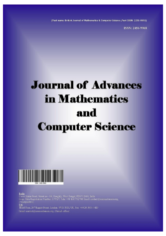

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Exact Solution of Space-Time Fractional Partial Differential Equations by Adomian Decomposition Method

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Authors' contributions

This work was carried out in collaboration between both authors. The second author gave the idea of main result. Numerical calculations, data explanations and graphical representation are done by first author. Both the authors gave equal contribution in the reading, writing of this paper and approved final manuscript.

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Abstract

The intention behind this paper is to achieve exact solution of one dimensional nonlinear fractional partial differential equation(NFPDE) by using Adomian decomposition method(ADM) with suitable initial value. These equations arise in gas dynamic model and heat conduction model. The results show that ADM is powerful, straightforward and relevant to solve NFPDE. To represent usefulness of present technique, solutions of some differential equations in physical models and their graphical representation are done by MATLAB software.

Keywords: Biological population model; Heat conduction model; fractional calculus; Adomian decomposition method; Mittage-Leffler function.

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1 Introduction

In current years, fractional calculus has been widely utilized for various applications in large number of well organized and technological fields such as biosciences, chemical sciences, biochemical and physical fields. Nonlinear partial differential equation(NPDE) appear in various fields of physics, engineering and applied mathematics. It has come to light that various facts in engineering, physics and other sciences can be expressed very gratefully by models using mathematical tool by fractional calculus [1, 2]. For better comprehension of phenomenon expressed by a given NFPDE, the solution of differential equations of fractional order must be elaborated. Fractional derivatives provide more perfect models of actual world problems than integer order derivatives. By virtue of their many applications in scientific research fields, FPDEs found to be an effective aid to describe certain physical phenomena, such as diffusion processes, electrical and rheological materials properties and viscoelasticity theories also in earthquake modeling, traffic flow models, diffusion model, control and relaxation processes [3, 4, 5, 6, 7].

Many researcher have concentrate to study the analytical or approximate solutions of NFPDEs by applying various numerical methods. In between these methods, ADM [8, 9, 10, 11, 12] is worldwide approach which can be used to solve fractional ordinary differential equations as well as FPDEs. ADM was at first suggested by Adomian [13, 14]. Wazwaz [15, 16] has applied ADM to solve variety of differential equations. While Shawagfeh [17] has employed Adomian decomposition method for solving NFPDEs, Daftardar-Gejji and Jafri have obtained solution of numerous problems [18, 19] by using Adomian decomposition method. Also Dhaigude and Birajdar [20, 21]extended the discrete ADM for obtaining the numerical solution of system of fractional partial differential equations. Chitalkar-Dhaigude and Bhadgaonkar in [22] have shown that the ADM is more convenient than the Charpits method to solve first-order nonlinear PDEs. Bhadgaonkar and Dhaigude [23] obtain exact analytical solution of nonlinear nonhomogeneous space-time FPDEs in Gas dynamics model, Advection model, Wave model and Klein-Gordon model by improved Adomian decomposition method coupled with fractional Taylor expansion series. B.Sontakke and R.Pandit [24, 25, 26] investigates the iterative solution of linear and NFPDEs using fractional ADM. Peng Guo [27] also solve fractional partial differential equations by ADM.

There are very few equations which can be solved by applying both space-time fractional order derivative. The gas dynamics equation and heat conduction equation are the most crucial nonlinear equation that plays a vital role in physical science and engineering. In this study, the use of ADM is extended to find analytical approximate solutions for the nonlinear fractional heat conduction problem and gas dynamics problem. The solutions of our model equations are deliberated in the form of convergent series with easily computable components. The space and time fractional derivatives are described in the Caputo sense. Gas dynamics equations [28, 29, 30, 31] are mathematical expressions based on the physical behaviours of conservation of mass,momentum, energy, etc. The nonlinear fractional gas dynamics equations are relevant in the shock fronts, rare factions, and contact disconnectedness. In [32] author solved gas dynamic equation for a time fractional derivative. The study of gas dynamics is often correlated with the flight of recent high-speed aircraft and atmospheric reentry of space-exploration vehicles. One-dimensional (1-D) flow refers to flow of gas through a duct or channel. Consider one dimensional space and time fractional gas dynamic equation.

$$D_t^{\alpha}u(x,t) + D_x^{\alpha}u(x,t) - u(x,t)(1-u(x,t)) = 0, \qquad 0 < \alpha \leq 1. \tag{1.1}$$

with initial condition $u(x,0) = f(x)$. If $\alpha = 1$ given equation reduces to classical one.

A heat conduction [33, 34] is a molecular transfer of thermal energy in solids, liquids and gases from the more energetic particles of a medium to the adjacent less energetic ones. The action of the heat conduction occur between the particles of the substance when they directly touch each-other and have unlike temperature. A heat transfer problem is said to be one-dimensional if the temperature in the medium varies in one direction only and thus heat is transferred in one direction, and the variation of temperature and thus heat transfer in other directions are negligible or zero. For example, heat transfer through the glass of a window can be considered to be one-dimensional. Likewise, heat transfer through a hot water pipe, Heat transfer to an egg dropped into boiling water can be considered to be one dimensional. Fakour, M. et al. [35, 36, 37, 38, 39] and Rahbari, A. et al.[40] are study more about heat conduction phenomenon. A heat conduction one dimensional space-time fractional order equation can be formulated by specifying the applicable differential equation and a set of proper initial condition.

$$D_t^{\alpha}u(x,t) = D_x^{2\alpha}u(x,t) + \frac{x^{\alpha}}{\Gamma(\alpha+1)}uD_x^{\alpha}u(x,t) - u^2(x,t) + u(x,t), \qquad 0 < \alpha \leq 1 \tag{1.2}$$

with initial condition $u(x,0) = f(x)$. If $\alpha = 1$ given equation reduces to classical one.
The paper is designed in such way: in section (2) few basic results about fractional calculus and related properties are given which are used in this paper, while in section (3) we clarify the steps of the ADM for solving nonlinear space and time FPDEs. The effectiveness and sharpness of the method is shown by obtaining solution of equations in physical models like gas dynamic model and heat conduction model in section (4). Section (5) is results and discussion. Section (6) is conclusion.

2 Preliminaries

In this section, we set up notations, basic definitions and main properties of Riemann-Liouville fractional integral operator(RLFIO), and Caputo fractional differential operator(CFDO) is also given. In this section, basic definitions on fractional calculus are discussed which are useful for further discussion.

Definition 2.1. [41] Let $f \in \mathcal{C}_{\alpha}$ and $\alpha \geq -1$, then RLFIO of $u(x,t)$ with respect to t of order α is indicated by $I_t^{\alpha}u(x,t)$ and is explained as

$$J_t^{\alpha}u(x,t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{(\alpha-1)}u(x,\tau)d\tau, \quad t > 0, \alpha > 0. \tag{2.1}$$

Definition 2.2. [41] Let $m-1 < \alpha < m$, $t \in R$ and $t > 0$. The CFDO for the function $f \in H^1([a,b],\mathbb{R}_+)$ with order $\alpha \geq 0$ is explained as

$$D_t^{\alpha}u(x,t) = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)}\displaystyle\int_0^t (t-\tau)^{m-\alpha-1}\frac{\partial^m u}{\partial \tau^m}d\tau, \\ \dfrac{\partial^m u}{\partial t^m}, \qquad \alpha = m \in N. \end{cases} \tag{2.2}$$

We have following properties of RLFIO and CFDO

$$D_t^{\alpha}t^{\mu} = \frac{\Gamma(\mu+1)}{\Gamma(\mu-\alpha+1)}t^{(\mu-\alpha)}, \tag{2.3}$$

$$J_t^{\alpha}t^{\mu} = \frac{\Gamma(\mu+1)}{\Gamma(\mu+\alpha+1)}t^{(\mu+\alpha)}, \quad \alpha > 0, \ \mu > -1. \tag{2.4}$$

Note that the relation between RLFIO and CFDO is given by:

$$J_t^{\alpha} D_t^{\alpha} u(x,t) = u(x,0) - \sum_{k=0}^{m-1} u^{(k)}(x,0) \frac{t^k}{k!}, \quad m-1 < \alpha \leq m. \tag{2.5}$$

Definition 2.3. The Mittage-Leffler function[42] for one parameter and two parameter is defined as follows

$$E_{\alpha}(t) = \sum_{n=0}^{\infty} \frac{t^n}{\Gamma(\alpha n + 1)}, \quad (\alpha \in \mathbf{C}, Re(\alpha) > 0),$$
$$E_{\alpha,\beta}(t) = \sum_{n=0}^{\infty} \frac{t^n}{\Gamma(\alpha n + \beta)}, \quad (\alpha, \beta \in \mathbf{C}, Re(\alpha, \beta) > 0).$$

When we apply CFDO on MLF we get

$$D_t^{\alpha} E_{\alpha}(at^{\alpha}) = aE_{\alpha}(at^{\alpha}), \tag{2.6}$$

where a is constant.

3 Analysis of Adomian Decomposition Method

In this section, we present the ADM to solve one-dimensional nonlinear space-time FPDEs. Consider the IVP for nonlinear space and time FPDE of order $0 < \alpha \leq 1$

$$L_t^{\alpha} u(x,t) + R(u(x,t)) + N(u(x,t)) = g(x,t), \tag{3.1}$$

with initial condition

$$u(x,0) = h(x) \tag{3.2}$$

where $L_t^{\alpha}(u)$ is the fractional differential operator of highest order fractional derivative with respect to t, $u(x,t)$ is unrecognized function which we want to determined, t is time variable, x is the space coordinate, $R(u)$ is linear differential operator, $N(u) = f(u(x,t))$ is nonlinear data and $g(x,t)$ is nonhomogeneous function.

Now, applying the RLFIO J_t^{α} on both side of equation(3.1) and use the IC (3.2), we attain:

$$u(x,t) = u(x,0) + J_t^{\alpha}\big[g(x,t) - R(u) - N(u)\big]. \tag{3.3}$$

The unrecognized function $u(x,t)$ can be expressed as an infinite series of the form

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{3.4}$$

The Adomian polynomials A_n for the nonlinear term $N(u)$ can be evaluated by using the following expression

$$N(u(x,t)) = \sum_{n=0}^{\infty} A_n(u_0, u_1, u_3, \cdots, u_n), \tag{3.5}$$

where

$$A_n = \frac{1}{n!} \frac{\partial^n}{\partial \lambda^n} \big[N \sum_{i=0}^{n} \lambda^i u_i\big]_{\lambda=0}, \qquad n = 0, 1, 2, 3, \ldots \tag{3.6}$$

where A_n are the nonlinear Adomian polynomials. By substituting decomposed series (3.4), (3.5) in (3.3) we attain

$$\sum_{n=0}^{\infty} u(x,t) = h(x) + J_t^{\alpha}\Big[g(x,t) - R\sum_{n=0}^{\infty} u_n(x,t) - \sum_{n=0}^{\infty} A_n(x,t)\Big], \tag{3.7}$$

Taking term by term comparison on both side of equation (3.7), we set recursion scheme like:

$$\begin{aligned}
u_0(x,t) &= h(x) + J_t^{\alpha} g(x,t),\\
u_1(x,t) &= J_t^{\alpha}\Big[-R(u_0) - A_0\Big],\\
u_2(x,t) &= J_t^{\alpha}\Big[-R(u_1) - A_1\Big],\\
u_3(x,t) &= J_t^{\alpha}\Big[-R(u_2) - A_2\Big],\\
&\vdots\\
u_{(k+1)}(x,t) &= J_t^{\alpha}\Big[-R(u_k) - A_k\Big], k \geq 0,
\end{aligned}$$

and so forth. Wherever every component can be determined by manipulating the preceding components and we can attain the solution in a series form by computing the components $u_n(x,t)$,$n \geq 0$. Eventually, we approximate the solution $u(x,t)$ by the reduced series. Then the solution $u(x,t)$ of IVP (3.1) − (3.2) is

$$\phi_{m+1} = \sum_{n=0}^{m} u_n(x,t) \tag{3.8}$$

which gives

$$\lim_{m\to\infty} \phi_{m+1} = u(x,t). \tag{3.9}$$

4 Numerical Application

The benefits and intensity of the ADM can be expressed by applying it to some physical models in space and time FPDEs.

Example 4.1. *Consider space-time fractional order gas dynamic equation*

$$D_t^{\alpha} u(x,t) = uD_x^{\alpha} u - u(1-u), \qquad 0 < \alpha \leq 1 \tag{4.1}$$

subject to initial condition

$$u(x,0) = E_{\alpha}(-x^{\alpha}) \tag{4.2}$$

Solution:- *Applying J_t^{α} on both side of (4.1) we have*

$$J_t^{\alpha} D_t^{\alpha} u(x,t) = J_t^{\alpha}\Big[-uD_x^{\alpha}u + u - u^2\Big]$$

$$u(x,t) = u(x,0) + J_t^{\alpha}\Big[-uD_x^{\alpha}u + u - u^2\Big] \tag{4.3}$$

Suppose that the solution $u(x,t)$ has the following series form

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{4.4}$$

Then equation (4.3) *has the form*

$$\sum_{n=0}^{\infty} u_n(x,t) = E_\alpha(-x^\alpha) + J_t^\alpha\left[\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} u_n - \sum_{n=0}^{\infty} B_n\right] \tag{4.5}$$

where A_n *and* B_n *are the Adomian polynomials to be determined from the nonlinear term* $uD_x^\alpha u$ *and* u^2 *. Comparing both side of equation* (4.5) *we have*

$$\begin{aligned}
u_0(x,t) &= E_\alpha(-x^\alpha)\\
u_1(x,t) &= J_t^\alpha\Big[-A_0 + u_0 - B_0\Big]\\
&= J_t^\alpha\Big[-u_0D_x^\alpha u_0 + u_0 - (u_0)^2\Big]\\
u_1(x,t) &= E_\alpha(-x^\alpha)\frac{t^\alpha}{\Gamma(\alpha+1)}\\
u_2(x,t) &= J_t^\alpha\Big[-A_1 + u_1 - B_1\Big]\\
&= J_t^\alpha\Big[-(u_1D_x^\alpha u_0 + u_0D_x^\alpha u_1) + u_1 - 2u_0u_1\Big]\\
u_2(x,t) &= E_\alpha(-x^\alpha)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\\
u_3(x,t) &= J_t^\alpha\Big[-A_2 + u_2 - B_2\Big]\\
&= J_t^\alpha\Big[-(u_2D_x^\alpha u_0 + u_1D_x^\alpha u_1 + u_0D_x^\alpha u_2) + u_2 - ((u_1)^2 + 2u_0u_2)\Big]\\
u_3(x,t) &= E_\beta(-x^\alpha)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}
\end{aligned}$$

and so on.Then solution of IVP is

$$\begin{aligned}
u(x,t) &= \sum_{n=0}^{\infty} u_n(x,y,t) = u_0 + u_1 + u_2 + \dots\\
&= E_\beta(-x^\beta)\left[1 + \frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \dots\right]\\
&= E_\alpha(-x^\alpha)\sum_{n=0}^{\infty}\frac{t^{n\alpha}}{\Gamma(n\alpha+1)}\\
u(x,t) &= E_\beta(-x^\alpha)E_\alpha(t^\alpha) \qquad (4.6)
\end{aligned}$$

It is exact solution of IVP (4.1)-(4.2). *If* $\alpha = 1$ *solution* (4.6) *reduces to*

$$u(x,t) = e^{-x+t} \tag{4.7}$$

which is an exact solution to the standard form gas dynamic equation.

Example 4.2. *Consider space-time fractional order heat conduction equation*

$$D_t^\alpha u(x,t) = D_x^{2\alpha}u(x,t) + \frac{x^\alpha}{\Gamma(\alpha+1)}uD_x^\alpha u(x,t) - u^2(x,t) + u(x,t), \qquad 0 < \alpha \le 1 \tag{4.8}$$

subject to initial condition

$$u(x,0) = \frac{x^{\alpha}}{\Gamma(\alpha+1)} \tag{4.9}$$

Solution:- *Applying* J_t^{α} *on both side of* (4.8) *we have*

$$\begin{aligned} J_t^{\alpha} D_t^{\alpha} u(x,t) &= J_t^{\alpha}\left[D_x^{2\alpha} u(x,t) + \frac{x^{\alpha}}{\Gamma(\alpha+1)} u D_x^{\alpha} u(x,t) - u^2(x,t) + u(x,t)\right] \\ u(x,t) &= u(x,0) + J_t^{\alpha}\left[D_x^{2\alpha} u(x,t) + \frac{x^{\alpha}}{\Gamma(\alpha+1)} u D_x^{\alpha} u(x,t) - u^2(x,t) + u(x,t)\right] \end{aligned} \tag{4.10}$$

Suppose that the solution $u(x,t)$ *has the following series form*

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{4.11}$$

Then equation (4.10) *has the form*

$$\sum_{n=0}^{\infty} u_n(x,t) = \frac{x^{\alpha}}{\Gamma(\alpha+1)} + J_t^{\alpha}\left[D_x^{2\alpha} \sum_{n=0}^{\infty} u_n + \frac{x^{\alpha}}{\Gamma(\alpha+1)} \sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n + \sum_{n=0}^{\infty} u_n\right] \tag{4.12}$$

where A_n *and* B_n *are the Adomian polynomials to be determined from the nonlinear term* $uD_x^{\alpha}u$ *and* u^2 . *Comparing both side of equation* (4.12) *we have*

$$\begin{aligned} u_0(x,t) &= \frac{x^{\alpha}}{\Gamma(\alpha+1)} \\ u_1(x,t) &= J_t^{\alpha}\left[D_x^{2\alpha} u_0 + \frac{x^{\alpha}}{\Gamma(\alpha+1)} A_0 - B_0 + u_0\right] \\ &= J_t^{\alpha}\left[0 - \frac{x^{\alpha}}{\Gamma(\alpha+1)} u_0 D_x^{\alpha} u_0 - (u_0)^2 + u_0\right] \\ u_1(x,t) &= \frac{x^{\alpha}}{\Gamma(\alpha+1)} \frac{t^{\alpha}}{\Gamma(\alpha+1)} \\ u_2(x,t) &= J_t^{\alpha}\left[D_x^{2\alpha} u_1 + \frac{x^{\alpha}}{\Gamma(\alpha+1)} A_1 - B_1 + u_1\right] \\ &= J_t^{\alpha}\left[0 - \frac{x^{\alpha}}{\Gamma(\alpha+1)} (u_0 D_x^{\alpha} u_1 + u_1 D_x^{\alpha} u_0) - (2u_0 u_1) + u_1\right] \\ u_2(x,t) &= \frac{x^{\alpha}}{\Gamma(\alpha+1)} \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} \\ u_3(x,t) &= J_t^{\alpha}\left[D_x^{2\alpha} u_2 + \frac{x^{\alpha}}{\Gamma(\alpha+1)} A_2 - B_2 + u_2\right] \\ &= J_t^{\alpha}\left[0 - \frac{x^{\alpha}}{\Gamma(\alpha+1)} (u_0 D_x^{\alpha} u_2 + u_1 D_x^{\alpha} u_1 + u_2 D_x^{\alpha} u_0) - (2u_0 u_2 + u_1^2) + u_2\right] \\ u_3(x,t) &= \frac{x^{\alpha}}{\Gamma(\alpha+1)} \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} \end{aligned}$$

and so on.Then solution of IVP is

$$
\begin{aligned}
u(x,t) &= \sum_{n=0}^{\infty} u_n(x,y,t) = u_0 + u_1 + u_2 + \dots \\
&= \frac{x^{\alpha}}{\Gamma(\alpha+1)}\left[1 + \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \dots\right] \\
&= \frac{x^{\alpha}}{\Gamma(\alpha+1)} \sum_{n=0}^{\infty} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} \\
u(x,t) &= \frac{x^{\alpha}}{\Gamma(\alpha+1)} E_{\alpha}(t^{\alpha}) \qquad (4.13)
\end{aligned}
$$

It is exact solution of IVP (4.8)-(4.9)*. If* $\alpha = 1$ *solution* (4.13) *reduces to*

$$u(x,t) = xe^{t} \qquad (4.14)$$

which is an exact solution to the standard form heat conduction equation.

5 Results and Discussion

Fig. 1 is the graphical behaviour of ADM solution (4.6) for different values of α such as $\alpha = 1, 0.8, 0.6, 0.4$ and exact solution (4.7) when $x = y = 0.25$. Figs. $2(a), (b)$ and $3(c), (d)$ shows the surface of the 4 terms of the improved ADM solution (4.6) for values of $\alpha = 1, 0.8, 0.6$ and surface of exact solution (4.7). It is clear from Fig. 1 and Figs. 2 to 3, in the limit while $\alpha \to 1$, (4.6) approaches to the exact solution (4.7). Fig. 4 is the graphical behaviour of ADM solution (4.13) for different values of α such as $\alpha = 1, 0.8, 0.6, 0.4$ and exact solution (4.14) when $x = 0.25$. Fig. $5(a), (b)$ and $6(c), (d)$ shows the surface of the 4 terms of the improved ADM solution (4.13) for values of $\alpha = 1, 0.8, 0.6$ and surface of exact solution (4.14). It is clear from Fig. 4 and Figs. 5 to 6, in the limit while $\alpha \to 1$, (4.13) approaches to the exact solution (4.14). We can see that the shape of curve of approximate solution for $\alpha = 1$ coincides with shape of the exact solution. Therefore, the improved ADM is an effective and sharp method which can be handled to detect exact analytical solution of fractional-order gas dynamics equation and heat conduction equation.

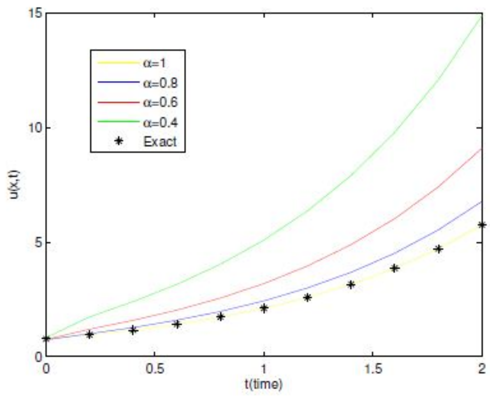

Fig. 1. 2D Graphical representation of solution (4.6) **of IVP** (4.1)-(4.2) **for different values of** α **such as** $\alpha = 1, 0.8, 0.6, 0.4$ **and exact when** $x = 0.25$**.**

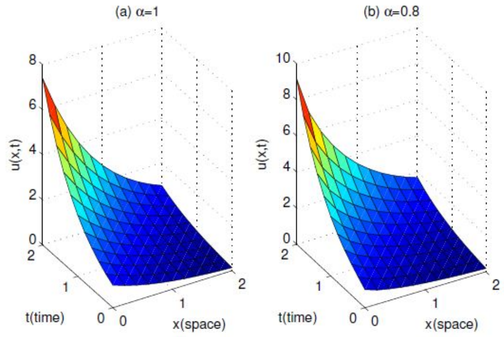

Fig. 2. 3D Graphical representation of solution (4.6) **of IVP** (4.1)-(4.2) **when** $\alpha = 1, 0.8$ **with respect to time**

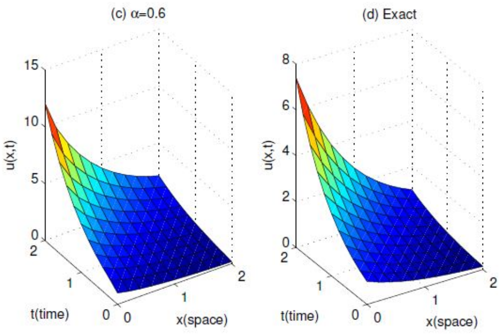

Fig. 3. 3D Graphical representation of solution (4.6) **of IVP** (4.1)-(4.2) **when** $\alpha = 0.6$ **and exact solution** (4.7) **with respect to time**

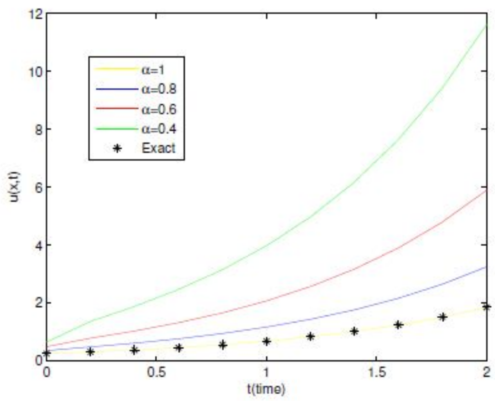

Fig. 4. 2D Graphical representation of solution (4.13) **of IVP** (4.8)-(4.9) **for different values of** α **such as** $\alpha = 1, 0.8, 0.6, 0.4$ **and exact when** $x = 0.25$.

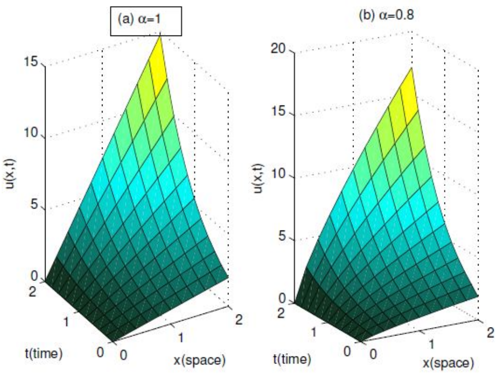

Fig. 5. 3D Graphical representation of solution (4.13) **of IVP** (4.8)-(4.9) **when** $\alpha = 1, 0.8$ **with respect to time**

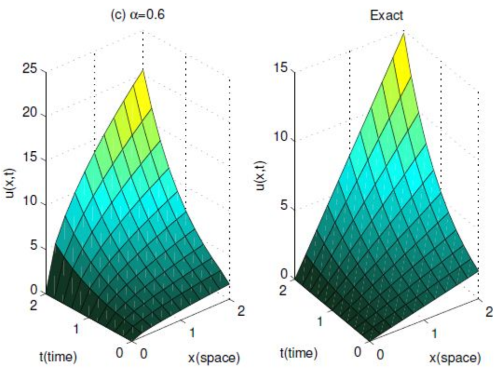

Fig. 6. 3D Graphical representation of solution (4.13) **of IVP** (4.8)-(4.9) **when** $\alpha = 0.6$ **and exact solution** (4.14) **with respect to time**

6 Conclusions

Equations arises in the nonlinear space and time fractional gas dynamic model and heat conduction model are studied successfully by virtue of ADM. The Caputo definition of fractional derivative is used to express fractional-order derivative. The solution of these models are in series form may have rapid convergence to a closed-form solution. One dimensional graphical demonstrations make sure the high accuracy of the generated results using ADM. It is a more appropriate way to solve such types physical models with the help of ADM.

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Competing Interests

Authors have declared that no competing interests exist.

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