



Ternary Mathematics and 3D Placement of Logical Elements Justification

Ruslan Pozinkevych^{1*}

¹Faculty of Informations Technologies and Mathematics, The Eastern European National University, 43021, Lutsk, Potapov str.9, Ukraine.

Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/AJRCOS/2021/V11i230257

Editor(s):

- (1) Dr. G. Sudheer, GVP College of Engineering for Women, India.
- (2) Dr. Hasibun Naher, BRAC University, Bangladesh.
- (3) Dr. Dariusz Jacek Jakóbczak, Koszalin University of Technology, Poland.

Reviewers:

- (1) G. Hannah Grace, Vellore Institute of Technology, India.
 - (2) Sunil Kumar Srivastava, Jaipur Engineering college and Research Centre, India.
- Complete Peer review History: <https://www.sdiarticle4.com/review-history/71745>

Original Research Article

Received 06 June 2021
Accepted 12 August 2021
Published 17 August 2021

ABSTRACT

Aims/ Objectives: The research presented in the following application aims to prove use of Ternary Maths for calculating machines and to simplify the process of calculating. In it we will try to justify the use of triplets and describe how it works.

An earlier research presented in "Logical Principles in Ternary Mathematics" [1,2,3] shows that we can transit from one expression of a number such as a "component form" to another, e.g a decimal, or still another, that is it's vector form [4]. The aim of our further research is to explain why we associate Triplets of numbers in such choice $\{-1,0,1\}$ and not the numbers 1,2,3 for example, or a set $\{1,2,3\}$. The explanation seems obvious as a set of decimal numbers consists of 10 entries not 3. At the same time we have to prove that the mentioned set of triplets is a unique and the only one to be used as a Ternary Set or a base, as we might call it, for our calculating machines.

Keywords: Base; Unique Representation; 3D placement; Logical Operators; Ternary Algebra.

1. INTRODUCTION

The idea why we even start to speak about Ternary Maths stems from the fact that any

numeric system can be reduced/expanded to a minimum/maximum numbers of entries [5].

The principle can be demonstrated by the

*Corresponding author: E-mail: galagut@protonmail.com;

following example Let's take a look at geometry. Suppose we have a quadrilateral that in its turn has 4 vertices If we add another 2 we must receive the out come of $6(4+2)=6$ Yet if we place the same vertices differently, we will obtain another quadrilateral again and so our addition principle will not work hence $4+2 \neq 6$ Another example from geometry Think of the trapezium If we add another vertex we might obtain a triangle and thus $4+1$ will be equal 3 The idea of Ternary Maths is quit similar we may reduce and extend the number of entries but of course their choice is not random Our research is aimed at establishing a connection between Ternary Principle and mathematical operations (addition, multiplication) for purposes of calculating [6,7].

This method will be used primarily by computing systems where logical elements are placed on xy, yz and zx planes Once the approach is justified the use of Ternary Maths will serve its purpose and make it a distinctive branch of Mathematics. So what gives us a right to speak about $(-1,0,1)$ as a uniques set? Well first of all we can choose eigenvectors

I would like to begin with the analysis of Ternary addition and take a look at the entries table.

We have managed to construct Truth Tables for (T, \bar{T}, F) entries. Here they are:

2. METHODOLOGY

Thus far the following results have been obtained; We have managed to construct Truth Tables for (T, \bar{T}, F) entries and here they are:

In this case the results are as follows: $(-1)+(-1)=(-1)$;

$$1+1=1$$

Its very important to mention it because in Ternary Mathematics this very principle plays the key role [8]

It's high time for us to present a definition of a ternary maths So what is actually a Ternary Maths?

A Ternary maths is a matrix presentation of a number whose orthonormal basis is a set of numbers $(-1,0,1)$

For the time being we are going to leave our "Ternary Multiplication" Table unchanged and concentrate on our "Ternary Addition"

Table 1. Ternary Addition

A	B	$A \oplus B$
T	T	T
F	F	F
\bar{T}	\bar{T}	\bar{T}
T	F	T
F	T	T
\bar{T}	T	F
\bar{T}	F	\bar{T}
F	\bar{T}	\bar{T}
T	\bar{T}	F

Table 2. Ternary Multiplication

A	B	$A * B$
T	T	T
F	F	F
\bar{T}	\bar{T}	T
T	F	F
F	T	F
\bar{T}	T	\bar{T}
\bar{T}	F	F
F	\bar{T}	F
T	\bar{T}	\bar{T}

**For the sake of propriety one should mention that Ternary addition works very much like addition of numbers $(-1,0,1)$ with an exception that, when we have negative one plus negative one it equals negative one Whereas positive one plus positive one equals positive one*

Let's group elements of the "Ternary Addition" results column into a $\begin{vmatrix} 1 \\ z & 1 \\ x & y & 1 \end{vmatrix}$ which is a proper orthogonal matrix the determinant of which is equal 1Its very important that $\Delta \neq 0$ and that $\Delta = 1$

Firstly because if matrix is orthogonal that means that the entries x,y,z are orthonormal and thus are linearly independent and do not lie on the plane [9] On the other hand we can always present this matrix in the form :

$$abc = \begin{vmatrix} a_1 \\ b_1 & b_2 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 b_2 c_3$$

Where :

$$a = a_1 e_1$$

$$b = b_1 e_1 + b_2 e_2$$

$$c = c_1 e_1 + c_2 e_2 + c_3 e_3$$

(e an eigenvector) [10,11].

This is a general approach towards 3 D placement of logical elements,the detailed explanation of which follows in our "Materials and Methods" discussion

3. MATERIALS AND METHODS

Our next step is to find the values of e (e_1, e_2, e_3) for which the evaluation will hold To be able to do this let's remind ourselves that our matrix

$$\begin{vmatrix} a_1 \\ b_1 & b_2 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

where entries $a_1 = b_2 = c_3 = 1$;

$$b_1 = c_1 = c_2 = (-1);$$

and

$$a_2 = a_3 = b_3 = 0$$

(see Table 1 Ternary Addition) is the last column of Ternary Addition entries rearranged into the 3X3 input

Even though we arranged our entries into the 3x3 matrix, our sought after matrix is not the

matrix of input but the matrix of coefficients, which is an inverse of an input matrix A

Let's call this matrix A^{-1}

It is also triangular yet has different entries altogether They are

$$a_1 = b_2 = c_3 = 1;$$

$$b_1 = 1; c_1 = 2; c_2 = 1;$$

In that case we obtain $a=b=c = 1$; and input entries of the orthonormal vectors:

$$\begin{aligned} e_1 &= 1; \\ e_2 &= 0; \\ e_3 &= (-1) \end{aligned}$$

This last part is probably the most important part in Ternary Mathematics principle We obtained 3 orthonormal vectors which we will write in the form of a column vector [12].

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

Now every matrix of coefficients can be written as γA^{-1}

$$\text{Where } \gamma = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

4. RESULTS AND DISCUSSION

Our goal was to present a unique unit [13] that would be an Eigenvalue for the Ternary Mathematics calculations This unit has been found It's a set of orthonormal entries (1,0-1) with which every matrix of coefficients can be presented in the form

$$\gamma A^{-1}$$

What has led to obtaining this result was a group of conversions, such as Gaussean eliminations; the fact that: $A^{-1} A = I$;

as well as algebra conversions and solving systems of linear equations

One of the steps we made on the way to solve simultaneous equations was to convert a matrix into it's inverse A^{-1}

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ x & 1 & 0 & 0 & 1 & 0 \\ y & z & 1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 - zR_2}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ x & 1 & 0 & 0 & 1 & 0 \\ y - xz & 0 & 1 & 0 & -z & 1 \end{bmatrix} \xrightarrow{\begin{matrix} R_3 - (y-xz)R_1 \\ R_2 - xR_1 \end{matrix}}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -x & 1 & 0 \\ 0 & 0 & 1 & -y + xz & -z & 1 \end{bmatrix}$$

The idea behind was to find an orthonormal basis e_1, e_2, e_3 and place these vectors in the formula to represent **abc**

By solving the system of three equations we found the values of e_1, e_2, e_3 first for the matrix A and then by analogy for the matrix A^{-1} . Of course they are different, therefore a special operator

has been chosen for the “unique” set $\gamma = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

We were finally ready to come up with the first postulate of Ternary Mathematics:

“A number or a single unit of counting in the Ternary system is presented as a product of a unique set γ and the matrix itself” [14].
(Pozinkevych Ruslan)

5. CONCLUSIONS

The main purpose of our research was to show that the system we use for calculations consists of a set of numbers $(-1,0,1)$ which is unique and corresponds to coordinates of orthonormal vectors. The chosen system allows to perform calculations in a matrix form which makes it easier to operate any numbers presenting them in the form of functions, which in their turn can be presented in the form of arguments and variables e.g. making it a system of linear equations.

Why do we choose to linearize the data? The reason is simple. Our system stands for a digital representation of a signal [15], such as for example, a sine with its upper, lower and medium levels corresponding to numbers -1 and 1 , whereas a medium entry corresponds to a 0 . We all know that the signal is coded and decoded in a digital form [16], thus Ternary approach to analyzing the signal data seems to be the most plausible. Besides, in a closed interval $[-1,1]$, 0 is the mean and median number of the set $(-1,0,1)$, which also indicates that a data set has a symmetrical distribution, making it ideal for discrete data analysis.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Duerlinger J. Sullogismos and Sullogizesqai in Aristotle's Organon. The American Journal of Philology. 1969;90(3):320-328.
2. Lukasiewicz J. Aristotle's syllogistic from the standpoint of modern formal logic; 1951.
3. Pozinkevych R. Logical Principles in Ternary Mathematics. Asian Journal of Research in Computer Science. 2021;49-54.
4. Dolciani MP. Modern introductory analysis. In Modern introductory analysis. 1967;660-660.
5. Hilbert D, Bernays P. Grundlagen der Mathematik II; 1974.
6. Russel B. Mysticism and logic and other essays. London, UK: George Allen & Unwin; 1956.
7. Wittgenstein L. Ludwig Wittgenstein. Rowman & Littlefield; 2003.
8. Pozinkevych R. Ternary Mathematics Principles Truth Tables and Logical Operators 3 D Placement of Logical Elements Extensions of Boolean Algebra. Asian Journal of Research in Computer Science. 2020;35-38.
9. Kravchuk OM. Workshop on analytical geometry; 2013.
10. Huss-Lederman S, Jacobson EM, Johnson JR, Tsao A, Turnbull T. Implementation of Strassen's algorithm for matrix multiplication. In Supercomputing'96: Proceedings of the 1996 ACM/IEEE Conference on Supercomputing. IEEE. 1996; 32-32.
11. Postnikov MM. Analytic geometry. Geometry lectures; 2009.
12. Macbeath AM. Elementary vector algebra. Oxford: Oxford University Press; 1964.
13. Cantor G. Contributions to the founding of the theory of transfinite numbers translated, and provided with an introduction and notes, by Philip EB Jourdain; 1952.
14. Cung VD, Danjean V, Dumas JG, Gautier T, Huard G, Raffin B, Trystram D. Adaptive

- and hybrid algorithms: classification and illustration on triangular system solving. In Transgressive Computing. Copias Coca, Madrid. 2006;131-148.
15. Harris D, Harris S. Digital design and computer architecture. Morgan Kaufmann; 2010.
 16. Li F, Nicopoulos C, Richardson T, Xie Y, Narayanan V, Kandemir M. Design and management of 3D chip multiprocessors using network-in-memory. In 33rd International Symposium on Computer Architecture (ISCA'06). IEEE. 2006;130-141.

© 2021 Pozinkevych; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:
The peer review history for this paper can be accessed here:
<https://www.sdiarticle4.com/review-history/71745>