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The Relativistic Cyclotron Radiation in the Circular Rotating Frame of the Moving Heavy Particle

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

In the current paper we tackle the task of determining the formula for the cyclotron radiation as measured from a frame co-moving with the particle being accelerated. In the case of cyclotrons, as opposed to synchrotrons, the magnetic field is constant, resulting into spiral trajectories for light particle, like electrons and into circular trajectories for heavier particles, like protons, as we will demonstrate in the current paper. This due to the fact that the braking force is a very small percentage of the accelerating (Lorentz) force, as will be shown later in our paper. These proofs have never been attempted before owing to the difficulty of dealing with rotating frames. Our paper is divided into two main sections, the first section deals with cyclotron radiation measured in the inertial frame of the lab, the second section deals with cyclotron radiation as measured in a frame co-rotating with the particle along a circular path, at a uniform speed.

Keywords: Cyclotron radiation (Bremsstrahlung); uniformly rotating frames; special relativity.

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1. INTRODUCTION - BREMSSTRAHLUNG

Bremsstrahlung is the electromagnetic radiation produced by the deceleration of a charged particle. The moving particle loses kinetic energy, which is converted into a photon, it is the process of producing the energy radiation [1]:

$$p = \frac{q^2\gamma^6}{6\pi\varepsilon_0 c}(\dot{\beta}^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\vec{\beta} = \frac{\vec{v}}{c}$$

$$\dot{\vec{\beta}} = \frac{\vec{a}}{c}$$

$$\vec{a} = \frac{d\vec{v}}{d\tau} \tag{1.1}$$

For the case of acceleration perpendicular to the velocity (as in the case of synchrotrons), the formula simplifies to:

$$p = \frac{q^2 a^2 \gamma^4}{6\pi\varepsilon_0 c^3} \tag{1.2}$$

Where p is the power measured in the frame of the lab. In the current paper we will make the attempt of finding the power as expressed in the frame co-moving with the particle, to our best knowledge, this has never been attempted before.

2. KINEMATICS IN UNIFORM ANGULAR VELOCITY ROTATION

In this section we introduce all the fundamental notions that will help discussing the case of the particle moving in an arbitrary plane, with the normal given by the constant angular velocity $\boldsymbol{\omega}(a,b,c)$, as in the case of a charged particle in circular motion in a synchrotron. According to Moller [2], the simpler case when $\boldsymbol{\omega}$ is aligned with the z-axis produces the transformation between the rotating frame $S'(\tau)$ attached to the particle and an inertial, non-rotating frame S attached to the center of rotation [2-7], [15,16] (see Fig.1):

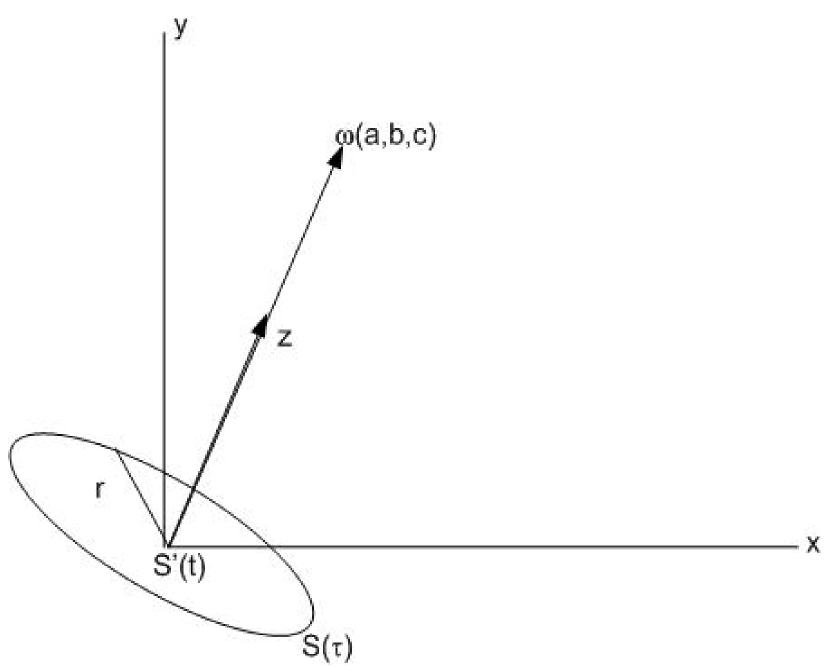

Fig. 1. Relationship between rotating and inertial frames

$$\begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix} = \mathbf{A} \begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} \tag{2.1}$$

Where [7]:

$$\mathbf{A} = \begin{bmatrix} \cos\alpha\cos\beta + \gamma\sin\alpha\sin\beta & \sin\alpha\cos\beta - \gamma\cos\alpha\sin\beta & 0 & -\frac{u\gamma}{c}\sin\beta \\ \cos\alpha\sin\beta - \gamma\sin\alpha\cos\beta & \sin\alpha\sin\beta + \gamma\cos\alpha\cos\beta & 0 & \frac{u\gamma}{c}\cos\beta \\ 0 & 0 & 1 & 0 \\ \frac{u\gamma}{c}\sin\alpha & -\frac{u\gamma}{c}\cos\alpha & 0 & \gamma \end{bmatrix}$$

$$\mathbf{A}^{-1} = \begin{bmatrix} \cos\alpha\cos\beta + \gamma\sin\alpha\sin\beta & \cos\alpha\sin\beta - \gamma\sin\alpha\cos\beta & 0 & -\frac{u\gamma}{c}\sin\alpha \\ \sin\alpha s\cos\beta - \gamma\cos\alpha\sin\beta & \sin\alpha\sin\beta + \gamma\cos\alpha\cos\beta & 0 & \frac{u\gamma}{c}\cos\alpha \\ 0 & 0 & 1 & 0 \\ -\frac{u\gamma}{c}\sin\beta & \frac{u\gamma}{c}\cos\beta & 0 & \gamma \end{bmatrix} \tag{2.2}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$
$$u = r\omega$$
$$\alpha = \omega\gamma\tau$$
$$\beta = \omega\gamma^2\tau \tag{2.3}$$

3. BREMSSTRAHLUNG IN A UNIFORMLY ROTATING FRAME

Assume that we have a particle of charge q and mass m moving in the x-y plane under the influence of a constant magnetic field $\mathbf{B}$ aligned with the z axis. The magnetic field is the only field present since the particle is to have a circular motion [4,8]. We know that in the frame of the lab, the expression of the Lorentz force acting on the particle is [8]:

$$\mathbf{F} = q\mathbf{v}\times\mathbf{B} = q\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r\omega\cos(\omega t) & r\omega\sin(\omega t) & 0 \\ 0 & 0 & B \end{bmatrix} \tag{3.1}$$

We would like to find out the expression of the force in the frame co-rotating with the charged particle. For this purpose we will resort to the fact [9] that four-force transforms like four-coordinate (2.1)

$$\begin{pmatrix} \gamma'(u')F_x' \\ \gamma'(u')F_y' \\ \gamma'(u')F_z' \\ \gamma'(u')\frac{\mathbf{F'.u'}}{c} \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} \gamma(u)F_x \\ \gamma(u)F_y \\ \gamma(u)F_z \\ \gamma(u)\frac{\mathbf{F.u}}{c} \end{pmatrix} \tag{3.2}$$

The term $\gamma(u)\frac{\mathbf{F.u}}{c}$ represents the power imparted by the magnetic field to the particle measured in the lab frame (divided by c) while the term $\gamma'(u')\frac{\mathbf{F'.u'}}{c}$ represents the power imparted by the magnetic field to the particle measured in the frame commoving with the particle (divided by c). Transformation (3.2) gives the general formulas for transforming four-force (proper force) in rotating frames.

We know from [8] that:

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} r\cos(\omega t) \\ r\sin(\omega t) \\ 0 \\ t \end{pmatrix} \tag{3.3}$$

$$\begin{aligned} u_x &= -r\omega\sin\omega t \\ u_y &= r\omega\cos\omega t \\ u_z &= 0 \\ u &= \sqrt{u_x^2+u_y^2+u_z^2} = r\omega = u_0 \\ \gamma(u) &= \gamma(u_0) \end{aligned} \tag{3.4}$$

$$\begin{aligned} F_x &= -qBr\omega\sin\omega t \\ F_y &= qBr\omega\cos\omega t \\ F_z &= 0 \end{aligned} \tag{3.5}$$

Substituting (3.4),(3.5) into (3.2) we obtain:

$$\begin{pmatrix} \gamma'(u')F_x' \\ \gamma'(u')F_y' \\ \gamma'(u')F_z' \\ P'/c \end{pmatrix} = \mathbf{A}^{-1}\begin{pmatrix} \gamma(u_0)F_x \\ \gamma(u_0)F_y \\ \gamma(u_0)F_z \\ \gamma(u_0)\frac{\mathbf{F.u}}{c} \end{pmatrix} = \mathbf{A}^{-1}\begin{pmatrix} \gamma(u_0)qBu_0\sin\omega t \\ -\gamma(u_0)qBu_0\cos\omega t \\ 0 \\ \gamma(u_0)\frac{qBu_0^2}{c} \end{pmatrix} \tag{3.6}$$

Therefore:

$$\begin{pmatrix} \gamma'(u')F_x' \\ \gamma'(u')F_y' \\ \gamma'(u')F_z' \\ P'/c \end{pmatrix} =$$

$$= \begin{bmatrix} \cos\omega t\cos\gamma\omega t+\gamma\sin\omega t\sin\gamma\omega t & \sin\omega t\cos\gamma\omega t-\gamma\cos\omega t\sin\gamma\omega t & 0 & -\frac{u_0\gamma\sin\gamma\omega t}{c} \\ \cos\omega t\sin\gamma\omega t-\gamma\sin\omega t\cos\gamma\omega t & \sin\omega t\sin\gamma\omega t+\gamma\cos\omega t\cos\gamma\omega t & 0 & \frac{u_0\gamma\cos\gamma\omega t}{c} \\ 0 & 0 & 1 & 0 \\ -\frac{u_0\gamma\sin\omega t}{c} & \frac{u_0\gamma\cos\omega t}{c} & 0 & \gamma \end{bmatrix}\begin{pmatrix} \gamma qBu_0\sin\omega t \\ -\gamma qBu_0\cos\omega t \\ 0 \\ \gamma\frac{qBu_0^2}{c} \end{pmatrix} \tag{3.7}$$

The final formula for the power imparted by the magnetic field to the particle measured in the frame commoving with the particle is:

$$P' = 2qB\gamma^2(u_0)u_0^2 \tag{3.8}$$

Expression (3.7) provides us with the expression of the force acting on the particle as measured in the frame of the particle (the proper force). For example:

$$\tilde{F}_x' = \gamma'(u')F_x' = \gamma^2(u_0)qBu_0(1-\frac{u_0^2}{c^2})\sin\gamma\omega t = qBu_0\sin\gamma\omega t$$
$$\tilde{F}_y' = \gamma'(u')F_y' = -\gamma^2(u_0)qBu_0(1-\frac{u_0^2}{c^2})\cos\gamma\omega t = qBu_0\cos\gamma\omega t$$
$$\tilde{F}_z' = 0 \tag{3.9}$$

The term $\gamma^2(u_0)qBu_0$ in (3.9) represents the non-fictitious component, the active Lorentz force while the term $-\gamma^2(u_0)qBu_0(\frac{u_0^2}{c^2})$ represents the fictitious component, the centrifugal "force" due to the calculations being done in the (uniformly) rotating frame.

We are now ready to derive the radiated power. From [4] we know that:

$$\omega = \frac{qB}{\gamma(v_0)m} \tag{3.10}$$

Substituting (3.4),(3.10) into (1.2) we obtain:

$$p = \frac{q^2a^2\gamma^4}{6\pi\varepsilon_0c^3} = \frac{q^2\gamma^6(u_0)u_0^2\omega^2}{6\pi\varepsilon_0c^3} = \frac{q^2B^2}{\gamma^2(u_0)m^2}\frac{q^2\gamma^6(u_0)u_0^2}{6\pi\varepsilon_0c^3} = \frac{q^4\gamma^4(u_0)u_0^2B^2}{6\pi\varepsilon_0c^3m^2} \tag{3.11}$$

The radiated power in this case is a constant that depends on the mass of the particle, m , its charge, q , its initial speed of injection into the synchrotron, u_0 and the magnitude of the magnetic field B . The constancy is due to the fact that $r\omega = u_0$. The braking force due to radiation acts in direct opposition to the direction of motion (direction of **u**):

$$f_x = +\frac{p}{u_0}\sin\omega t$$
$$f_y = -\frac{p}{u_0}\cos\omega t$$
$$f_z = 0$$
$$\frac{f_x}{f_y} = -\tan\omega t \tag{3.12}$$

We know that:

$$\begin{pmatrix} \gamma'(u')f_x' \\ \gamma'(u')f_y' \\ \gamma'(u')f_z' \\ p'/c \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} \gamma(u_0)f_x \\ \gamma(u_0)f_y \\ \gamma(u_0)f_z \\ \gamma(u_0)\frac{\mathbf{f.u}}{c} \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} \gamma(u_0)\frac{p}{u_0}\sin\omega t \\ -\gamma(u_0)\frac{p}{u_0}\cos\omega t \\ 0 \\ \gamma(u_0)\frac{p}{c} \end{pmatrix} = \gamma(u_0)p\mathbf{A}^{-1} \begin{pmatrix} \frac{\sin\omega t}{u_0} \\ -\frac{\cos\omega t}{u_0} \\ 0 \\ \frac{1}{c} \end{pmatrix} \tag{3.13}$$

Therefore:

$$\begin{pmatrix} \gamma'(u')f_x' \\ \gamma'(u')f_y' \\ \gamma'(u')f_z' \\ p'/c \end{pmatrix} =$$

$$= \gamma(u_0)p \begin{bmatrix} \cos\omega t\cos\gamma\omega t + \gamma\sin\omega t\sin\gamma\omega t & \sin\omega t\cos\gamma\omega t - \gamma\cos\omega t\sin\gamma\omega t & 0 & -\frac{u_0\gamma\sin\gamma\omega t}{c} \\ \cos\omega t\sin\gamma\omega t - \gamma\sin\omega t\cos\gamma\omega t & \sin\omega t\sin\gamma\omega t + \gamma\cos\omega t\cos\gamma\omega t & 0 & \frac{u_0\gamma\cos\gamma\omega t}{c} \\ 0 & 0 & 1 & 0 \\ -\frac{u_0\gamma\sin\omega t}{c} & \frac{u_0\gamma\cos\omega t}{c} & 0 & \gamma \end{bmatrix} \begin{pmatrix} \frac{\sin\omega t}{u_0} \\ -\frac{\cos\omega t}{u_0} \\ 0 \\ \frac{1}{c} \end{pmatrix} \tag{3.14}$$

From (3.14) we get the radiated power measured in the frame co-moving with the particle:

$$p' = 0 \tag{3.15}$$

This should come as no surprise since the particle is not accelerating in its own frame of reference. We also get the braking force due to radiation:

$$\begin{aligned} \bar{f}_x' &= \gamma'(u')f_x' = -\gamma^2(u_0)\frac{p}{u_0}(1-\frac{u_0^2}{c^2})\sin\gamma\omega t = -\frac{p}{u_0}\sin\gamma\omega t \\ \bar{f}_y' &= \gamma'(u')f_y' = \gamma^2(u_0)\frac{p}{u_0}(1-\frac{u_0^2}{c^2})\cos\gamma\omega t = \frac{p}{u_0}\cos\gamma\omega t \\ \bar{f}_z' &= 0 \\ \frac{\bar{f}_x'}{\bar{f}_y'} &= -\tan\gamma\omega t \end{aligned} \tag{3.16}$$

Notice the similarity between expressions (3.16) and (3.13). Also notice how the ratio between the force components in the x and y directions has been increased by the contribution of the gamma factor. We can see that the particle experiences a braking force in the frame co-moving with it. This is extremely important for practical reasons: despite of the absence of radiating power as measured in the co-moving frame, the particle is still being slowed down, independent of the frame of reference used for calculations, thus requiring external energy to be imparted in order to maintain its speed [10-14].

4. BRAKING FORCE AS A PERCENTAGE OF THE LORENTZ FORCE

The total radiated power goes as m^{-4}, which accounts for why electrons lose energy due to bremsstrahlung radiation much more rapidly than heavier charged particles (e.g., muons, protons, alpha particles). This is the reason why the TeV energy electron-positron colliders cannot use circular tunnels (requiring constant acceleration), while a proton-proton colliders (such as LHC) can utilize circular tunnels (and, consequently, the acting Lorentz force is dependent only on the magnetic field, as explained in (3.1)). The electrons lose energy due to bremsstrahlung at a rate $(m_p / m_e)^4 \approx 10^{13}$ times higher than protons do [10-14].

Another way of looking at the issue is by calculating the ratio between the braking force and the active (Lorentz) force. From (3.12) we know that the braking force is:

$$f = \frac{p}{u_0} = \frac{q^4 \gamma^4 (u_0) u_0 B^2}{6\pi\varepsilon_0 c^3 m^2} \tag{4.1}$$

The Lorentz force is:

$$F = qBu_0 \tag{4.2}$$

Therefore, their ratio is:

$$r = \frac{f}{F} = \frac{q^3 \gamma^4 (u_0) B}{6\pi\varepsilon_0 c^3 m^2} \tag{4.3}$$

When comparing the ratios for the cases of an electron vs. a proton, for the same conditions in terms of magnetic field and initial particle injection sped we find out that:

$$\frac{r_e}{r_p} = (\frac{m_p}{m_e})^2 \tag{4.4}$$

The reaction force in the case of an electron is much larger (about 10^7) than that the one for a proton for the case of equal particle accelerating Lorentz forces.

We could to the above calculations in the frame co-rotating with the particle. From (3.9) we obtain:

$$\tilde{F}' = \gamma^2 (u_0) qBu_0 (1 + \frac{u_0^2}{c^2}) \tag{4.5}$$

From (3.16) we obtain:

$$\tilde{f}' = \frac{p}{u_0} \tag{4.6}$$

The interesting result is that we obtain the same exact result as the one calculated in the frame of the lab, the ratio of forces depends only on the inverse ratio of masses, as shown in (4.4). Even more interestingly, the braking force is the same in both frames.

In the case of cyclotrons, as opposed to synchrotrons [17], the magnetic field is constant, resulting into spiral trajectories for light particle, like electrons, and into circular trajectories for heavier particles, like protons, as we have demonstrated in the current paper. This due to the fact that the braking force is a very small percentage of the accelerating (Lorentz) force, as shown in our paper.

5. CONCLUSIONS

We derived the Bremsstrahlung formula for the cyclotron radiation as measured from the point of view of a frame co-rotating with the charged particle. These proofs have never been attempted before owing to the difficulty of dealing with rotating frames. The formula is of great interest to particle physicists showing that the particles experienced a braking force despite of the absence of radiating power as measured in the co-moving frame, thus requiring external energy application in order to maintain speed.

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The present paper is an updated study of the previously published paper.

In the current paper we have applied the same methodology, of electrodynamics in rotating frames [3,7], as the one employed in describing the physics of synchrotrons [17], with the key difference that cyclotrons, as opposed to synchrotrons are capable of maintaining circular trajectories for heavy particles without having to resort to ever increasing the kinetic energy of the particles via using an ever increasing Lorentz force due to an ever increasing magnetic field, a constant magnetic field is sufficient.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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