Asian Research Journal of Mathematics

6(1): 1-12, 2017; Article no.ARJOM.35737

ISSN: 2456-477X

The Impact of Consumption on an Investor's Strategy under Stochastic Interest Rate and Correlating Brownian Motions

Silas A. Ihedioha[1*], Ubani, Sunday Iheanyi[2] and Njoku, Iheanyi Odochi[2]

[1]Department of Mathematics, Plateau State University Bokkos, P.M.B. 2012, Jos, Plateau State, Nigeria.
[2]Federal College of Agriculture, Ishiagu, Ebonyi State, Nigeria.

Authors' contributions

This work was carried out in collaboration between all authors. Authors SAI designed, analyzed, interpreted and prepared the first draft of the manuscript. Authors USI and NIO managed the literature searches and analyses and prepared the final draft of the manuscript. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2017/35737
Editor(s):
(1) Radoslaw Jedynak, Computer Science and Mathematics, Kazimierz Pulaski University of Technology and Humanities, Poland.
Reviewers:
(1) Carlos Humberto Martins, State University of Maringa, Brazil.
(2) Hussin Jose Hejase, Al Maaref University, Lebanon.
(3) Victor Gumbo, University of Botswana, Botswana.
Complete Peer review History: http://www.sciencedomain.org/review-history/20674

Received: 27th July 2017
Accepted: 25th August 2017
Published: 26th August 2017

Original Research Article

Abstract

In this work, we consider that an investor's portfolio comprises of two assets- a risk-free asset driven by Ornstein-Uhlenbeck Stochastic interest rate of return model and the second asset a risky stock with a price process governed by the geometric Brownian motion. It is also considered that there are withdrawals for consumption and taxes, transaction costs and dividends are in involved. The aim was to investigate the effect of consumption on an investor's trading strategy under correlating Brownian motions. The relating Hamilton-Jacobi-Bellman (HJB) equation was obtained using maximum principle. The application of elimination of variable dependency gave the optimal investment strategy for the investor's problem. Among the findings is that more fund should be made available for investment on the risky asset when there is consumption to keep the investor solvent.

Keywords: Consumption; Hamilton-Jacobi-Bellman (HJB) equation; optimal investment; Ornstein-Uhlenbeck; stochastic; interest rate.

MSC 2010: 62P05, 65C 30.

**Corresponding author: E-mail: silasihedioha@yahoo.com;*

1 Introduction

In the field of mathematical finance asset allocation problems in continuous time framework are among the most widely studied problems, and dates back to Merton [1,2]. In the Merton's original work provided explicit solutions on how one's expected utility is maximized while trading on stocks and consumption taking place as the underlying assets follow the Black–Scholes–Merton model with specific utility preference. After these pioneer works, many researches have been done and more are going on in many facets of Mathematical Finance. Among them, some allow for imperfections in the financial markets, Magill and Constantinides [3]. In the case of transaction costs, Guasoni and Muhle-Karbe [4], have made contributions. For investment under drawdown constraint, contributors include, Elie and Touzi [5]. In the case of trading with price impact we have, Cuoco and Cvitani´c [6] etc.

In the area of the volatility being stochastic, contributors include Zariphopoulou [7], Chacko and Viceira [8], Fouque et al. [9] and Lorig and Sircar [10].

Empirical studies have shown that non-Markovian (dependence) structure models in long-term investment which is much related to daily data and long range dependence exhibits in both return and volatility describe the data better, (Cont [11], Chronopoulou and Viens [12]).

The introduction of transaction costs into the investment and consumption problems follow from the works of Shreve and Soner [13], Akian et al. [14], and Janeˇcek and Shreve [15]. Investigators into optimal consumption problem with borrowing constraints include, Fleming and Zariphopoulou [16], Vila and Zariphopoulou [17], Ihedioha [18] and Yao and Zhang [19].

The mentioned models were studies under the assumption that the risky asset's price dynamics was driven by the geometric Brownian motion (GBM) and the risk-free asset with a rate of return that is assumed constant. Some authors have studied the problem under the extension of geometric Brownian motion (GBM) called the constant elasticity of variance (CEV) model. The constant elasticity of variance (CEV) model has an advantage that the volatility rate has correlation with the risky asset price. Cox and Ross [20] originally proposed the use of constant elasticity of variance (CEV) model as an alternative diffusion process for pricing European option; Cox and Ross [20]. Schroder [21], Lo et al. [22], Phelim and Yisong [23], and Davydov and Linetsky [24] have applied it to analyze the option pricing formula. Further applications of the constant elasticity of variance (CEV) model, in the recent years, has been in the areas of annuity contracts and the optimal investment strategies in the utility framework using dynamic programming principle.

Detailed discussions can be found in, Xiao et al. [25], Gao [26,27], Gu et al. [28], Lin and Li [29], Gu et al. [30], Jung and Kim [31] and Zhao and Rong [32].

The cases of portfolio maximization when the price of the risk-free asset is driven by Ornstein-Uhlenbeck model and the risky asset by the geometric Brownian motion and the rate of return of the risk-free asset is constant and the risky asset governed by constant elasticity of variance have been investigated by Ihedioha ([32], [33]).

This paper aims at investigating and giving a closed form solution to the impact of consumption on an investor's investment strategy when the rate of return of the risky asset is governed by the geometric Brownian motion and the risk-free asset driven by the Ornstein-Uhlenbeck Stochastic interest rate of return model and under correlating Brownian motions. Dynamic programming principle, specifically, the maximum principle is applied to obtain the HJB equation for the value function.

The rest of this paper is organized as follows: In section 2 is the problem formulation and the model. In section 3, maximum principle is applied to obtain, the HJB equation, the optimal investment strategy and the impact of consumption investigated. Section 4 concludes the paper.

2 The Problem Formulation

Two cases are considered in the work, thus;

1. When there is no consumption
2. When there is consumption

Case 1: When there is no consumption

Adopting the formulation in Ihedioha [33], we assume that an investor trades two assets in an economy continuously-c riskless asset (bond) and a risky asset (stock), Let the price of the riskless asset be denoted by $P(t)$ with a rate of return$r(t)$ which is stochastic and driven by the Orinstein-Uhlenbeck model. That is

$$dP(t) = r(t)P(t)dt \tag{1}$$

where

$$dr(t) = \alpha\big(\beta - r(t)\big)dt + \sigma dz_1(t) \colon r(0) = r_0 \tag{2}$$

where α is the speed of mean reversion, β the mean level attracting the interest rate and σ the constant volatility of the interest rate. $Z_1(t)$ is a standard Brownian motion. Also, let the price of the risky asset be denoted by$P_1(t)$ with the process

$$dP_1(t) = P_1(t)[\mu dt + \lambda dZ_2(t)], \tag{3}$$

where μ and λ are constants and μ the drift parameter while λ is the diffusion parameter.$z_2(t)$ is another standard Brownian motion.

Through this work, we assume a probability space $(\Omega, \mathcal{F}, \rho)$ and a filtration $\{\mathcal{F}_t\}$. Uncertainty in the models are generated by the Brownian motions $Z_1(t)$and $Z_2(t)$.

Let $\pi(t)$ to the amount of money the investor decides to put in the risky asset at time t,then the balance $[X(t) - \pi(t)]$ is the amount to be invested in the riskless assets, where$w(t)$ is the total amount of money available for investment.

Assumption:

We assume that transaction cost, tax and dividend are paid on the amount invested in the risky asset at constant rates, σ, θ and d respectively. Therefore for any policy π, the total wealth process of the investor follows the stochastic differential equation (SDE)

$$dX^{\pi}(t) = \pi(t)\frac{dP_1(t)}{P_1(t)} + [X(t) - \pi(t)]\frac{dP(t)}{P(t)} - (\vartheta + \theta - d)\pi(t)dt. \tag{4}$$

Applying (1) and (3) in (4) gives

$$dX^{\pi}(t) = \{[(\mu + d) - (r(t) + \vartheta + \theta)]\pi(t) + r(t)X(t)\}dt + \lambda\pi(t)dZ_2(t). \tag{5}$$

Suppose the investor has a utility function $U(.)$ which is strictly concave and continuously differentiable on $(-\infty, +\infty)$ and wishes to maximize his expected utility of terminal wealth, then his problem can therefore be written as

$$\underset{\pi}{^{Max}}\epsilon[U(X(T))] \tag{6}$$

subject to (5).

This work assumes a probability space $(\Omega, \mathcal{F}, \mathrm{P})$ and a filtration $\{\mathcal{F}_t\}_{t\geq 0}$, and uncertainties in the models are generated by the Brownian motions $Z_1(t)$ and $Z_2(t)$.

Case 2: When there is consumption

Also, adopting Ihedioha [34], further assumptions is that consumption withdrawals are made from the risk-free account, therefore for any trading strategy $(\pi(t), K(t))$ the total wealth process of the investor follows the stochastic differential equation (SDE)

$$dX(t) = \pi(t)\frac{dP_1(t)}{P_1(t)} + [X(t) - \pi(t)]\frac{dP(t)}{P(t)} - [(\vartheta + \theta - d)\pi(t) + K(t)]dt, \tag{7}$$

where $K(t)$ is the rate of consumption.

Applying (2) and (3) in (4) obtains:

$$dX(t) = \pi(t)[\mu dt + \lambda dZ_2(t)] + [X(t) - \pi(t)]r(t)dt - [(\vartheta + \theta - d)\pi(t) + K(t)]dt. \tag{8}$$

which becomes

$$dX(t) = \{[(\mu + d) - (r + \vartheta + \theta)]\pi(t) + r(t)X(t) - K(t)\}dt + \lambda\pi(t)dZ_2(t). \tag{9}$$

Definition: (admissible strategy). An investment and consumption $(\pi(t), K(t))$ strategy is said to be admissible if the following conditions are satisfied:

i. $(\pi(t), k(t))$ is $\mathcal{F}_t$ −progressively measurable and

ii. $\int_0^T \pi^2(t)dt < \infty, \int_0^T k(t)dt < \infty \;;\; \forall\, T > 0$ (10)

iii. $E\left[\int_0^T (\lambda^2\pi^2(t))dt\right] < \infty$ (11)

iv. For $\forall\ (\pi(t), k(t))$,the stochastic differential equation (9) has a unique solution, Chang et al. [35].

Assuming the set of all admissible investment and consumption strategies $(\pi(t), k(t))$ is denoted by $B = \left[\left(\pi(t), k(t)\right): 0 \leq t \leq T\right]$,then the investor`s problem can be stated mathematically as:

$$\mathrm{Max}_{[\pi(t),k(t)]\in B}\, E[(U(X(T))]. \tag{12}$$

This study considers the power utility function given by

$$U\big(X(t)\big) = \frac{X^{1-\phi}}{1-\phi}; \phi \neq 1. \tag{13}$$

Using the classical tools of stochastic optimal control where consumption is involved, define the value function at time t as:

$$G(t, r(t), P_1(t), X(t)) = \sup_B E\left[\int_0^T e^{-\varrho\tau}\frac{K^{1-\phi}}{1-\phi}d\tau + e^{-\varrho T}\frac{{X_T}^{1-\phi}}{1-\phi}\right];$$

$$P_1(t) = p_1; X(t) = x; r(t) = r, K(t) = k; 0 < t < T \tag{14}$$

Therefore the investor`s problem becomes

$$G(t,r,p_1,x) = \sup_{[\pi(t),k(t)] \in B} E\left[\int_0^T e^{-\varrho\tau}\frac{k^{1-\phi}}{1-\phi}d\tau + e^{-\varrho T}\frac{x^{1-\phi}}{1-\phi}\right] \tag{15}$$

subject to (9).

3 The Optimal Investment Strategy for the Power Utility Function

Here we obtain the explicit strategies for the optimization problem using the maximum principle and stochastic control.

3.1 When there is no consumption

Define the value function as

$$G(t,r,p_1,x) = {}^{Max}_{\pi}[\epsilon(U(w)] = 0; U(T,W) = U(w), 0 < t < T$$

$$r(t) = r, X(t) = x, P_1(t) = p_1, \tag{16}$$

then the Hamilton-Jacobi-Bellman equation (HJB) is

$$G_t + \alpha(\beta - r)G_r + \mu p_1 G_{p_1} + \{[(\mu + d) - (r + \vartheta + \theta)]\pi + rx\}G_x + \lambda^2 p_1 \pi G_{p_1 x} + \rho\sigma p_1 G_{rp_1} + \rho\sigma\lambda\pi G_{rx} + \frac{1}{2}\left[\sigma^2 G_{rr} + \lambda^2 {p_1}^2 G_{p_1 p_1} + \lambda^2\pi^2 G_{xx}\right] = 0 \tag{17}$$

where the Brownian motions have correlation coefficient ρ.

$G_t, G_{p_1}, G_x \: and \: G_r$, are first partial derivatives with respect to $t, s, wandr$ respectively. Also $G_{rp_1}, G_{rx}, G_{p_1x}, G_{rr}, G_{p_1p_1} \: and \: G_{xx}$ are second partial derivatives.

Differentiating (17) with respect to π gives

$$[(\mu + d) - (r + \vartheta + \theta)]G_x + \lambda^2 G_{p_1 x} + \rho\sigma\lambda G_{rx} + \lambda^2\pi G_{xx} = 0, \tag{18}$$

and the optimal strategy

$$\pi^*_{d,\vartheta,\theta} = \frac{-[(\mu+d)-(r+\vartheta+\theta)]G_x}{\lambda^2 G_{xx}} - \frac{p_1 G_{p_1 x}}{G_{xx}} - \frac{\rho\sigma\lambda G_{rx}}{\lambda^2 G_{xx}}, \tag{19}$$

To eliminate the dependency on x, let the solution to the HJB equation (17) be

$$G(t,r,p_1,x) = H(t,r,p_1)\frac{x^{1-\phi}}{1-\phi}, \tag{20}$$

with boundary condition

$$H(T,r,p_1) = 1, \tag{21}$$

Then we obtain from (20)

$$G_x = x^{-\phi}H,\ G_{xx} = -\phi x^{-\phi-1}H,\ G_{p_1 x} = x^{-\phi}H_{p_1}, G_{rx} = x^{-\phi}H_r. \tag{22}$$

Applying the equivalent of $G_x, G_{xx}, G_{p_1x}, and\ G_{rx}$ from equation (19) and (22) gives

$$\pi^*_{d,\vartheta,\theta} = \frac{[(\mu+d)-(r+\vartheta+\theta)]x}{\lambda^2} + \frac{p_1xH_{p_1}}{\phi H} + \frac{\rho\sigma xH_r}{\lambda\phi H}. \tag{23}$$

To eliminate dependency on p_1, we further conjecture that

$$H(t,r,p_1) = \frac{{p_1}^{1-\phi}}{1-\phi} I(t,r), \tag{24}$$

where

$$I(T,r) = \frac{1-\phi}{{p_1}^{1-\phi}}. \tag{25}$$

We obtain from (24)

$$H_r = \frac{{p_1}^{1-\phi}}{1-\phi} I_r, H_{p_1} = {p_1}^{-\phi} I. \tag{26}$$

Using (24) and (26) in (23) gives

$$\pi^*_{d,\vartheta,\theta} = \left[\frac{[(\mu+d)-(r+\vartheta+\theta)]x}{\lambda^2} + \frac{(1-\phi)x}{\phi} + \frac{\rho\sigma xI_r}{\lambda\phi I}\right]. \tag{27}$$

We conjecture further that

$$I(t,r) = \frac{r^{1-\phi}}{1-\phi} J(t), \tag{28}$$

to eliminate dependency on r such that at the terminal time T,

$$J(T) = \frac{(1-\phi)^2}{(rp_1)^{1-\phi}}. \tag{29}$$

From (28) we obtain,

$$I_r = r^{-\phi} J. \tag{30}$$

Therefore equation (27) becomes

$$\pi^*_{d,\vartheta,\theta} = x\left[\frac{[(\mu+d)-(r+\vartheta+\theta)]}{\lambda^2} + \frac{1-\phi}{\phi} + \frac{(1-\phi)\rho\sigma}{\lambda\phi r}\right], \tag{31}$$

the optimal investment in the risky asset.

3.2 When there is consumption

The derivation of Hamilton-Jacobi-Bellman (HJB) partial differential starts with the Bellman;

$$G(t,r,p_1,x) = sup_\pi \left\{\frac{K^{1-\phi}}{1-\phi} + \frac{1}{1+\zeta} E[G(t+\Delta t, r', x')]\right\}. \tag{32}$$

The actual utility over time interval of length Δt is $\frac{C^{1-\phi}}{1-\phi}\Delta t$ and the discounting over such period is expressed as $\frac{1}{1+\zeta\Delta t}$, $\zeta > 0$.

Therefore, the Bellman equation becomes;

$$G(t,r,p_1,x) = sup_{\pi}\left\{\frac{K^{1-\phi}}{1-\phi}\Delta t + \frac{1}{1+\vartheta\Delta t}E[G(t+\Delta t, r', p_1', x')]\right\}. \tag{33}$$

The multiplication of (13) by $(1 + \zeta\Delta t)$ and rearranging terms obtains;

$$\vartheta G(t,r,p_1,x)\Delta t = sup_{\pi}\left\{\frac{K^{1-\phi}}{1-\phi}\Delta t\ (1+\zeta\Delta t) + E(\Delta G)\right\}. \tag{34}$$

Dividing (14) by Δt and taking limit to zero, obtains the Bellman equation;

$$\zeta G = sup_{\pi}\left\{\frac{K^{1-\phi}}{1-\phi} + \frac{1}{dt}E(dG)\right\}. \tag{35}$$

Applying the maximum principle obtains the corresponding Hamilton-Jacob-Bellman equation (HJB) as

$$\frac{k^{1-b}}{1-b} + G_t + \mu p_1 G_{p_1} + \alpha(\beta - r)G_r + \{[(\mu+d)-(r+\vartheta+\theta)]\pi + rx - k\}G_x + \rho\sigma x p_1 G_{rp_1} + \rho\sigma\lambda\pi G_{rx} + \lambda^2\pi p_1 x G_{p_1} + \frac{1}{2}\left[\sigma^2 G_{rr} + \lambda^2 {p_1}^2 G_{p_1p_1} + \lambda^2\pi^2 G_{xx}\right] - \zeta G = 0. \tag{36}$$

$G_t, G_{p_1} and\ G_x$ are first partial derivatives $G_{rp_1}, G_{rx}, G_{p_1x}, G_{rr}, G_{p_1p_1} and\ G_{xx}$ are second order partial derivatives.

Differentiating (36) with respect to π gives the optimal investment in the risky asset as;

$$\pi^* = \frac{-[(\mu+d)-(r+\vartheta+\theta)]G_x}{\lambda^2 G_{xx}} - \frac{\rho\sigma}{\lambda}\frac{G_{rx}}{G_{xx}} - \frac{p_1 G_{p_1}}{G_{xx}}. \tag{37}$$

To cope with this, it is conjectured that a solution of the form

$$G(t,r,p_1,x) = \frac{x^{1-\phi}}{1-\phi}J(t,r,p_1), \tag{38}$$

such that

$$J(T,r,p_1) = 1, \tag{39}$$

eliminates the dependency on x.

From (38) we obtain

$$G_x = x^{-b}J, G_{xx} = -\phi x^{-\phi-1}J, G_s = \frac{x^{1-\phi}}{1-\phi}J_{p_1}, G_{rx} = x^{-\phi}J_r. \tag{40}$$

Applying the equivalents of G_x, G_{rx} , G_{p_1x}, and G_{xx} from (40) to (37) yields

$$\pi^* = \frac{[(\mu+d)-(r+\vartheta+\theta)]x}{\phi\lambda^2} + \frac{\rho\sigma x J_r}{\phi\lambda J} + \frac{p_1 x J_{p_1}}{\phi\ J}. \tag{41}$$

To continue we conjecture that

$$J(t,r,p_1) = H(t,r)\frac{{p_1}^{1-\phi}}{1-\phi}, \tag{42}$$

such that

$$H(T,r) = \frac{1-b}{{p_1}^{1-b}}\,, \tag{43}$$

at the terminal time T, and dependency on p_1 eliminated.

Obtained from (42) are

$$J_r = \frac{{p_1}^{1-\phi}}{1-\phi} H_r; J_{p_1} = {p_1}^{-\phi} H. \tag{44}$$

The application of the equivalents of J_r and J_{p_1} from (44) and (42) to (41) gives

$$\pi^* = \frac{[(\mu+d)-(r+\vartheta+\theta)]x}{\phi\lambda^2} + \frac{1-\phi}{\phi} x + \frac{\rho\sigma x}{\phi\lambda}\frac{H_r}{H}, \tag{45}$$

as the optimal investment is the risky asset.

To eliminate the dependency on r, the conjecture that

$$H(t,r) = I(t)\frac{r^{1-\phi}}{1-b}, \tag{46}$$

is used such that

$$I(T) = \frac{(1-\phi)^2}{(rp_1)^{1-\phi}}, \tag{47}$$

at the terminal time T.

From (46) we obtain

$$H_r = r^{-\phi} I. \tag{48}$$

Applying the equivalent of H_r from (48) to (45) yields

$$\pi^* = \frac{[(\mu+d)-(r+\vartheta+\theta)]x}{\phi\lambda^2} + \frac{(1-\phi)x}{\phi} + \frac{(1-\phi)\rho\sigma x}{r\phi\lambda}$$

$$= \frac{x}{\phi}\left[\frac{[(\mu+d)-(r+\vartheta+\theta)]}{\lambda^2} + (1-\phi)\left(1+\frac{\rho\sigma}{r\lambda}\right)\right]. \tag{49}$$

3.3 The effect of the consumption

We shall assume that $\phi \neq 1$ and $\phi > 0$.

Let π^{*NC} and π^{*C} denote the optimal investment in the risky asset when there is no consumption and when there is consumption respectively. Therefore we have the following:

1. When there is no consumption; equation (31) gives;

$$\pi^{*NC}_{d,\vartheta,\theta} = x\left[\frac{[(\mu+d)-(r+\vartheta+\theta)]}{\lambda^2} + \frac{(1-\phi)}{\phi} + \frac{(1-\phi)\rho\sigma}{\lambda r\phi}\right]. \tag{50}$$

2. When there is consumption, equation (49) becomes,

$$\pi^{*C}_{d,\vartheta,\theta} = \frac{x}{\phi}\left[\frac{[(\mu+d)-(r+\vartheta+\theta)]}{\lambda^2} + (1-\phi)\left(1+\frac{\rho\sigma}{r\lambda}\right)\right]. \tag{51}$$

Taking ratio gives:

$$\frac{\pi^{*NC}_{d,\vartheta,\theta}}{\pi^{*C}_{d,\vartheta,\theta}} = \frac{\left[x\left[\frac{[(\mu+d)-(r+\vartheta+\theta)]}{\lambda^2}+\frac{1-\phi}{\phi}+\frac{(1-\phi)\rho\sigma}{\lambda r\phi}\right]\right.}{\left[\frac{x}{\phi}\left[\frac{[(\mu+d)-(r+\vartheta+\theta)]}{\lambda^2}+(1-\phi)\left(1+\frac{\rho\sigma}{r\lambda}\right)\right]\right]}. \tag{52}$$

Notice:

1. $$\lim_{\phi\to 1}\left[\frac{\pi^{*NC}_{d,\vartheta,\theta}}{\pi^{*C}_{d,\vartheta,\theta}}\right] = 1. \tag{53}$$

2. $$\lim_{\phi\to\infty}\left[\frac{\pi^{*NC}_{d,\vartheta,\theta}}{\pi^{*C}_{d,\vartheta,\theta}}\right] = 1 - \left[\frac{r[(\mu+d)-(r+\vartheta+\theta)]}{\lambda(\lambda r+\rho\sigma)}\right]. \tag{54}$$

Since the investor holds the risky asset as long as $[(\mu+d)-(r+\vartheta+\theta)] > 0$ and λ, r, ρ, σ are all positive constants, then

$$\left[\frac{r[(\mu+d)-(r+\vartheta+\theta)]}{\lambda(\lambda r+\rho\sigma)}\right] = k\,, \tag{55}$$

is positive, therefore,

$$\lim_{\phi\to\infty}\left[\frac{\pi^{*NC}_{d,\vartheta,\theta}}{\pi^{*C}_{d,\vartheta,\theta}}\right] = 1 - k. \tag{56}$$

This implies that the limit of the investment in risky asset when there is no consumption is less than that of when there is consumption. Put in another way, when there is consumption, more fund is required for investment in the risky asset to keep the investor solvent.

3.4 Findings

1. When there is no consumption:

Equation (31)

$$\pi^{*NC}_{d,\vartheta,\theta} = x\left[\frac{[(\mu+d)-(r+\vartheta+\theta)]}{\lambda^2}+\frac{1-\phi}{\phi}+\frac{(1-\phi)\rho\sigma}{\lambda\phi r}\right]$$

shows that the investment in the risky a fraction of the total amount available for investment which becomes dependent on x, ρ, σ, λ,r and ϕ whenever $[(\mu+d)-(r+\vartheta+\theta)] = 0$.

2. When there is consumption:

It can be seen from equation (49)

$$\pi^{*C}_{d,\vartheta,\theta} = \frac{x}{\phi}\left[\frac{[(\mu+d)-(r+\vartheta+\theta)]}{\lambda^2}+(1-\phi)\left(1+\frac{\rho\sigma}{r\lambda}\right)\right].$$

that the optimal investment is a ratio of the total amount available for investment and the relative risk aversion coefficient.

3. From the effect of consumption, more fund is required for investment on the risky asset when
There is consumption to keep the investor solvent.

4 Conclusions

This work investigated the effect of consumption on the investment strategy of an investor. It assumed that the price process of the risk less asset has a rate of return that is driven Ornstein-Uhlenbeck model. Using the maximum principle and conjectures on elimination of variables obtained the optimal investment strategy of investor who has power utility preference where taxes, transaction costs and dividend payments are charged and paid.

It was found that the investment in the risky a fraction of the total amount available for investment which becomes dependent on x, ρ, σ, λ,r and ϕ whenever $[(\mu + d) - (r + \vartheta + \theta)] = 0$, when there was no consumption, while when there was consumption, the optimal investment in the risky asset was a ratio of the total amount available for investment and the relative risk aversion coefficient. Also, consumption resulted that more fund is required for investment on the risky asset if the investor is to remain in business.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Merton RC. Lifetime portfolio selection under uncertainty: The continuous-time case. The Review of Economics and Statistics. 1969;51(3):247–257.

[2] Merton RC. Optimum consumption and portfolio rules in a continuous-time model. Journal of Economic Theory. 1971;3(4):373–413.

[3] Magill MJ, Constantinides GM. Portfolio selection with transactions costs. Journal of Economic Theory. 1976;13:245–263.

[4] Guasoni P, Muhle-Karb J. Portfolio choice with transaction costs: A user's guide. In Paris-Princeton Lectures on Mathematical Finance. Springer. 2013;169–201.

[5] Elie R, Touzi N. Optimal lifetime consumption and investment under a drawdown constraint. Finance and Stochastics. 2008;12:299–330.

[6] Cuoco D, Cvitani´c J. Optimal consumption choices for a large investor. Journal of Economic Dynamics and Control. 1998;22(3):401–436.

[7] Zariphopoulou T. Optimal investment and consumption models with non-linear stock dynamics. Mathematical Methods of Operations Research. 1999;50(2):271–296.

[8] Chacko G, Viceira LM. Dynamic consumption and portfolio choice with stochastic volatility in incomplete markets. Review of Financial Studies. 2005;18(4):1369–1402.

[9] Fouque JP, Sircar R, Zariphopoulou T. Portfolio optimization & stochastic volatility asymptotics. Mathematical Finance; 2015.

[10] Lorig M, Sircar R. Portfolio optimization under local-stochastic volatility: Coefficient Taylor series approximations and implied sharpe ratio. SIAM Journal on Financial Mathematics. 2016;7(1):418–447.

[11] Cont R. Long range dependence in financial markets. In Fractals in Engineering. Springer. 2005;159–179.

[12] Chronopoulou A, Viens FG. Estimation and pricing under long-memory stochastic volatility. Annals of Finance. 2012;8(2):379–403.

[13] Shreve SE, Soner HM. Optimal investment and consumption with transaction costs. The Annals of Applied Probability. 1994;4(3):609–692.

[14] Akian M, Menaldi JL, Sulem A. On an investment consumption model with transaction costs. SIAM Journal on Control and Optimization. 1996;34(1):329–364.

[15] Janeˇcek K, Shreve SE. Asymptotic analysis for optimal investment and consumption with transaction costs. Finance and Stochastics. 2004;8(2):181–206.

[16] Fleming WH, Zariphopoulou T. An optimal investment/consumption model with borrowing. Mathematics of Operations Research. 1991;16(4):802–822.

[17] Vila JL, Zariphopoulou T. Optimal consumption and portfolio choice with borrowing constraints. Journal of Economic Theory. 1997;77(2):402–431.

[18] Ihedioha SA. Investor's power utility optimization with consumption, tax, dividend and transaction cost under constant elasticity of variance model. Asian Research Journal of Mathematics. 2017;4(2): 1-12.

[19] Yao R, Zhang HH. Optimal consumption and portfolio choices with risky housing and borrowing constraints. The Review of Financial Studies. 2005;18(1):197–239.

[20] Cox JC, Ross SA. The valuation of options for alternative stochastic processes. Journal of Financial Economics. 1976;3(1-2):145–166.

[21] Schroder M. Computing the constant elasticity of variance option pricing formula. Journal of Finance. 1989;44(1):211–219.

[22] Lo CF, Yuen PH, Hui CH. Constant elasticity of variance option pricing model with time-dependent parameters. International Journal of Theoretical and Applied Finance. 2000;3(4):661–674.

[23] Phelim PB, Yisong ST. Pricing look back and barrier options under the CEV process. Journal of Financial and Quantitative Analysis. 1999;34(2):241–264.

[24] Davydov D, Linetsky V. Pricing and hedging path dependent options under the CEV process. Management Science. 2001;47(7):949–965.

[25] Xiao J, Hong Z, Qin C. The constant elasticity of variance (CEV) model and the Legendre transform-dual solution for annuity contracts. Insurance: Mathematics and Economics. 2007;40(2):302–310.

[26] Gao J. Optimal investment strategy for annuity contracts under the constant elasticity of variance (CEV) model. Insurance: Mathematics and Economics. 2009;45(1):9–18.

[27] Gao J. An extended CEV model and the Legendre transform dual-asymptotic solutions for annuity contracts. Insurance: Mathematics and Economics. 2010;46(3):511–530.

[28] Gu M, Yang Y, Li S, Zhang J. Constant elasticity of variance model for proportional reinsurance and investment strategies. Insurance: Mathematics and Economics. 2010;46(3):580–587.

[29] Lin X, Li Y. Optimal reinsurance and investment for a jump diffusion risk process under the CEV model. North American Actuarial Journal. 2011;15(3):417–431.

[30] Gu A, Guo X, Li Z, Zeng Y. Optimal control of excess of loss reinsurance and investment for insurers under a CEV model. Insurance: Mathematics and Economics. 2012;51(3):674–684.

[31] Jung EJ, Kim JH. Optimal investment strategies for the HARA utility under the constant elasticity of variance model. Insurance: Mathematics and Economics. 2012;51(1):667–673.

[32] Zhao H, Rong X. Portfolio selection problem with multiple risky assets under the constant elasticity of variance model. Insurance: Mathematics and Economics. 2012;50(1):179–190.

[33] Ihedioha SA. Optimal asset allocation problem for an investor with ornstein-uhlenbeck stochastic interest rate model. Asian Journal of Mathematics and Computer Research. 2017;19(1):33-41.

[34] Ihedioha SA. Investor's power utility optimization with consumption, tax, dividend and transaction cost under constant elasticity of variance model. Asian Research Journal of Mathematics. 2017;4(2): 1-12.

[35] Chang H, Rong X, Zhao H, Zhang C. Optimal investment and consumption decisions under constant elasticity of variance model. Hindawi Publishing Corporation, Mathematical Problems in Engineering. 2013;2013:1-11.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://sciencedomain.org/review-history/20674