k_1\lambda_1 + k_1^2 + k l_1^2}{-2b\lambda_1 k_1^3}\right)^{1/2}(k_1 x + l_1 y + \lambda_1 t)}{C_3 \cosh\left(\frac{k_1\lambda_1 + k_1^2 + k l_1^2}{-2b\lambda_1 k_1^3}\right)^{1/2}(k_1 x + l_1 y + \lambda_1 t) + C_4 \sinh\left(\frac{k_1\lambda_1 + k_1^2 + k l_1^2}{-2b\lambda_1 k_1^3}\right)^{1/2}(k_1 x + l_1 y + \lambda_1 t)}\right)^{\frac{1}{n}}, \tag{18}$$

where C_3, C_4 are arbitrary constants.

II. $\frac{\lambda_1 k_1 + k_1^2 + kl_1^2}{2b\lambda_1 k_1^3} > 0$

We obtain the hyperbolic function solutions

$$u(x,y,t) = \left(-\frac{k_1\lambda_1 + k_1^2 + kl_1^2}{ak_1^2}\right)^{\frac{1}{2n}}$$
$$\left(\frac{-C_5 \sin\left(\frac{k_1\lambda_1 + k_1^2 + kl_1^2}{2b\lambda_1 k_1^3}\right)^{1/2}(k_1x + l_1y + \lambda_1 t) + C_6 \cos\left(\frac{k_1\lambda_1 + k_1^2 + kl_1^2}{2b\lambda_1 k_1^3}\right)^{1/2}(k_1x + l_1y + \lambda_1 t)}{C_5 \cos\left(\frac{k_1\lambda_1 + k_1^2 + kl_1^2}{2b\lambda_1 k_1^3}\right)^{1/2}(k_1x + l_1y + \lambda_1 t) + C_6 \sin\left(\frac{k_1\lambda_1 + k_1^2 + kl_1^2}{2b\lambda_1 k_1^3}\right)^{1/2}(k_1x + l_1y + \lambda_1 t)}\right)^{\frac{1}{n}}, \tag{19}$$

where C_5, C_6 are arbitrary constants.

III. $\frac{\lambda_1 k_1 + k_1^2 + kl_1^2}{2b\lambda_1 k_1^3} = 0$

We obtain the rational solutions

$$u(x,y,t) = \left(\frac{-2b\lambda_1 k_1^3}{ak_1^2}\right)^{\frac{1}{2n}} \left(\frac{C_7}{C_7(k_1x + l_1y + \lambda_1 t) + C_8}\right)^{\frac{1}{n}}, \tag{20}$$

where C_7, C_8 are arbitrary constants. The solutions (18), (19) and (20) are presented in the following figures.

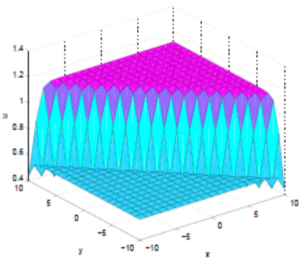

Fig. 1: The solution (18) for $t = 0$, $a = b = \frac{1}{2}, k_1 = l_1 = k = 1$, $\lambda_1 = -1, n = 3, m = 8$, $C_3 = 2, C_4 = -1$.

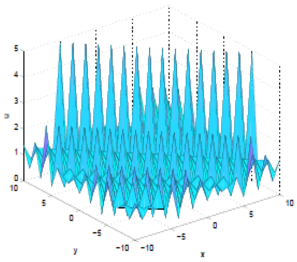

Fig. 2: The solution (19) for $t = 0$, $a = -\frac{1}{2}, b = \frac{1}{2}, n = 3, m = 8$, $k_1 = l_1 = \lambda_1 = k = 1$, $C_5 = C_6 = 1$.

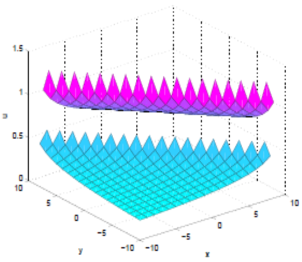

Fig. 3: The solution (20) for $t = 0$, $a = b = \frac{1}{2}, k_1 = l_1 = k = 1$, $\lambda_1 = -2, n = 3, m = 8$, $C_7 = 1, C_8 = 0$.

Case 3. $\frac{2n}{m-n+1} - 2 = 0$ (i.e. $n = \frac{m+1}{2}$)

$\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}-1}$ coeff:

$$-b\lambda_1 k_1^3 \frac{2n}{m-n+1}(\frac{4n}{m-n+1} - 1)\lambda\mu D^n = 0, \tag{21}$$

$\left(\frac{G'}{G}\right)^{\frac{2n}{m-n+1}-2}$ coeff:

$$-b\lambda_1 k_1^3 \frac{2n}{m-n+1}(\frac{2n}{m-n+1} - 1)\mu^2 D^n - g = 0. \tag{22}$$

Solving (7)-(9), (21)-(22) and substituting $n = \frac{m+1}{2}$ into the results, we get

$$\lambda = 0, \quad g = \frac{3(k_1\lambda_1 + k_1^2 + kl_1^2)^2}{16ak_1^2}, \quad \mu = \frac{k_1\lambda_1 + k_1^2 + kl_1^2}{8b\lambda_1 k_1^3}, \quad D = \left(\frac{-6b\lambda_1 k_1^3}{ak_1^2}\right)^{\frac{1}{n}}. \tag{23}$$

Substituting (23) into (5) and (6), similarly, we have the following three cases.

I. $\frac{k_1\lambda_1+k_1^2+kl_1^2}{8b\lambda_1 k_1^3} < 0$

We obtain the hyperbolic function solutions

$$u(x,y,t) = \left(\frac{3(k_1\lambda_1 + k_1^2 + kl_1^2)}{4ak_1^2}\right)^{\frac{1}{n}}$$
$$\left(\frac{C_9 \sinh\left(\frac{k_1\lambda_1+k_1^2+kl_1^2}{-8b\lambda_1 k_1^3}\right)^{1/2}(k_1x + l_1y + \lambda_1 t) + C_{10}\cosh\left(\frac{k_1\lambda_1+k_1^2+kl_1^2}{-8b\lambda_1 k_1^3}\right)^{1/2}(k_1x + l_1y + \lambda_1 t)}{C_9 \cosh\left(\frac{k_1\lambda_1+k_1^2+kl_1^2}{-8b\lambda_1 k_1^3}\right)^{1/2}(k_1x + l_1y + \lambda_1 t) + C_{10}\sinh\left(\frac{k_1\lambda_1+k_1^2+kl_1^2}{-8b\lambda_1 k_1^3}\right)^{1/2}(k_1x + l_1y + \lambda_1 t)}\right)^{\frac{2}{n}}, \tag{24}$$

where C_9, C_{10} are arbitrary constants.

II. $\frac{k_1\lambda_1+k_1^2+kl_1^2}{8b\lambda_1 k_1^3} > 0$

We obtain the hyperbolic function solutions

$$u(x,y,t) = \left(-\frac{3(k_1\lambda_1 + k_1^2 + kl_1^2)}{4ak_1^2}\right)^{\frac{1}{n}}$$
$$\left(\frac{-C_{11} \sin\left(\frac{k_1\lambda_1+k_1^2+kl_1^2}{8b\lambda_1 k_1^3}\right)^{1/2}(k_1x + l_1y + \lambda_1 t) + C_{12}\cos\left(\frac{k_1\lambda_1+k_1^2+kl_1^2}{8b\lambda_1 k_1^3}\right)^{1/2}(k_1x + l_1y + \lambda_1 t)}{C_{11} \cos\left(\frac{k_1\lambda_1+k_1^2+kl_1^2}{8b\lambda_1 k_1^3}\right)^{1/2}(k_1x + l_1y + \lambda_1 t) + C_{12}\sin\left(\frac{k_1\lambda_1+k_1^2+kl_1^2}{8b\lambda_1 k_1^3}\right)^{1/2}(k_1x + l_1y + \lambda_1 t)}\right)^{\frac{2}{n}}, \tag{25}$$

where C_{11}, C_{12} are arbitrary constants.

III. $\frac{k_1\lambda_1+k_1^2+kl_1^2}{8b\lambda_1 k_1^3} = 0$

We obtain the rational solutions

$$u(x,y,t) = \left(\frac{-6b\lambda_1 k_1^3}{ak_1^2}\right)^{\frac{1}{n}} \left(\frac{C_{13}}{C_{13}(k_1x + l_1y + \lambda_1 t) + C_{14}}\right)^{\frac{2}{n}}, \tag{26}$$

where C_{13}, C_{14} are arbitrary constants. The solutions (24), (25) and (26) are presented in the following figures.

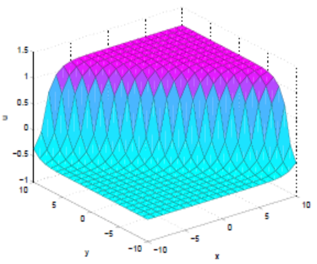

Fig. 4: The solution (24) for $t=0$, $a=b=\frac{1}{2}, k_1=l_1=k=1$, $\lambda_1=-1, n=3, m=5$, $C_9=2, C_{10}=-1$.

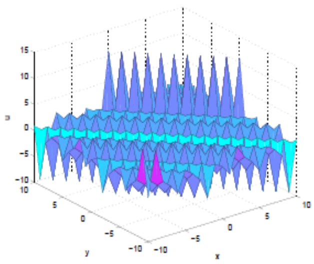

Fig. 5: The solution (25) for $t=0$, $a=b=\frac{1}{2}, n=3, m=5$, $k_1=l_1=\lambda_1=k=1$, $C_{11}=C_{12}=1$.

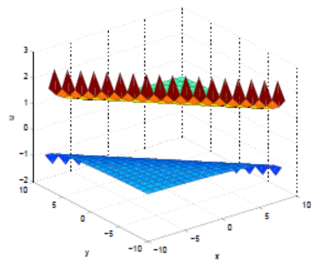

Fig. 6: The solution (26) for $t=0$, $a=b=\frac{1}{2}, k_1=l_1=k=1$, $\lambda_1=-2, n=3, m=5$, $C_{13}=1, C_{14}=0$.

2.2 $m+1=n$

In this condition, (4) thus can be converted into

$$(k_1\lambda_1+k_1^2+kl_1^2-ak_1^2)u^n-b\lambda_1k_1^3(u^n)''=g.$$

Obviously, we have the exact solutions as following cases.

Case 1. $\frac{k_1\lambda_1+k_1^2+kl_1^2-ak_1^2}{-b\lambda_1k_1^3}<0$

$$\begin{aligned}u(x,y,t)= & \left[C_{15}e^{\left(\frac{k_1\lambda_1+k_1^2+kl_1^2-ak_1^2}{b\lambda_1k_1^3}\right)^{1/2}(k_1x+l_1y+\lambda_1t)}\right.\\ & \left.+C_{16}e^{-\left(\frac{k_1\lambda_1+k_1^2+kl_1^2-ak_1^2}{b\lambda_1k_1^3}\right)^{1/2}(k_1x+l_1y+\lambda_1t)}+\frac{g}{k_1\lambda_1+k_1^2+kl_1^2-ak_1^2}\right]^{\frac{1}{n}}.\end{aligned}\tag{27}$$

Case 2. $\frac{k_1\lambda_1+k_1^2+kl_1^2-ak_1^2}{-b\lambda_1k_1^3}>0$

$$\begin{aligned}u(x,y,t)= & \left[C_{17}\cos\left(\frac{k_1\lambda_1+k_1^2+kl_1^2-ak_1^2}{-b\lambda_1k_1^3}\right)^{1/2}(k_1x+l_1y+\lambda_1t)\right.\\ & \left.+C_{18}\sin\left(\frac{k_1\lambda_1+k_1^2+kl_1^2-ak_1^2}{-b\lambda_1k_1^3}\right)^{1/2}(k_1x+l_1y+\lambda_1t)+\frac{g}{k_1\lambda_1+k_1^2+kl_1^2-ak_1^2}\right]^{\frac{1}{n}}.\end{aligned}\tag{28}$$

Case 3. $\frac{k_1\lambda_1+k_1^2+kl_1^2-ak_1^2}{-b\lambda_1k_1^3}=0$

$$u(x,y,t)=\left[\frac{-g}{2b\lambda_1k_1^3}(k_1x+l_1y+\lambda_1t)^2+C_{19}(k_1x+l_1y+\lambda_1t)+C_{20}\right]^{\frac{1}{n}},\tag{29}$$

where $C_{15},...,C_{20}$ are arbitrary constants. The solutions (27), (28) and (29) are presented in the following figures.

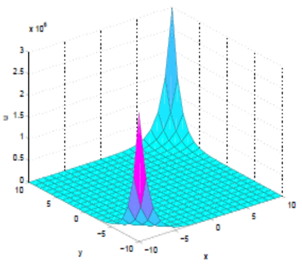

Fig. 7: The solution (27) for $t = 0$, $a = b = \frac{1}{2}, n = 3, m = 2,$ $k_1 = l_1 = \lambda_1 = k = 1,$ $g = 0, C_{15} = C_{16} = 1.$

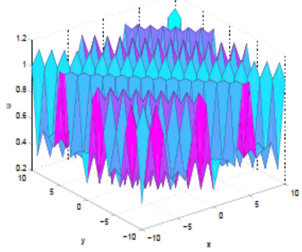

Fig. 8: The solution (28) for $t = 0$, $a = b = \frac{1}{2}, k_1 = l_1 = k = 1,$ $\lambda_1 = -1, n = 3, m = 2,$ $g = 0, C_{17} = C_{18} = 1.$

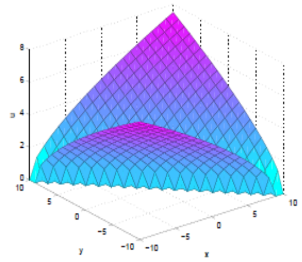

Fig. 9: The solution (29) for $t = 0$, $a = 1, b = \frac{1}{2}, k_1 = l_1 = k = 1,$ $\lambda_1 = -1, n = 3, m = 2,$ $g = 1, C_{19} = 1, C_{20} = 0.$

3 THE IMPROVED G'/G-EXPANSION METHOD TO THE GENERALIZED KP-BBM EQUATION

In order to obtain closed form solutions, we let $g = 0$ and use the transformation

$$u(\xi) = v^{\frac{2}{m-n+1}}(\xi), \qquad m + 1 \neq n,$$

which will reduce (4) into the following ODE

$$(k_1\lambda_1 + k_1^2 + kl_1^2)v^2 - ak_1^2v^4 - b\lambda_1 k_1^3\frac{2n}{m-n+1}(\frac{2n}{m-n+1} - 1)(v')^2 + b\lambda_1 k_1^3\frac{2n}{m-n+1}vv'' = 0. \tag{30}$$

Suppose that the solutions of (30) can be expressed by a polynomial of G'/G as follows

$$v(\xi) = \sum_{i=0}^{N} a_i \left(\frac{G'}{G}\right)^i, \tag{31}$$

where a_i are real constants with $a_N \neq 0$ and $G = G(\xi)$ satisfies (5). N is a positive integer which can be determined by balancing the highest order derivative term with the highest order nonlinear term after substituting (31) into (30).

Balancing vv'' and v^4 of (30), we have $N + (N + 2) = 4N$, i.e., $N = 1$. Therefore, (31) can be rewritten as

$$v(\xi) = a_0 + a_1\left(\frac{G'}{G}\right). \tag{32}$$

Combining (5) and (32), we deduce

$$\begin{aligned} v' &= -a_1\left(\frac{G'}{G}\right)^2 - \lambda a_1\left(\frac{G'}{G}\right) - \mu a_1, \\ v'' &= 2a_1\left(\frac{G'}{G}\right)^3 + 3\lambda a_1\left(\frac{G'}{G}\right)^2 + (\lambda^2 a_1 + 2\mu a_1)\left(\frac{G'}{G}\right) + \lambda\mu a_1, \\ v^2 &= a_1^2\left(\frac{G'}{G}\right)^2 + 2a_0a_1\left(\frac{G'}{G}\right) + a_0^2, \\ v^4 &= a_1^4\left(\frac{G'}{G}\right)^4 + 4a_0a_1^3\left(\frac{G'}{G}\right)^3 + 6a_0^2a_1^2\left(\frac{G'}{G}\right)^2 + 4a_0^3a_1\left(\frac{G'}{G}\right) + a_0^4, \end{aligned}$$

and

$$\begin{aligned} (v')^2 &= a_1^2\left(\frac{G'}{G}\right)^4 + 2\lambda a_1^2\left(\frac{G'}{G}\right)^3 + (2\mu a_1^2 + \lambda^2 a_1^2)\left(\frac{G'}{G}\right)^2 + 2\lambda\mu a_1^2\left(\frac{G'}{G}\right) + \mu^2 a_1^2, \\ vv'' &= 2a_1^2\left(\frac{G'}{G}\right)^4 + (2a_0a_1 + 3\lambda a_1^2)\left(\frac{G'}{G}\right)^3 + (3\lambda a_0a_1 + \lambda^2 a_1^2 + 2\mu a_1^2)\left(\frac{G'}{G}\right)^2 \\ &\quad + (\lambda\mu a_1^2 + \lambda^2 a_0a_1 + 2\mu a_0a_1)\left(\frac{G'}{G}\right) + \lambda\mu a_0a_1. \end{aligned}$$

Substituting the above v^2, v^4, $(v')^2$ and vv'' into (30), collecting all terms with the same powers of G'/G and setting each coefficient to zero, we have a system of algebraic equations for a_0, a_1, λ and μ as follows.
$\left(\frac{G'}{G}\right)^4$ coeff:

$$-ak_1^2a_1^4 - b\lambda_1k_1^3a_1^2\frac{2n}{m-n+1}(\frac{2n}{m-n+1} - 1) + 2b\lambda_1k_1^3a_1^2\frac{2n}{m-n+1} = 0,$$

$\left(\frac{G'}{G}\right)^3$ coeff:

$$-4ak_1^2a_0a_1^3 - 2b\lambda_1\lambda k_1^3a_1^2\frac{2n}{m-n+1}(\frac{2n}{m-n+1} - 1) + b\lambda_1k_1^3\frac{2n}{m-n+1}(2a_0a_1 + 3\lambda a_1^2) = 0,$$

$\left(\frac{G'}{G}\right)^2$ coeff:

$$\begin{aligned} &-6ak_1^2a_0^2a_1^2 - b\lambda_1k_1^3\frac{2n}{m-n+1}(\frac{2n}{m-n+1} - 1)(2\mu a_1^2 + \lambda^2 a_1^2) \\ &+b\lambda_1k_1^3\frac{2n}{m-n+1}(3\lambda a_0a_1 + \lambda^2 a_1^2 + 2\mu a_1^2) + a_1^2(\lambda_1k_1 + k_1^2 + kl_1^2) = 0, \end{aligned}$$

$\left(\frac{G'}{G}\right)$ coeff:

$$\begin{aligned} &-4ak_1^2a_0^3a_1 - 2b\lambda_1\lambda\mu k_1^3a_1^2\frac{2n}{m-n+1}(\frac{2n}{m-n+1} - 1) \\ &+b\lambda_1k_1^3\frac{2n}{m-n+1}(\lambda\mu a_1^2 + a_0a_1\lambda^2 + 2a_0a_1\mu) + 2a_0a_1(\lambda_1k_1 + k_1^2 + kl_1^2) = 0, \end{aligned}$$

$\left(\frac{G'}{G}\right)^0$ coeff:

$$-ak_1^2a_0^4 - b\lambda_1k_1^3\mu^2a_1^2\frac{2n}{m-n+1}(\frac{2n}{m-n+1} - 1) + b\lambda_1\lambda\mu k_1^3a_0a_1\frac{2n}{m-n+1} + a_0^2(\lambda_1k_1 + k_1^2 + kl_1^2) = 0.$$

Solving the above algebraic system with the aid of Matlab, we have

$$a_0 = \pm\frac{\lambda\sqrt{2abn\lambda_1 k_1(3m-5n+3)}}{2a(m-n+1)}, \qquad a_1 = \pm\frac{\sqrt{2abn\lambda_1 k_1(3m-5n+3)}}{a(m-n+1)},$$
$$\mu = \frac{1}{4}\lambda^2, \qquad \lambda_1 k_1 + k_1^2 + kl_1^2 = 0. \tag{33}$$

Substituting a_0 and a_1 into (32), which thus can be written as

$$v(\xi) = \pm\frac{\sqrt{2abn\lambda_1 k_1(3m-5n+3)}}{a(m-n+1)}\left(\frac{\lambda}{2}+\frac{G'}{G}\right). \tag{34}$$

Combining (5) and $\mu = \frac{1}{4}\lambda^2$ of (33), we derive

$$\frac{G'}{G} = -\frac{\lambda}{2} + \frac{C_{21}}{C_{21}\xi + C_{22}}. \tag{35}$$

Substituting (35) into (34), finally, we obtain the rational solutions

$$u(x,y,t) = \left[\pm\frac{\sqrt{2abn\lambda_1 k_1(3m-5n+3)}}{a(m-n+1)}\left(\frac{C_{21}}{C_{21}(k_1 x + l_1 y + \lambda_1 t) + C_{22}}\right)\right]^{\frac{2}{m-n+1}}, \tag{36}$$

where C_{21}, C_{22} are arbitrary constants. The solution $u(x,y,t)$ is presented in Fig. 12 at the end of the paper.

4 THE FIRST INTEGRAL METHOD TO THE GENERALIZED KP-BBM EQUATION

For simplicity, we propose a transformation $u = \varphi^{\frac{2}{m-n+1}}$, $(m+1 \neq n)$. Then, (4) is reduced to

$$(\lambda_1 k_1 + k_1^2 + kl_1^2)\varphi^2 - ak_1^2\varphi^4 - b\lambda_1 k_1^3\frac{2n}{m-n+1}(\frac{2n}{m-n+1}-1)\varphi'^2$$
$$-b\lambda_1 k_1^3\frac{2n}{m-n+1}\varphi\varphi'' - g\varphi^{2-\frac{2n}{m-n+1}} = 0. \tag{37}$$

Let $x = \varphi, y = \frac{\mathrm{d}\varphi}{\mathrm{d}\xi}$, (37) is equivalent to the two dimensional autonomous system

$$\begin{cases} x' = y, \\ y' = \frac{(\lambda_1 k_1 + k_1^2 + kl_1^2)x^2 - ak_1^2x^4 - gx^{2-\frac{2n}{m-n+1}} - b\lambda_1 k_1^3\frac{2n}{m-n+1}(\frac{2n}{m-n+1}-1)y^2}{b\lambda_1 k_1^3\frac{2n}{m-n+1}x}. \end{cases} \tag{38}$$

Making the transformation $d\eta = \frac{d\xi}{b\lambda_1 k_1^3\frac{2n}{m-n+1}x}$, (38) thus becomes

$$\begin{cases} \frac{dx}{d\eta} = b\lambda_1 k_1^3\frac{2n}{m-n+1}xy, \\ \frac{dy}{d\eta} = (\lambda_1 k_1 + k_1^2 + kl_1^2)x^2 - ak_1^2x^4 - gx^{2-\frac{2n}{m-n+1}} - b\lambda_1 k_1^3\frac{2n}{m-n+1}(\frac{2n}{m-n+1}-1)y^2. \end{cases} \tag{39}$$

Then, we apply the Division Theorem[10] to seek the first integral of system (39). Suppose that $x = x(\eta), y = y(\eta)$ are the nontrivial solutions to (39), and $p(x,y) = a_0(x) + a_1(x)y$ is an irreducible polynomial in $C[x,y]$, where $a_i(x), (i = 0,1)$ are polynomials of x and $a_i(x) \neq 0$. Let $p(x(\eta), y(\eta)) = 0$

be the first integral to system (39). $\frac{dp}{d\eta}$ is a polynomial in x, y and $\frac{dp}{d\eta}\big|_{(39)} = 0$. According to the Division Theorem, there exists a polynomial $g(x) + h(x)y$ in $C[x, y]$, such that

$$\begin{aligned} \frac{dp}{d\eta}\Big|_{(39)} &= \left(\frac{\partial p}{\partial x}\frac{dx}{d\eta} + \frac{\partial p}{\partial y}\frac{dy}{d\eta}\right)\Big|_{(39)} \\ &= \left[a_0'(x) + a_1'(x)y\right] b\lambda_1 k_1^3 \frac{2n}{m-n+1} xy \\ &+ a_1(x)\left[(\lambda_1 k_1 + k_1^2 + kl_1^2)x^2 - ak_1^2 x^4 - gx^{2-\frac{2n}{m-n+1}} - b\lambda_1 k_1^3 \frac{2n}{m-n+1}(\frac{2n}{m-n+1} - 1)y^2\right] \\ &= \left[g(x) + h(x)y\right]\left[a_0(x) + a_1(x)y\right]. \end{aligned} \tag{40}$$

Comparing the coefficients of y^i on both sides of (40), we have

$$b\lambda_1 k_1^3 \frac{2n}{m-n+1} x a_1'(x) = h(x)a_1(x) + b\lambda_1 k_1^3 \frac{2n}{m-n+1}(\frac{2n}{m-n+1} - 1)a_1(x), \tag{41}$$

$$b\lambda_1 k_1^3 \frac{2n}{m-n+1} x a_0'(x) = h(x)a_0(x) + g(x)a_1(x), \tag{42}$$

$$g(x)a_0(x) = \left[(\lambda_1 k_1 + k_1^2 + kl_1^2)x^2 - ak_1^2 x^4 - gx^{2-\frac{2n}{m-n+1}}\right] a_1(x). \tag{43}$$

According to (41), we deduce that $a_1(x)$ is a constant and $h(x) = -b\lambda_1 k_1^3 \frac{2n}{m-n+1}(\frac{2n}{m-n+1} - 1)$. For simplicity, taking $a_1(x) = 1$. Balancing the degrees of $g(x)$ and $a_0(x)$, we can deduce that $\deg(g(x)) = \deg(a_0(x))$. Since $a_0(x), g(x)$ are polynomials and $m, n \in N^+$, we derive that only when $2 - \frac{2n}{m-n+1} = 0$, $2 - \frac{2n}{m-n+1} = 1$ and $2 - \frac{2n}{m-n+1} = 6$, there exists exact solutions. Therefore, there are three cases as follows.

4.1 $2 - \frac{2n}{m-n+1} = 0$

(37) becomes

$$(\lambda_1 k_1 + k_1^2 + kl_1^2)\varphi^2 - ak_1^2\varphi^4 - b\lambda_1 k_1^3 \frac{2n}{m-n+1}(\frac{2n}{m-n+1} - 1)\varphi'^2 - b\lambda_1 k_1^3 \frac{2n}{m-n+1}\varphi\varphi'' - g = 0,$$

Substituting $2 - \frac{2n}{m-n+1} = 0$ into (41)-(43), so, we can have the following expressions

$$2b\lambda_1 k_1^3 x a_1'(x) = h(x)a_1(x) + 2b\lambda_1 k_1^3 a_1(x), \tag{44}$$

$$2b\lambda_1 k_1^3 x a_0'(x) = h(x)a_0(x) + g(x)a_1(x), \tag{45}$$

$$g(x)a_0(x) = \left[(\lambda_1 k_1 + k_1^2 + kl_1^2)x^2 - ak_1^2 x^4 - g\right] a_1(x). \tag{46}$$

Accordingly, we deduce that $a_1(x) = 1$, $h(x) = -2b\lambda_1 k_1^3$ and $\deg(g(x)) = \deg(a_0(x))$. Then, from (46), we derive $\deg(g(x)) = \deg(a_0(x)) = 2$. We suppose that

$$a_0(x) = A_0 + A_1 x + A_2 x^2, \ (A_2 \neq 0). \tag{47}$$

Combining (47) and (45), we derive that

$$g(x) = 2b\lambda_1 k_1^3 (A_0 + 2A_1 x + 3A_2 x^2), \tag{48}$$

where $A_i, (i = 0, 1, 2)$ all are real constants that will be determined later. Substituting (47), (48) and $a_1(x) = 1$ into (46) and setting all the coefficients of powers x to be zero which allows a system of nonlinear algebraic equations to be obtained. Solving the system equations, we can get

$$A_0 = \pm\sqrt{\frac{-g}{2b\lambda_1 k_1^3}}, \quad A_2 = \pm\sqrt{\frac{-a}{6b\lambda_1 k_1}}, \quad A_1 = 0, \quad g = \frac{3(\lambda_1 k_1 + k_1^2 + kl_1^2)^2}{16ak_1^2}. \tag{49}$$

Using the conditions (49) in $p(x, y) = a_0(x) + a_1(x)y = 0$, we obtain

$$y = \mp\sqrt{\frac{-g}{2b\lambda_1 k_1^3}} \mp \sqrt{\frac{-a}{6b\lambda_1 k_1}} x^2. \tag{50}$$

Combining the transformations $d\eta = \frac{d\xi}{2b\lambda_1 k_1^3 x}$ and $x = \varphi, y = \frac{\mathrm{d}\varphi}{\mathrm{d}\xi}$, (50) can be converted into

$$\frac{d\varphi}{d\xi} = \mp\sqrt{\frac{-g}{2b\lambda_1 k_1^3}} \mp \sqrt{\frac{-a}{6b\lambda_1 k_1}} \varphi^2.$$

Solving this first order ODE, we have

$$\varphi(\xi) = \mp\sqrt{\frac{3g}{ak_1^2}} \tan\left(\sqrt{\frac{ag}{12b^2\lambda_1^2 k_1^4}}(\xi + C_{23})\right),$$

and

$$\varphi(\xi) = \mp\sqrt{\frac{3g}{ak_1^2}} \tanh\left(\sqrt{\frac{ag}{12b^2\lambda_1^2 k_1^4}}(\xi + C_{24})\right).$$

Finally, we obtain the exact solution

$$u(x, y, t) = \left[\mp\sqrt{\frac{3g}{ak_1^2}} \tan\left(\sqrt{\frac{ag}{12b^2\lambda_1^2 k_1^4}}\left((k_1 x + l_1 y + \lambda_1 t) + C_{23}\right)\right)\right]^{\frac{2}{n}},$$

and

$$u(x, y, t) = \left[\mp\sqrt{\frac{3g}{ak_1^2}} \tanh\left(\sqrt{\frac{ag}{12b^2\lambda_1^2 k_1^4}}\left((k_1 x + l_1 y + \lambda_1 t) + C_{24}\right)\right)\right]^{\frac{2}{n}},$$

where C_{23}, C_{24} are arbitrary constants. The solution $u(x, y, t)$ are presented in Fig. 10, where the formula of $u(x, y, t)$ are that with plus signs.

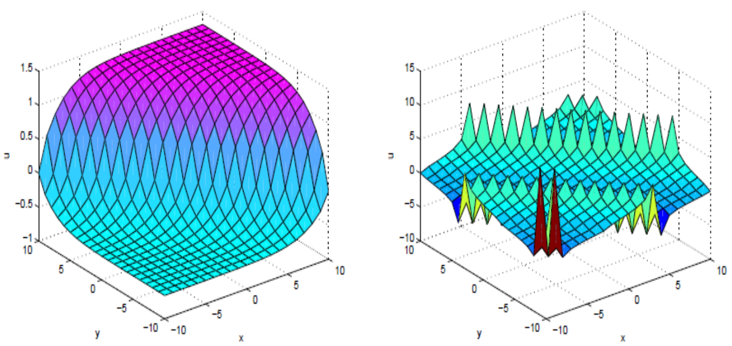

Fig. 10: The solution $u(x, y, t)$ for $t = 0, a = b = \frac{1}{2}, k_1 = l_1 = k = 1, \lambda_1 = -1, n = 3, m = 5, C_{23} = C_{24} = 0$.

4.2 $2-\frac{2n}{m-n+1}=1$

Now, (37) becomes

$$(\lambda_1 k_1+k_1^2+kl_1^2)\varphi^2-ak_1^2\varphi^4-b\lambda_1k_1^3\frac{2n}{m-n+1}(\frac{2n}{m-n+1}-1)\varphi'^2-b\lambda_1k_1^3\frac{2n}{m-n+1}\varphi\varphi''-g\varphi=0.$$

Similarly, we can get

$$b\lambda_1k_1^3xa_1'(x)=h(x)a_1(x)+b\lambda_1k_1^3a_1(x), \tag{51}$$

$$b\lambda_1k_1^3xa_0'(x)=h(x)a_0(x)+g(x)a_1(x), \tag{52}$$

$$g(x)a_0(x)=\left[(\lambda_1k_1+k_1^2+kl_1^2)x^2-ak_1^2x^4-gx\right]a_1(x). \tag{53}$$

Then, we derive that $h(x)=-b\lambda_1k_1^3,\ a_1(x)=1$ and $\deg(a_0(x))=\deg(g(x))=2$. Thus, we suppose

$$a_0(x) \quad = \quad A_0+A_1x+A_2x^2,\ \ (A_2\neq 0). \tag{54}$$

From (52) and (54), we have

$$g(x)=b\lambda_1k_1^3(A_0+2A_1x+3A_2x^2),$$

where $A_i,(i=0,1,2)$ are all real constants that need to be determined later. We substitute $a_0(x)$, $a_1(x)$ and $g(x)$ into (53) and compare all the coefficients of powers x of the both sides of (53). After setting them to be zero, we can have a system of nonlinear algebraic equations. By solving these equations, we derive the corresponding solutions as follows

$$A_2=\pm\sqrt{\frac{-a}{3b\lambda_1k_1}},\quad A_0=A_1=0,\quad g=0,\quad \lambda_1k_1+k_1^2+kl_1^2=0. \tag{55}$$

Using the conditions (55) in $p(x,y)=a_0(x)+a_1(x)y=0$, we obtain

$$y=\mp\sqrt{\frac{-a}{3b\lambda_1k_1}}x^2. \tag{56}$$

According to $d\eta=\frac{d\xi}{b\lambda_1k_1^3x}$ and $x=\varphi, y=\frac{\mathrm{d}\varphi}{\mathrm{d}\xi}$, (56) can be reduce to

$$\frac{d\varphi}{d\xi}=\mp\sqrt{\frac{-a}{3b\lambda_1k_1}}\varphi^2. \tag{57}$$

Solving (57), we have

$$\varphi(\xi)=\left(\pm\sqrt{\frac{-a}{3b\lambda_1k_1}}\xi+C_{25}\right)^{-1}.$$

Therefore, we obtain the rational solutions

$$u(x,y,t)=\left(\pm\sqrt{\frac{-a}{3b\lambda_1k_1}}(k_1x+l_1y+\lambda_1t)+C_{25}\right)^{-\frac{1}{n}},$$

where C_{25} is an arbitrary constant. In addition, we find the figure is similar to Fig. 3 when letting $t=0, a=b=\frac{1}{2}, k_1=l_1=k=1, \lambda_1=-2, n=3, m=8, C_{25}=0$ and taking the plus sign.

4.3 $2-\frac{2n}{m-n+1}=6$

Accordingly, (37) is reduced to

$$(\lambda_1 k_1+k_1^2+kl_1^2)\varphi^2-ak_1^2\varphi^4-b\lambda_1 k_1^3\frac{2n}{m-n+1}(\frac{2n}{m-n+1}-1)\varphi'^2-b\lambda_1 k_1^3\frac{2n}{m-n+1}\varphi\varphi''-g\varphi^6=0.$$

Similarly, we have

$$-4b\lambda_1 k_1^3 x a_1'(x)=h(x)a_1(x)+20b\lambda_1 k_1^3 a_1(x), \tag{58}$$
$$-4b\lambda_1 k_1^3 x a_0'(x)=h(x)a_0(x)+g(x)a_1(x), \tag{59}$$
$$g(x)a_0(x)=\left[(\lambda_1 k_1+k_1^2+kl_1^2)x^2-ak_1^2x^4-gx^6\right]a_1(x). \tag{60}$$

Then, we derive that $h(x)=-20b\lambda_1 k_1^3,\ a_1(x)=1$ and $\deg(a_0(x))=\deg(g(x))=3$. Thus, we suppose

$$a_0(x)\quad=\quad A_0+A_1x+A_2x^2+A_3x^3,\ \ (A_3\neq 0). \tag{61}$$

From (59) and (61), we derive that

$$g(x)=4b\lambda_1 k_1^3(5A_0+4A_1x+3A_2x^2+2A_3x^3), \tag{62}$$

where $A_i,(i=0,1,2,3)$ are real constants which will be determined later. Then, we can also obtain a system of nonlinear algebraic equations. After solving that, we have

$$A_0=A_2=0,\quad A_1=\pm\sqrt{\frac{\lambda_1 k_1+k_1^2+kl_1^2}{16b\lambda_1 k_1^3}},\quad A_3=\pm\sqrt{\frac{-g}{8b\lambda_1 k_1^3}}\quad g=\frac{-2a^2k_1^4}{9(\lambda_1 k_1+k_1^2+kl_1^2)}. \tag{63}$$

Using the conditions (63) in $p(x,y)=a_0(x)+a_1(x)y=0$, we obtain

$$y=\mp\sqrt{\frac{\lambda_1 k_1+k_1^2+kl_1^2}{16b\lambda_1 k_1^3}}x\mp\sqrt{\frac{-g}{8b\lambda_1 k_1^3}}x^3. \tag{64}$$

According to $d\eta=\frac{d\xi}{-4b\lambda_1 k_1^3 x}$ and $x=\varphi, y=\frac{\mathrm{d}\varphi}{\mathrm{d}\xi}$, (64) can be reduce to

$$\frac{d\varphi}{d\xi}=\mp\sqrt{\frac{\lambda_1 k_1+k_1^2+kl_1^2}{16b\lambda_1 k_1^3}}\varphi\mp\sqrt{\frac{-g}{8b\lambda_1 k_1^3}}\varphi^3. \tag{65}$$

Solving (65), we have

$$\varphi(\xi)=\pm\left(\pm\sqrt{\frac{-2g}{\lambda_1 k_1+k_1^2+kl_1^2}}+C_{26}e^{\pm\frac{1}{2}\sqrt{\frac{\lambda_1 k_1+k_1^2+kl_1^2}{b\lambda_1 k_1^3}}\xi}\right)^{-\frac{1}{2}}. \tag{66}$$

Substituting (66) and $2-\frac{2n}{m-n+1}=6$ (i.e. $n=2(m+1)$) into $u=\varphi^{\frac{2}{m-n+1}}$, thus, we obtain the exact solutions

$$u(x,y,t)=\pm\left(\pm\sqrt{\frac{-2g}{\lambda_1 k_1+k_1^2+kl_1^2}}+C_{26}e^{\pm\frac{1}{2}\sqrt{\frac{\lambda_1 k_1+k_1^2+kl_1^2}{b\lambda_1 k_1^3}}(k_1x+l_1y+\lambda_1 t)}\right)^{\frac{2}{n}}, \tag{67}$$

where C_{26} is an arbitrary constant. The solution $u(x,y,t)$ is presented in the Fig. 13, where the formula of $u(x,y,t)$ is that with plus signs.

Remark : In the subsection 4.1, 4.2 and 4.3, when we let $a_1(x)=Ax^i$, ($i\in N^+, i\geq 1$, A is a constant), accordingly, we can find the corresponding $h(x)$. However, the final solutions u are the same as those we obtain in the subsection 4.1, 4.2 and 4.3.

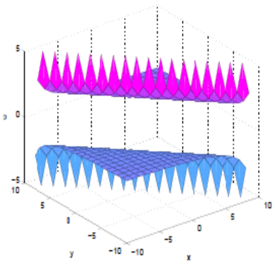

Fig. 11: The solution (14) for $t = 0$, $a = b = \frac{1}{2}, k_1 = l_1 = k = 1$, $\lambda_1 = -2, n = 3, m = 4$, $C_1 = 1, C_2 = 0$.

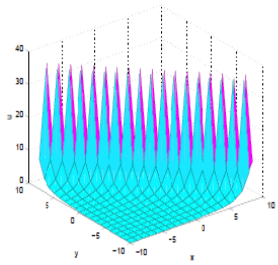

Fig. 12: The solution (36) for $t = 0$, $a = b = \frac{1}{2}, k_1 = l_1 = k = 1$, $\lambda_1 = -2, n = m = 3$, $C_{21} = 1, C_{22} = 0$.

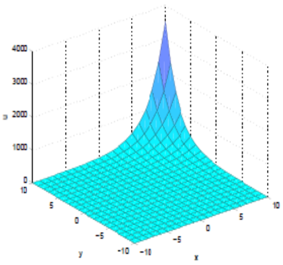

Fig. 13: The solution (67) for $t = 0$, $a = b = \frac{1}{2}, n = 6, m = 2$, $k_1 = l_1 = \lambda_1 = k = 1$, $C_{26} = 1$.

5 CONCLUSION

In this work, the G'/G-expansion method and the first integral method were successfully used to establish the exact solutions of the generalized KP-BBM equation. The rational solutions, the periodic solutions and the hyperbolic function solutions are obtained under some parametric conditions. As far as we know, the solutions that we found are new solutions that are not found in other papers, such as the literature[7]. Certainly, the solution of system (2) should be studied further, which will be left to a further discussion.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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