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Realization and Implementation of Polynomial Chaotic Sun System

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Authors' contributions

The authors of this report have worked together on every aspect during the research.

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ABSTRACT

In this study, the circuit realization and its corresponding implementation by means of analog components is presented for a new chaotic system. In particular, the polynomial chaotic Sun system, which has 12 terms, twelve parameters and six nonlinearities, is considered. A relation for converting the chaotic ODE system parameters into circuit parameters is provided. The circuit realization of such system is simulated by PSPice-A/D. Next, the circuit is implemented by means of analog electronic components such as operational amplifiers and multipliers. The signals measured from experiments agreed with numerical simulations.

Keywords: Chaotic systems; simulation; circuit model; analog circuit implementation.

1. INTRODUCTION

Chaos is a phenomenon that could be modeled with nonlinear systems of equations [1]. A nonlinear, aperiodic and continuous-time system is said to be chaotic if it exhibits sensitive dependence on initial conditions; this behavior makes it practically impossible to predict a future state of the system given that we cannot with pinpoint accuracy ascertain the initial states [2].

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A positive Lyapunov exponent is also an indication that a nonlinear continuous system is chaotic. Resurgence of interests in chaotic systems started after an MIT professor, E. Lorenz, in 1963, applied it to weather forecasting [3]. With chaos stabilization and synchronization [4], engineering applications have soared. More recently, chaotic systems have been used in private communications [5,6]. Chaotic systems have also found applications in many other areas such as ecology [7,8], robotics [9], lasers [10,11], neural networks [12,13], chemical reaction [14], cryptosystem [15], finance and economy [16,17], medicine and biology [18,19], and so on. Polynomial chaotic Sun system [20] is one example of very many such systems. The present system on discourse can find applications in chaotic scenarios where many quadratic interactions including self-interactions are present.

Several electronic circuits with chaotic responses have been designed and realized [21,22, 23,24,5], however the polynomial chaotic Sun system is of interest for a couple of reasons including the fact that it has twelve parameters and six nonlinearities. These many parameters and nonlinearities compared to contemporary chaotic systems imply the system holds potential to accommodate real or more complex quadratic interactions. For instance, in applying chaos to communication, more parameter degrees of freedom can be exploited as extra layers of security although that would fundamentally require more resources; more so, in economics or in chemical reactions for instance, many nonlinearities would allow for representing more interactions amongst competing variables. Hence, the polynomial chaotic Sun system deserves investigation at different levels.

Engineering applications of chaotic systems often involve design and hardware implementation, hence the motivation for the present study. In this report, we present an electronic circuit model and implementation of the novel polynomial chaotic Sun system. To the best of our knowledge, circuit realization and or implementation of this system has never been reported.

2. METHODOLOGY

The polynomial chaotic Sun system is stated as:

$$\begin{aligned}\dot{x}_1 &= p_1x_1 + p_2x_2 + p_3x_2x_3 + p_4{x_2}^2\\ \dot{x}_2 &= p_5x_1 + p_6x_2 + p_7x_3 + p_8x_1x_2 + p_9{x_2}^2\\ \dot{x}_3 &= p_{10}x_3 + p_{11}x_1x_2 + p_{12}{x_2}^2,\end{aligned} \quad (1)$$

where the parameter values are $p_1 = -2, p_2 = 10, p_3 = -1, p_4 = 2, p_5 = 18, p_6 = -8, p_7 = 8, p_8 = -1, p_9 = -2, p_{10} = -2, p_{11} = 2, p_{12} = 1$. x_k $(k = 1,2,3)$, are time dependent state variables. A dot on a state variable implies derivative with respect to time, for example $\dot{x}_k = {dx_k}/{dt}$.

The amplitudes and scales of the state variables differ markedly and are also too large for a DC supply that integrated circuits (IC) can handle [22]; to take care of these issues, we make the transformations given in eq. (2). The transformation was necessary because *PSPice-A/D* simulation would not give a response that agreed with *Matlab* for a voltage supply within safe range for electronic components like the integrated circuits. Also, running the *Matlab max* and *min* functions on the numerical data stored in x_1 , x_2 and x_3 reveals the wide differences between these variables.

$$x_1 = 6x;\ \ x_2 = 4y;\ \ x_3 = 11z\ . \quad (2)$$

Now, we obtain a more physically viable polynomial chaotic Sun system which reads:

$$\begin{aligned}\dot{x} &= p_1x + \frac{2}{3}p_2y + \frac{22}{3}p_3yz + \frac{8}{3}p_4y^2\\ \dot{y} &= \frac{3}{2}p_5x + p_6y + \frac{11}{4}p_7z + 6p_8xy + 4p_9y^2\\ \dot{z} &= p_{10}z + \frac{24}{11}p_{11}xy + \frac{16}{11}p_{12}y^2\ .\end{aligned} \quad (3)$$

Numerical data from the solutions of eq. (3) is plotted in Fig. 1 and the dynamics agree with the chaotic Sun system as originally reported in Ref. [20].

Now, let v_x , v_y and v_z be physical signals (e.g. electric voltages) corresponding to the mathematical objects x, y and z respectively. Then the electronic circuit model corresponding to eq. (3) is given as follows.

$$\begin{aligned}\dot{v}_x &= -\frac{1}{R_1C_1}v_x + \frac{1}{R_2C_1}\frac{r_1}{r_2}v_y - \frac{1}{R_3C_1}v_yv_z + \frac{1}{R_4C_1}{v_y}^2\\ \dot{v}_y &= \frac{1}{R_5C_2}\frac{r_3}{r_4}v_x - \frac{1}{R_6C_2}v_y + \frac{1}{R_7C_2}\frac{r_5}{r_6}v_z - \frac{1}{R_8C_2}v_xv_y - \frac{1}{R_9C_2}{v_y}^2\\ \dot{v}_z &= -\frac{1}{R_{10}C_3}v_z + \frac{1}{R_{11}C_3}v_xv_y + \frac{1}{R_{12}C_3}{v_y}^2\ ,\end{aligned} \quad (4)$$

where R_i, $(i = 1,2,\dots,12)$ and r_j, $(j = 1,2,\dots 6)$ are electronic resistances to be computed and

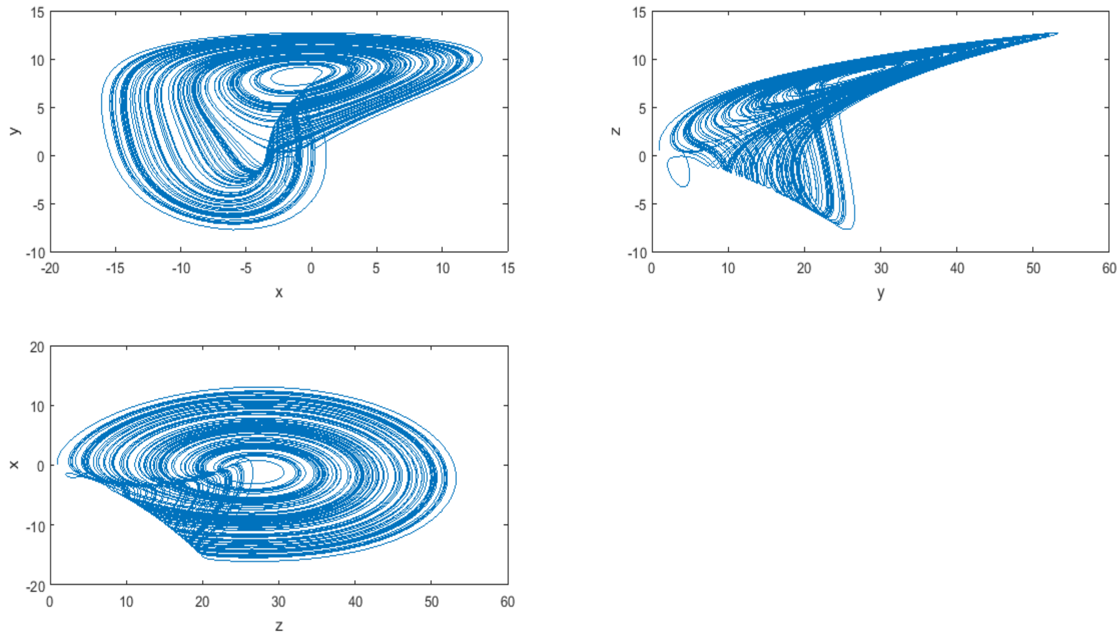

Fig. 1. Phase portraits from numerical solutions of the scaled polynomial chaotic Sun system. *Matlab ode45* with automatic time step and initial conditions $x(0) = 0.1, y(0) = 0.5, z(0) = 1.0$ were used

selected respectively and C_k , $(k = 1,2,3)$ are capacitances of capacitors. A circuit realization for the model is displayed in Fig. 2 and choosing a scale of 2500, the formula given in eq. (5) provides the relationship between parameters of the chaotic Sun system and the electronic circuit parameters. Indeed the formula applies to any such system of ordinary differential equations for which one wishes to convert the equation parameters to electronic parameters for any desired scale.

$$R_i = \frac{1/C_k}{s \times |p_i| \times 2500} \times \frac{1}{10^{n-1}}, \qquad (5)$$

where for a reasonably chosen circuit capacitance, $C_k (= 1nF$ in our case), R_i is a circuit resistance corresponding to system parameter p_i, s is a scalar from the transformation and n is the (polynomial) power of the i^{th} term; for example, looking at eq. (1) and eq. (3), it is clear that for $p_4 = 2$, and $s = 8/3$, the power of the corresponding variable y , is $n = 2$ and the corresponding circuit resistance R_4 is thus calculated as:

$$R_4 = \frac{{}^{1}/_{10^{-9}}}{8/3 \times 2 \times 2500} \times \frac{1}{10} = 7.5k\,;$$

other R_i are computed in a similar fashion using eq. (5). Each r_i is chosen to be $100k\Omega$ as indicated in the circuit schematics; they only serve to provide the appropriate gains after signal inversion[1].

3. SCHEMATIC MODEL SIMULATION

We have designed *PSPice* top-level part as shown in Fig. 3; each contains the circuit design of Fig. 2 except that terminals for the capacitors are mapped out to the top-level; initial stored voltages of the capacitors of a part were set at $(0.01,0.00,0.01)\,volts$ and in the other part were set at $(0.02,0.01,0.02)\,volts$. The simulation results are given in Fig. 4 (*Middle-Right, Bottom-Left, Bottom-Right*) and they show sensitive dependence on initial stored voltages. Also in Fig. 4 are phase portraits from *PSPice-A/D* simulations, which are in concordance with phase portraits from *Matlab* solutions as shown in Fig. 1.

We present in the following section hardware implementation of the circuit realization.

[1] *Netlist by PSpice-A/D makes no distinction between R_i and r_i. We have adopted the labels on the schematics after simulation for convenience because r_i are not derivable from eq. (5) but chosen to give the appropriate gain.*

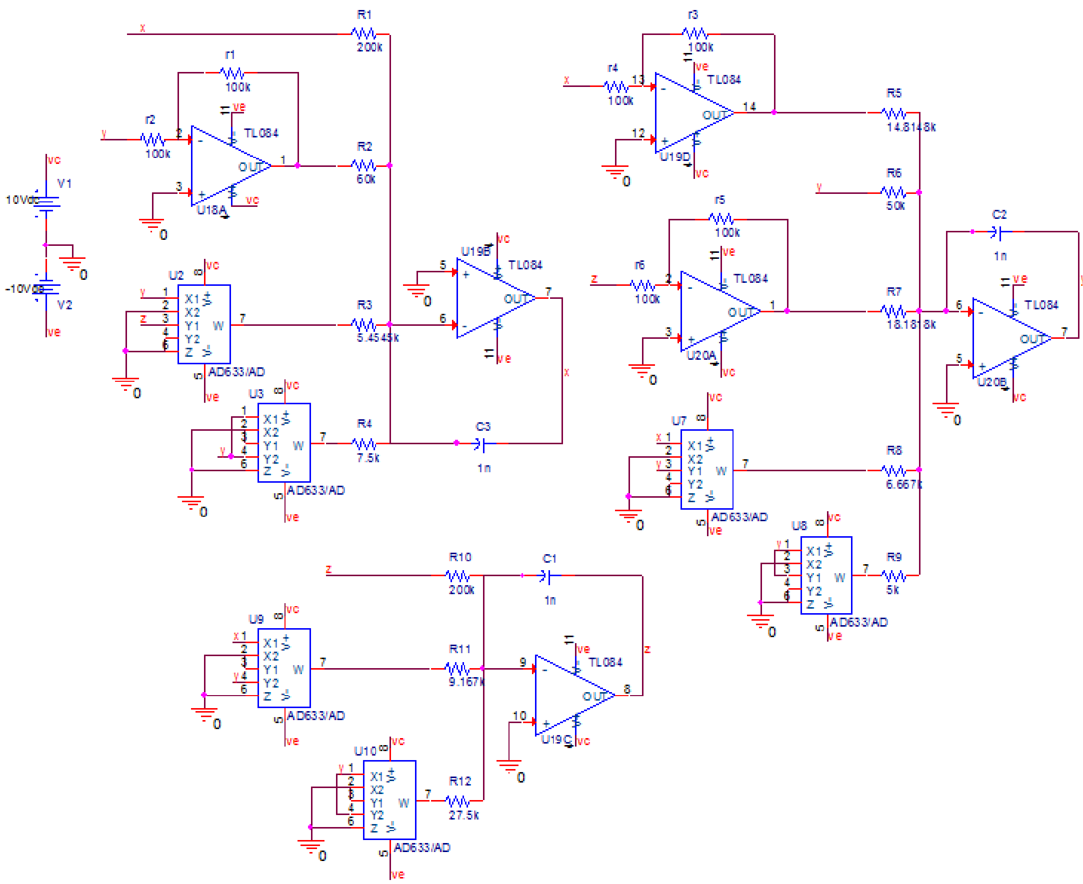

Fig. 2. Circuit realization of the polynomial chaotic Sun system. *OrCAD-Capture* was used to layout the schematic

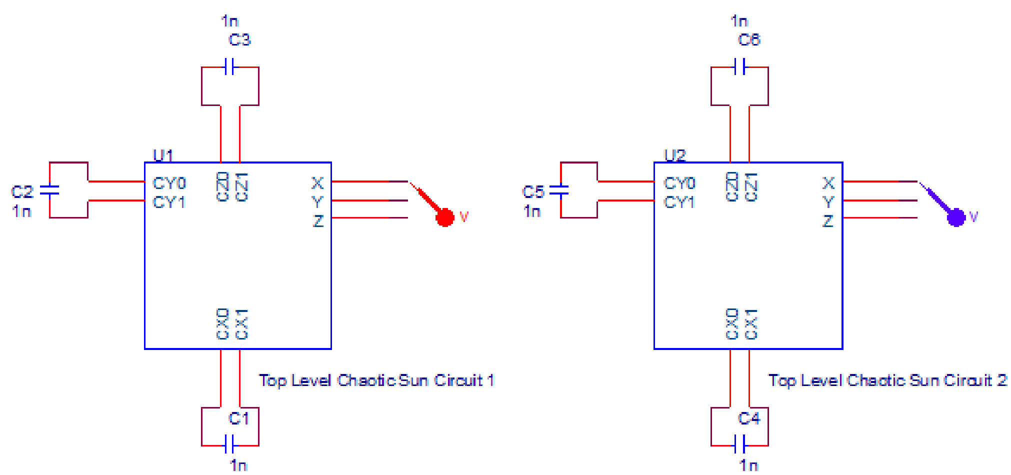

Fig. 3. Each of the top-level parts contain the chaotic Sun system as given in Fig. 2, with the capacitors ported to the outside. The initial stored voltage in each can be varied slightly in order to observe for the circuit, *sensitive dependence*

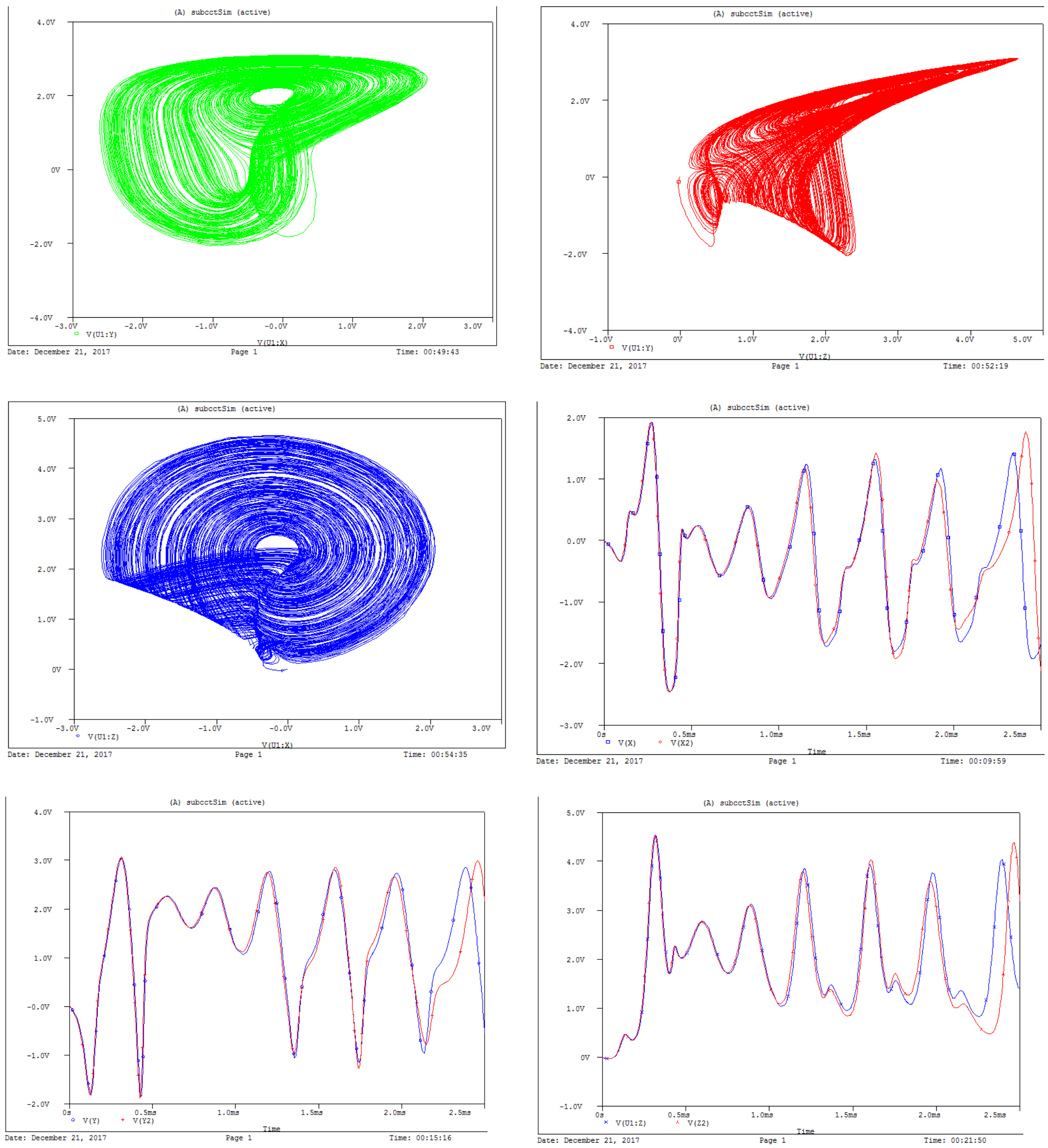

Fig. 4. *PSPice-A/D* simulation results. *Top-Left*: yx-portrait. Top-Right: yz-portrait. *Middle-Left*: zx–portrait. *Middle-Right*: Time series plots of x & x2, *Bottom-Left::* Time series plots of y & y2, *Bottom-Right:* Time series plots of z & z2, with initial stored voltages of capacitors at (0.01, 0.00, 0.01) volts and (0.02, 0.01, 0.01) volts for the two chaotic systems shown in Fig. 3

4. IMPLEMENTATION

The chaotic Sun system was realized in electronic circuit using AD633JN for multiplication, TL084CN, containing four operational amplifiers (op-amps), for signal inversion, multi-turn trimpots were used to meticulously tune the circuit parameters and three capacitors each of $1nF$ together with op-amps implemented the mathematical integration and the outputs on the oscilloscope are displayed in Fig. 5.

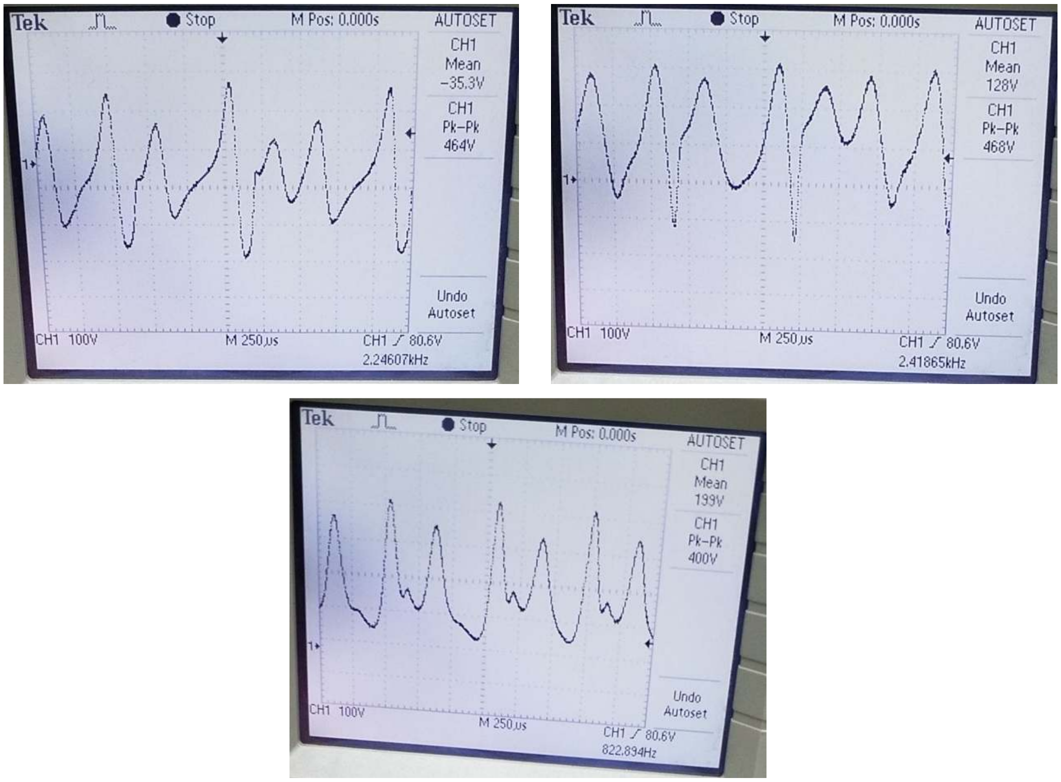

Fig. 5. Circuit implementation outputs as seen on the oscilloscope. *Top-Left*: v_x signal. *Top-Right*: v_y signal. *Bottom*: v_z signal

5. CONCLUSION

The output signals of Fig. 4 agree completely with numerical results from the abstract mathematical model plotted in Fig. 1 and with Ref. [20] Also, the electronic circuit implementation gives results that are consistent with the schematic model simulations shown in Fig. 4. Hence the circuit realization effectively represents the polynomial chaotic Sun system. Also, the circuit operation is near room temperature. Furthermore, the circuit implementation of the polynomial chaotic Sun system can find applications in situations where many variables interact in pairs and with themselves; more parameters compared to contemporary chaotic systems give more access points to experiment with the system's properties. Finally, in a forthcoming paper, we are investigating an FPGA [2] realization of the analog circuit presented in this report.

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[2] *FPGA – Field Programmable Gate Arrays.*

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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