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A Computational Method for the Solution of Nonlinear Burgers' Equation Arising in Longitudinal Dispersion Phenomena in Fluid Flow through Porous Media

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

This paper discusses the Modified Variational Iteration Method (MVIM) for the solution of nonlinear Burgers' equation arising in longitudinal dispersion phenomena in fluid flow through porous media. The method is an elegant combination of Taylor's series and the variational iteration method (VIM). Using Maple 18 for implementation, it is observed the procedure provides rapidly convergent approximation with less computational efforts. The result shows that the concentration $C(x,t)$ of the contaminated water decreases as distance x increases for the given time t.

Keywords: Modified variational iteration method; Burger's equation; porous media; partial differential equation.

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1 Introduction

Burgers' equation is the approximation for the one-dimensional propagation of weak shock waves in a fluid. It can also be used in the description of the variation in vehicle density in highway traffic.

The equation is one of the fundamental model equations in fluid mechanics which demonstrates the coupling between the dissipation effect of C_{xx} and convection process of CC_x. Burgers introduced the equation to describe the behavior of shock waves, traffic flow and acoustic transmission.

Many authors; [1-6] have worked on different methods to solve the Burgers' equation numerically. Wazwaz [1] studied Travelling wave solution of generalized forms of Burgers, Burgers-KdV and Burger's-Huxley equations. Patel and Mehta [2] applied Hope-Cole transformation to present solution of Burgers' equation for longitudinal dispersion of miscible fluid flow through porous media. Meher and Mehta [3] used Backlund Transformations to solve Burger's equation arising in longitudinal dispersion of miscible fluid flow through porous media and Joshi, et al. [4] used theoretic approach to find the solution of Burgers' equation for longitudinal dispersion phenomena occurring in miscible phase flow through porous media. Olayiwola et al. [5] also presented the modified variational iteration method for the numerical solution of generalized Burger's-Huxley equation. Recently, Kunjan and Twinkle [6] used mixture of new integral transform and Homotopy Perturbation Method to find the solution of Bugers' equation arising in the longitudinal dispersion phenomenon in fluid flow through porous media.

In this paper, a modified variational iteration method is presented to discuss the solution of the problem.

2 Modified Variational Iteration Method (MVIM)

The idea of variational iteration can be traced to Inokuti [7]. The variational iteration method was proposed by J. H He [8-9], In this paper, a Modified Variational Iteration Method proposed by Olayiwola [5,10-13] is presented for the solution of the Burgers' equation.

To illustrate the basic concept of the MVIM, we consider the following general nonlinear partial differential equation:

$$Lu(x,t)+Ru(x,t)+Nu(x,t)=g(x,t) \tag{1}$$

where L is a linear time derivative operator, R is a linear operator which has partial derivative with respect to x, N is a nonlinear operator and g is an inhomogeneous term. According to MVIM, we can construct a correction functional as follows:

$$u_0(x,t)=u(x,0)+g_1(x)t \tag{2}$$

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t \lambda\left[Lu_n+R\tilde{u}_n+N\tilde{u}_n-g\right]d\tau \tag{3}$$

where $g_1(x)$ can be evaluated by substituting $u_0(x,t)$ in (1) and evaluate at $t=0$.

λ is a Lagrange multiplier which can be identified optimally via Variational Iteration Method. The subscript n denote the nth approximation, $\tilde{u}_n$ is considered as a restricted variation i.e, $\delta\tilde{u}_n=0$.

3 Problem Formulation

In [14-17] and according to Darcy's law, the equation of continuity of fluid is given as:

$$\frac{\partial \rho}{\partial t} + \nabla * (\rho \bar{v}) = 0 \tag{4}$$

The equation of diffusion for a fluid flow through a homogeneous porous medium without decreasing or increasing the dispersing material is:

$$\frac{\partial C}{\partial t} + \bar{v}\left(\rho \bar{D} \nabla \left(\frac{C}{\rho}\right)\right) \tag{5}$$

In a lamina flow through homogeneous porous medium at a constant temperature, ρ is a constant. Then,

$$\nabla * \bar{v} = 0 \tag{6}$$

Therefore, equation (5) becomes:

$$\frac{\partial C}{\partial t} + \bar{v} \nabla C = \nabla * \left(\bar{D} \nabla C\right) \tag{7}$$

When the seepage velocity is along x-axis, then $D_L \approx \gamma, D_{i,j} = 0$

Hence, equation (7) becomes:

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = \gamma \frac{\partial^2 C}{\partial x^2} \tag{8}$$

As $x \geq 0, D_L > 0$

$$u = \frac{C(x,t)}{C_0} \tag{9}$$

Equation (8) then becomes:

$$\frac{\partial C}{\partial t} + C \frac{\partial C}{\partial x} = \gamma \frac{\partial^2 C}{\partial x^2} \tag{10}$$

This is the nonlinear Burgers equation for longitudinal dispersion of miscible fluid flows through porous media where:

C_0 = initial concentration of contaminant in liquid
C = concentration of contaminant in liquid phase
ρ = density of the mixure
$\bar{v}$ = pore seepage velocity vector
$\bar{D}$ = tensor coefficients of dispersion with component $D_{i,j}$
u = velocity component along x-axis
γ = coefficient of longitudinal dispersion

4 Solution of the Problem Using MVIM

In this section, the reliability of the MVIM is tested by applying it to find and discuss the behavior of solution of nonlinear Burgers equation for longitudinal dispersion phenomena in fluid flow through a porous media.

The initial and boundary condition for problem (10) is:

$$C(x,0) = e^{-x},\ 0 \le x \le 1,\ \ 0.001 \le t \le 0.01 \tag{11}$$

$$C(0,t) = 1 \tag{12}$$

The correction functional becomes:

$$C_{n+1}(x,t) = C_n(x,t) + \int_0^t \lambda \left[\frac{\partial C_n(x,\tau)}{\partial \tau} + C\frac{\partial C_n(x,\tau)}{\partial x} - \gamma \frac{\partial^2 C_n(x,\tau)}{\partial x^2} \right] d\tau \tag{13}$$

from equations (1-2)

$$C_0(x,t) = e^{-x} + \left(\gamma e^{-x} + e^{-2x}\right)t \tag{14}$$

When $n = 4$ Equations (13)-(14) gives:

$$\begin{aligned} C_5(x,t) &= e^{-x} + \left(\gamma e^{-x} + e^{-2x}\right)t + \left(\frac{1}{3}\gamma^2 e^{-x} + 3\gamma e^{-2x} + \frac{3}{2}e^{-3x}\right)t^2 + \\ &\left(\frac{1}{6}\gamma^3 e^{-x} + \frac{8}{3}e^{-4x} + \frac{14}{3}\gamma^2 e^{-2x} + \frac{17}{2}\gamma e^{-3x}\right)t^3 + \\ &\left(\frac{1}{24}\gamma^4 e^{-x} + \frac{125}{24}e^{-5x} + 5\gamma^3 e^{-2x} + \frac{101}{4}\gamma^2 e^{-3x} + \frac{71}{3}\gamma e^{-4x}\right)t^4 + O\left(t^5\right) \end{aligned} \tag{15}$$

Equation (15) represents the concentration of the longitudinal dispersion at any given distance x and time t. This solution is identical to solution obtained in [6] when $\gamma = 1$.

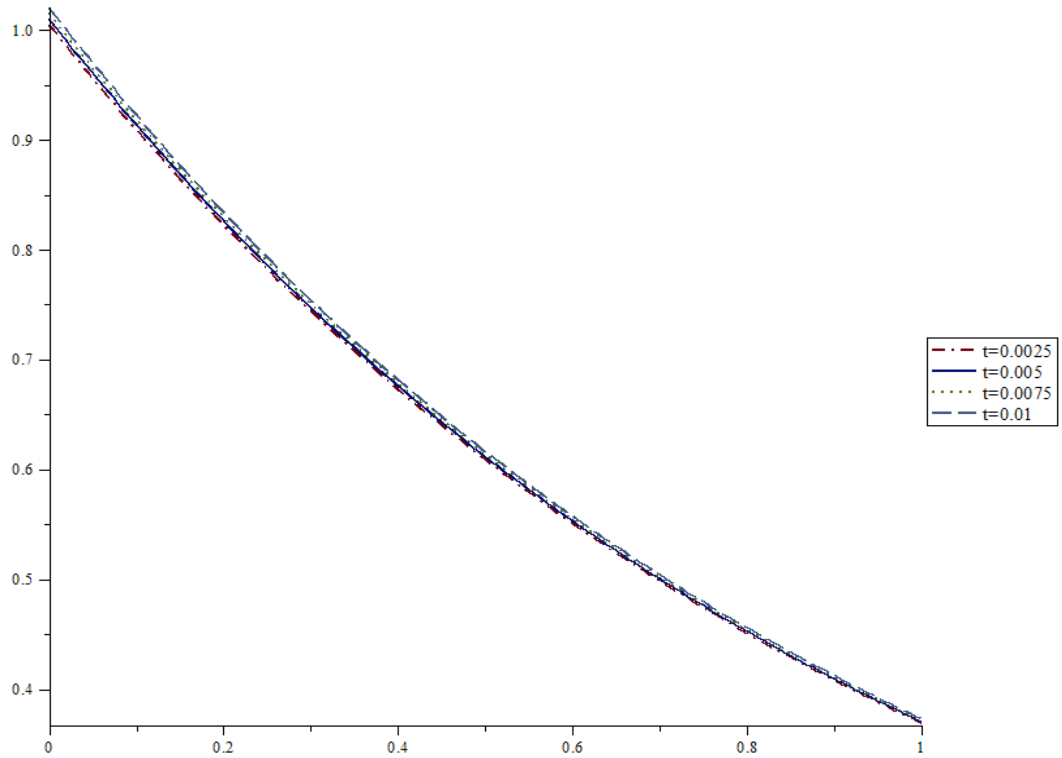

Fig. 1. Graph of $C(x,t)$ against x for various values of t

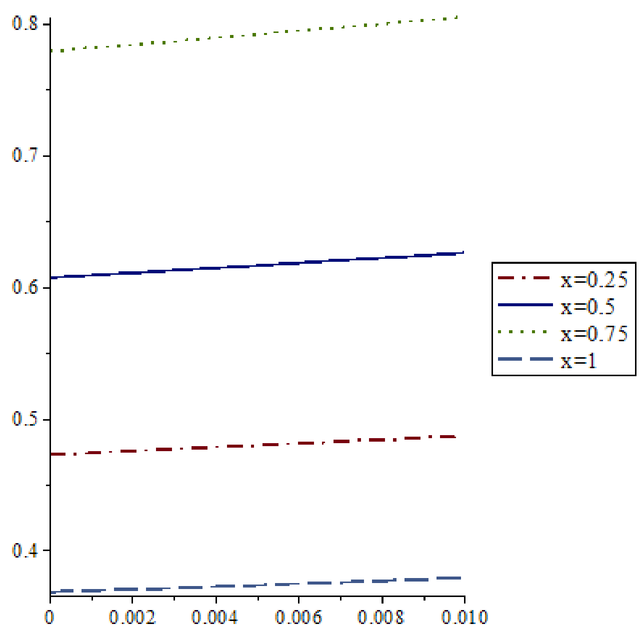

Fig. 2. Graph of $C(x,t)$ against t for various values of x

5 Conclusion

The graphs show that the numerical solution of concentration of a given dissolved substance in unsteady unidirectional seepage flows through semi-infinite, homogeneous, isotopic porous media subject to the source concentrations vary negatively exponentially with distance and slightly increase with time. This helps to predict the possible contamination of groundwater and it is in agreement with the physical phenomenon of longitudinal dispersion in miscible fluid thorough isotopic porous media subject to a defined initial and boundary conditions.

Disclaimer

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Competing Interests

Author has declared that no competing interests exist.

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