British Journal of Mathematics & Computer Science
14(5): 1-11, 2016, Article no.BJMCS.23900
ISSN: 2231-0851

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Optimization Models of Components of Educational Process

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/BJMCS/2016/23900
Editor(s):
(1) Dariusz Jacek Jakobczak, Chair of Computer Science and Management in this department, Technical University of Koszalin, Poland.
Reviewers:
(1) Rachana Pathak, University of Lucknow, Lucknow, India.
(2) Nityanand P. Pai, Manipal University, Manipal, India.
(3) Zaifang Zhang, Shanghai University, China.
(4) Zhi Shang, Louisiana State University, USA.
Complete Peer review History: http://sciencedomain.org/review-history/13346

Original Research Article

Received: 28th December 2015
Accepted: 5th February 2016
Published: 18th February 2016

Abstract

Research objects are components of educational process: a macrosystem of educational process for formation of competence of a pupil (of an educational group), systems representing mathematical models of competencys, ontologies, textbooks. Research objectives: to construct mathematical models of components of educational process, to investigate a question of stability of these systems and their possible classifications, to consider research methods of models (quadratic forms) on convexity – concavity, on positive and negative definiteness.

Keywords: Mathematical model; macrosystem; competency tree; Markov process; stream density; matrix of system.

1 Introduction

Process of education and of formation of the personality happens in the complicated, dynamical system. Educational process makes the operating actions on this system. Problems of mathematical modeling of educational process are considered in the works of many authors. There are many different models used in the researches: difference equations [1], Markov chains [2,3,4], Bayesian approach [5], differential

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equations [6]. Important components of educational process are a macrosystem of competence formation [7], competences systems [8], ontologies [9], textbooks (manuals) [10].

The article considers algebraic, matrix and differential models of components of educational process, method of research of their stability and method of basic data correction with stability preservation, offers variants of components classification of educational process (through convexity – concavity, though positive and negative definiteness of the corresponding quadratic forms, and also through curvature, torsion and a gradient of the vector decision).

This work is of actual significance as it is connected with optimum functioning of the components of educational process the practical importance of work is determined by high extent of direct use in educational process the scientific importance consists in application of steady solutions for functioning of educational system.

2 Macrosystem of Educational Process for Formation of Competence

2.1 Model formulation

Fig. 1 shows the generalized scheme of the educational process action on formation of competence of a pupil (an educational group) with a discipline. This system is a macrosystem of educational process.

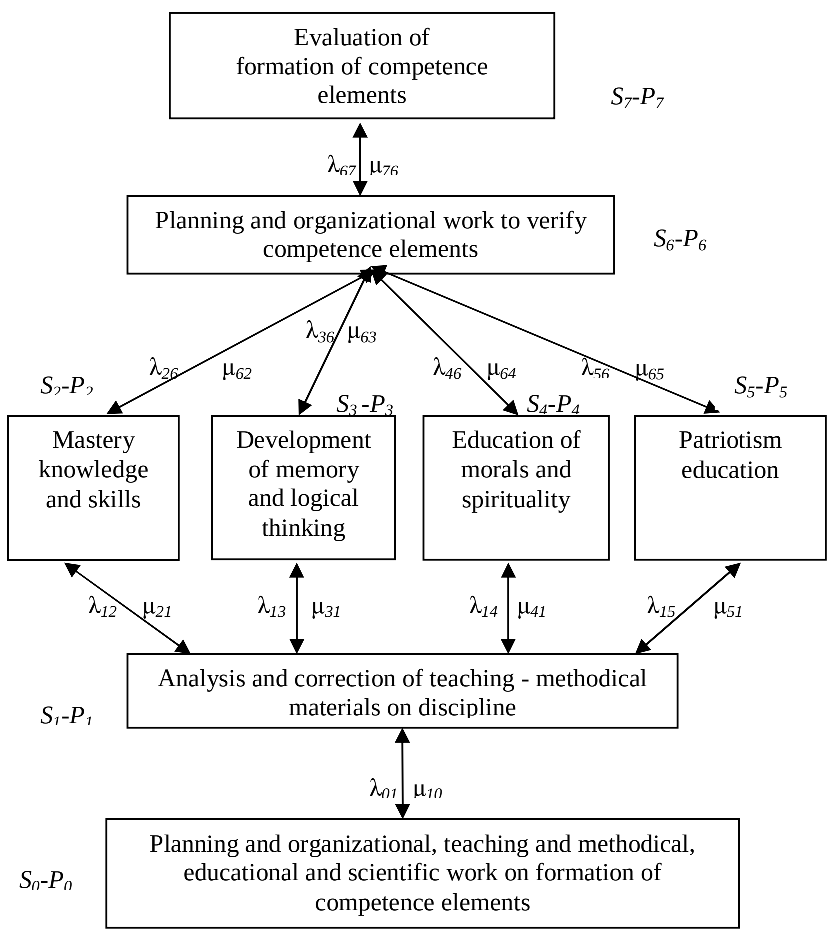

Fig. 1. Macrosystem of educational process

Fig. 1 denotes states of system through S_i and shows structural links of the system. The system has five horizontal levels. These levels are connected with formation of competence elements. This system has the following assumption:

1) The system can be only in one of structural links at the same level,
2) Correction of a task solution on this structural link on this temporary interval comes to correction of the work of the corresponding link (of one only) of the previous horizontal level,
3) At present time only one level of system is active, the operating signals of information stream with density λ_{ij} (μ_{ji}) translate the system of this level to other level. It can be orders, instructions, questions, service instructions, etc. These information units are weighed in accordance with their priority.

A conclusion can be drawn that the system is only in one status at any moment. At any moment on the third level (from the bottom upwards) there is only one of the actions: 1) development of knowledge and skills, 2) development of memory and logical thinking, 3) education of morals and spirituality, 4) patriotism education. These four indexes are weighed according to their importance. Importance is estimated by experts. If a signal of the insufficient competence of a pupil (an educational group) in some of these indexes comes from the top level, then the corresponding work is performed at the previous levels. Let P_i denotes probability of the status S_i. The process proceeding in system can be considered as Markov's process as transition from status S_i to status S_j doesn't depend on the system's transfer to status S_i.

2.2 Method of solution

Basic data may be described as a continuous Markov's chain. The density λ_{ij} of streams of the main units is evaluated. These units characterize educational work of educational institution. These are streams of informative units. They are connected with the integrated criterion of complexity, importance and of mastery extent [11]. Streams can have a two–way direction: from S_i to S_j and from S_j to S_i ($j>i$). Symbol μ_{ji} denotes corresponding densities. These streams have higher level responses to the influence of subordinate levels. Parameters λ_{ij} , μ_{ji} $(i=\overline{0,6}, j=\overline{1,7})$ correspond to streams density of standard and real quality indexes. They can depend on the time. This is Kolmogorov's system:

$$\begin{cases} P_0'(t) = \mu_{10} P_1(t) - \lambda_{01} P_0(t); \\ P_1'(t) = \lambda_{01} P_0(t) + \mu_{21} P_2(t) + \mu_{31} P_3(t) + \mu_{41} P_4(t) + \mu_{51} P_5(t) - \\ - \lambda_{12} P_1(t) - \lambda_{13} P_1(t) - \lambda_{14} P_1(t) - \lambda_{15} P_1(t) - \mu_{10} P_1(t); \\ P_2'(t) = \lambda_{12} P_1(t) + \mu_{62} P_6(t) - \lambda_{26} P_2(t) - \mu_{21} P_2(t); \\ P_3'(t) = \lambda_{13} P_1(t) + \mu_{63} P_6(t) - \lambda_{36} P_3(t) - \mu_{31} P_3(t); \\ P_4'(t) = \lambda_{14} P_1(t) + \mu_{64} P_6(t) - \mu_{41} P_4(t) - \lambda_{46} P_4(t); \\ P_5'(t) = \lambda_{15} P_1(t) + \mu_{65} P_6(t) - \lambda_{56} P_5(t) - \mu_{51} P_5(t); \\ P_6'(t) = \lambda_{26} P_2(t) + \lambda_{36} P_3(t) + \lambda_{46} P_4(t) + \lambda_{56} P_5(t) - \\ - \mu_{62} P_6(t) - \mu_{63} P_6(t) - \mu_{64} P_6(t) - \mu_{65} P_6(t) - \lambda_{67} P_6(t) + \mu_{76} P_7(t); \\ P_7'(t) = \lambda_{67} P_6(t) - \mu_{76} P_7(t); \\ P_0(t) + P_1(t) + P_2(t) + P_3(t) + P_4(t) + P_5(t) + P_6(t) + P_7(t) = 1. \end{cases} \quad (1)$$

Density λ_{ij} , μ_{ji} $(i=\overline{0,6},\ j=\overline{1,7})$ is considered as constant. The common decision of system is made with usage of the decision method of linear uniform systems of the differential equations with constant coefficients:

$$\begin{cases} P_0(t)=c_1x_{01}+c_2x_{02}+\ldots+c_6x_{06}+c_7x_{07}, \\ P_1(t)=c_1x_{11}+c_2x_{12}+\ldots+c_6x_{16}+c_7x_{17}, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ P_7(t)=c_1x_{71}+c_2x_{72}+\ldots+c_6x_{76}+c_7x_{77}, \end{cases}$$

where $x_{ij}=P_{ij}e^{\lambda_j t}$ $(j=\overline{1,7})$ is the decision of system (1), corresponding to eigenvalue λ_j $(j=\overline{1,7})$ and vector $(P_{1j},P_{2j},\ldots,P_{7j})$ of the matrix of this system

$$A_0=\begin{pmatrix} -\lambda_{01} & \mu_{10} & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{01} & \begin{matrix}-\lambda_{12}-\lambda_{13}- \\ -\lambda_{14}-\lambda_{15}-\mu_{10}\end{matrix} & \mu_{21} & \mu_{31} & \mu_{41} & \mu_{51} & 0 & 0 \\ 0 & \lambda_{12} & -\lambda_{26}-\mu_{21} & 0 & 0 & 0 & \mu_{62} & 0 \\ 0 & \lambda_{13} & 0 & -\lambda_{36}-\mu_{31} & 0 & 0 & \mu_{63} & 0 \\ 0 & \lambda_{14} & 0 & 0 & -\lambda_{46}-\mu_{51} & 0 & \mu_{64} & 0 \\ 0 & \lambda_{15} & 0 & 0 & 0 & -\lambda_{56}-\mu_{51} & \mu_{65} & 0 \\ 0 & 0 & \lambda_{26} & \lambda_{36} & \lambda_{46} & \lambda_{56} & \begin{matrix}-\mu_{62}-\mu_{63}- \\ -\mu_{64}-\mu_{65}-\lambda_{67}\end{matrix} & \mu_{76} \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{67} & -\mu_{76} \end{pmatrix} \quad (2)$$

If λ_{ij} , μ_{ji} depend on the time, other methods of the decision are used. Vector function $F(t)=(P_{1j}(t),P_{2j}(t),\ldots,P_{7j}(t))$ is the important functional characteristic of system (1). For this function such characteristics as curvature, torsion, and gradient are defined. These characteristics are used for analysis and classification of the system (1).

In the stationary regime (at rather big period of time t) all P_i $(i=\overline{0,7})$ are constant, all $P_i'(t)$ $(i=\overline{0,7})$ are equal to zero, and i.e. there is a uniform system of the algebraic equations. Probabilities P_i $(i=\overline{0,7})$ are found in solving of this system. The probability P_i characterizes average relative time of the system's stay in this status. Let vector $(P_1,\ldots,P_7)$ is used as a reference, and basic data are corrected in case of essential differences, i.e. density λ_{ij} and μ_{ji} of transferring streams.

2.3 Research of steady functioning

There is a problem to define deviations from standard as possible. This task is connected with concept of the system's stability and is one of the major tasks defining normal functioning of a civil society. This task is aimed on preservation of ethical, legal, and economic standards regardless different negative influences. The model is structurally steady if quite small changes in the model's structure result in such behavior which is qualitatively similarly to behavior of the initial model [12]. There is a problem of steady functioning of the considered system (Fig. 1). The functional system can be described as a system of linear differential equations

$$\frac{dx_i}{dt} = a_{i1}x_1 + a_{i2}x_2 + ... + a_{in}x_n, \ i = \overline{1,n}, \tag{3}$$

all x_i are functions of time *t*. We will designate: $\bar{x}(t) = \{x_1(t), x_2(t),..., x_n(t)\}$, $\overline{x^0}(t) = \{x_1^0(t), x_2^0(t),..., x_n^0(t)\}$, $\|\bar{x}(t)\|$ is a norm of the vector and $\|\bar{x}(t)\| = \sqrt{x_1^2(t) + x_2^2(t) + ... + x_n^2(t)}$.

The decision $\overline{x^0}(t)$ is steady if decisions $\bar{x}(t)$ enough close to it at any initial moment t_0 entirely sink into any narrow ε - tube constructed round the decision $\overline{x^0}(t)$. The decision $\overline{x^0}(t)$ is called asymptotically steady as $t \to \infty$, if:

1) This decision is steady;
2) There is such $\Delta = \Delta(t_0) > 0$ for any $t_0 \in (0, \infty)$ that all decisions $\bar{x}(t)$ $(t_0 \le t < \infty)$ with the condition $\|\bar{x}(t_0) - \overline{x^0}(t_0)\| < \Delta$ have a property: $\lim_{t \to \infty} \|\bar{x}(t) - \overline{x^0}(t)\| = 0$.

The linear system (3) is called steady (asymptotically steady), if all its decisions $\bar{x}(t)$ are steady (are asymptotically steady) as $t \to \infty$. If the system has *n* levels, each of which contains *n* elements, then a_{ij} $(i, j = \overline{1,n})$ is a generalizing characteristic of functioning of the element S_{ij}. If at some level the number of elements is less than *n* then corresponding a_{ij} are equal to zero. Generally a_{ij} are functions, in specific case they are numbers. The following matrix is applied in the analysis of the system on stability.

$$A = \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix} \tag{4}$$

The matrix *A* has exactly *n* eigenvalues $\eta_1, \eta_2, ..., \eta_n$, which are equation roots

$$\begin{vmatrix} (a_{11} - \eta) & a_{12} & ... & a_{1n} \\ a_{21} & (a_{22} - \eta) & ... & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & ... & (a_{nn} - \eta) \end{vmatrix} = 0. \tag{5}$$

This equation represents the algebraic equation:

$$\eta^n + b_1\eta^{n-1} + ... + b_{n-1}\eta + b_n = 0. \tag{6}$$

If all elements a_{ij} are numbers, b_i $(i = \overline{1,n})$ are numbers.

It is known from the theory of stability:

- The system (3) with the matrix *A* is steady if and only if all eigenvalues of the matrix *A* have material (valid) parts smaller or equal to zero,
- The system is asymptotically steady if and only if all eigenvalues of the matrix *A* have negative material parts,

- The system is unstable if and only if its matrix *A* has at least one eigenvalue λ for which the material part is both positive or equal to zero, thus the rank of the matrix $A-\eta I$ is less than multiplicity of eigenvalue η, where *I* - is a unit matrix.

The equations of type (6) of high degrees have no general expressions for roots. Then the following square matrix (Hurwitz's matrix) of coefficients of the algebraic equation may be constructed:

$$\Gamma = \begin{pmatrix} b_1 & 1 & 0 & 0 & \dots & 0 \\ b_3 & b_2 & b_1 & 1 & \dots & 0 \\ b_5 & b_4 & b_3 & b_2 & \dots & 0 \\ \hdotsfor{6} \\ 0 & 0 & 0 & 0 & \dots & b_n \end{pmatrix}$$

If $m > n$, that $b_m = 0$.

According to Hurwitz's stability criterion, the system (3) with the matrix *A* is asymptotically steady, if and only if all main diagonal minors of the corresponding of Hurwitz's matrix:

$$\Delta_1 = b_1,\ \Delta_2 = \begin{vmatrix} b_1 & 1 \\ b_3 & b_2 \end{vmatrix},\ \Delta_3 = \begin{vmatrix} b_1 & 1 & 0 \\ b_3 & b_2 & 1 \\ b_5 & b_4 & b_3 \end{vmatrix}, \dots,\ \Delta_n = b_n \cdot \Delta_{n-1}$$

are positive, i.e. $\Delta_i > 0,\ \mathrm{i} = 1,2,\dots,n$.

The system (1) is a linear system of type (3), which matrix has a form (2). It is possible to investigate system (1) on stability at specific values of density using the mentioned criteria. Stability of system shown in Fig. 1 will follow from stability of system (1). Let density λ_{ij}, μ_{ji} of streams of informative units be constant. The standard variant of the system's functioning is:

$$\lambda_{ij} = \mu_{ji}\ (i = \overline{0,6},\ j = \overline{1,7})$$

This condition is formulated as follows: real understanding of training material corresponds to the standard understanding (for example, the number of the understanding didactic units corresponds to the standard). Therefore, the matrix A_0in this case is symmetric and the square form corresponding to it and representing algebraic model of this system is convex if and only if all main minors of the matrix A_0 are non-negative, and square form is concave if the first minor is not positive, the second – non-negative, the third – not positive, etc. Thus there are three types of systems: 1) the system is represented through a convex function; 2) the system is represented through a concave function; 3) the system is represented through a function of a general form.

Quadratic forms of such systems can be characterized with such indexes as curvature and torsion, and accordingly to classify systems. There are three types of representation of an educational process system: 1) using positively one-signed quadratic form, 2) using negatively one-signed quadratic form, 3) alternating form. Thus the system can be considered as stable in case of negative one-signed quadratic form of the matrix $A_{0,}$ as the system with the matrix A_0 is steady if and only if all eigenvalues of the matrix A_0 have material parts negative or equal to zero. If the matrix A_0 isn't symmetric, let x_{ji} be a difference $\mu_{ji} - \lambda_{ij}$ if $\mu_{ji} > \lambda_{ij}$, and $-x_{ij}$, if $\mu_{ji} < \lambda_{ij}$. Generally let this difference be $\tilde{x}_{ij}$.

The following optimum values $\tilde{x}_{10}, \tilde{x}_{21}, \tilde{x}_{31}, \tilde{x}_{41}, \tilde{x}_{51}, \tilde{x}_{62}, \tilde{x}_{63}, \tilde{x}_{64}, \tilde{x}_{65}, \tilde{x}_{76}$ are found for the analyzed task so that the system with the matrix A_0 is steady and at the same time the sum $T = \tilde{x}^2{}_{10} + \tilde{x}^2{}_{21} + \tilde{x}^2{}_{31} + \tilde{x}^2{}_{41} + \tilde{x}^2{}_{51} + \tilde{x}^2{}_{62} + \tilde{x}^2{}_{63} + \tilde{x}^2{}_{64} + \tilde{x}^2{}_{65} + \tilde{x}^2{}_{76}$ of squares of these variables is minimum. Let M be a set of these variables. Let us consider the following method of this task's solution.

The following characteristic equation (6) is found for matrix A_0. The following Hurwitz's matrix Γ is built on the basis of this equation and criterion of Hurwitz's stability is determined. This criterion consists of positivity of the main minors of the matrix Γ. Thus, each minor $\Delta_k \quad (k = \overline{1,8})$ is presented in the form of the algebraic sum $\varphi_k = \varphi_k(\tilde{x}_{10}, \tilde{x}_{21}, \tilde{x}_{31}, \tilde{x}_{41}, \tilde{x}_{51}, \tilde{x}_{62}, \tilde{x}_{63}, \tilde{x}_{64}, \tilde{x}_{65}, \tilde{x}_{76})$ of certain products of variables $\tilde{x}_{ij}$ $(\tilde{x}_{ij} \in \mathrm{M})$ with numerical coefficients. These coefficients are expressed through products or algebraic sums of products of nonzero elements λ_{ij}, μ_{ji} of the matrix A_0. Thus, the following problem of nonlinear programming is solved. An extremum of function T is investigated under conditions $\varphi_k > 0 \ (k = \overline{1,8})$. For decision it is possible for example to use a method of internal penalties in combination with Newton's method, a method of expanded lagranzhian [13].

In case of the minimum value of function T there is a minimum correction of the matrix A_0 within its stability. It is stability of the corresponding system. In case of the maximum value of T there is the greatest possible correction of the matrix A_0 with stability preservation. In solution of this task, the symmetric matrix is considered as a standard matrix. This matrix satisfies to the equality of streams' densities $\mu_{ji} = \lambda_{ij}$. However, generally the system corresponding to such matrix cannot be steady.

A little simplified (for reduction of calculations) macrosystem of educational process with states S_1, S_2, S_3, S_6 and the corresponding connections is considered as an example where all λ and μ are equal. The differential system is converted to the form:

$$\begin{cases} P_1'(t) = \lambda P_2(t) + \lambda P_3(t) - 2\lambda P_1(t); \\ P_2'(t) = \lambda P_1(t) + \lambda P_6(t) - 2\lambda P_2(t); \\ P_3'(t) = \lambda P_1(t) + \lambda P_6(t) - 2\lambda P_3(t); \\ P_6'(t) = \lambda P_2(t) + \lambda P_3(t) - \lambda P_6(t) - 2\lambda P_6(t); \\ P_1(t) + P_2(t) + P_3(t) + P_6(t) = 1. \end{cases}$$

In this case the equation (6) is:

$$\eta^4 + 8\eta^3 + 20\eta^2 + 16\eta = 0.$$

The corresponding system is steady according to Hurwitz's criterion because all main minors are positive. It is supposed that all λ are equal, and all μ are different and distinct from λ.

Let $\mu_{ji} = \lambda + \tilde{x}_{ji}$ and $\tilde{x}_{ji} = \lambda \cdot \tilde{y}_{ji}$.

Equation (6) in this case has coefficients that depend on the algebraic sum of the variables $\tilde{y}_{ji}$ and their products with numerical coefficients. There are no coefficients for simplification of the record. Statistical supervision shows $a \le y_{ji} \le b$.

The maximum of the sum

$$T = \tilde{y}^2{}_{21} + \tilde{y}^2{}_{31} + \tilde{y}^2{}_{62} + \tilde{y}^2{}_{63}$$

is derived at Hurwitz's restrictions concerning positivity of the main minors with use of Mat lab (for example $a = -2{,}2; \quad b = 2{,}2$):

$$\tilde{y}_{21} = 1,\ \tilde{y}_{31} = 2{,}1,\ \tilde{y}_{62} = -1{,}1,\ \tilde{y}_{63} = 0{,}093,\ T_{\max} = 6{,}63.$$

Thus maximum deviations of density of streams are received, at preservation of stability.

A density μ_{21} has deviation equal to λ. It means that density μ_{21} can be increased by λ. A density μ_{31} has deviation equal to $2{,}1\lambda$, density can be increased by $2{,}1\lambda$. Density λ_{26} can be increased by $1{,}1\lambda$, μ_{63} can be increased by $0{,}093\lambda$.

3 A Mathematical Model Competency

Stability of other components of educational process is investigated. Fig. 2 shows the competency tree PK-5 of the discipline "The mathematical analysis" (program 0800000 "Economics").

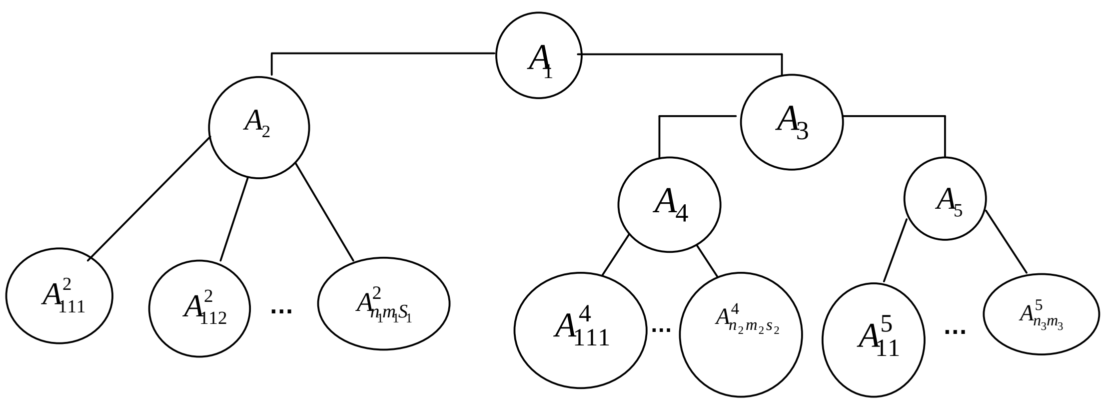

Fig. 2. Competency tree

Competence PK-5 = A_1, A_2 is an ability to choose instrumental means for processing of economic data according to a formulated problem; A_3 is an ability to analyze results of calculations and to justify the received conclusions; $A^2{}_{ijk}$ is an ability to choose i -type instrumental means ($i = \overline{1, n_1}$) for processing of economic data of j type ($j = \overline{1, m_1}$) according to t type ($t = \overline{1, s_1}$) of problem; A_4 is an ability to analyze the results of calculations; A_5 is an ability to justify the received conclusions; $A^4{}_{ijt}$ is an analysis with application of mathematical apparatus i $(i = \overline{1, n_2})$ for the result j $(j = \overline{1, m_2})$ of the calculation t $(t = \overline{1, s_2})$; $A^5{}_{ij}$ is an justification with application of the i - type mathematical apparatus $(i = \overline{1, n_3})$ for a conclusion of j - type $(j = \overline{1, m_3})$. Transition is characterized from top to downward by density λ_{ij}, from bottom upward by density μ_{ji}, $i \in \{111, \ldots, n_2 m_2 s_2\}, \quad j \in \{11, \ldots, n_3 m_3\}$.

This tree can be turned into a system if it is turned into an information network. For this purpose let us input the new A_φ top corresponding to a specific pupil. We connect this top to all hanging tops of the tree. Each edge is replaced with a pair of opposite directed arches. Weights of these arches are equal to densitys of information streams according to the planned and understood educational information of this pupil. These are streams of key concepts, examples, tasks, lists, etc. They are weighted according to degree of their importance, complexity and understanding. It is an information network. This system is considered only in one status at any moment, and it is possible to consider that Markov's process takes place in this network. Such assumption is well defined. First, this competency consists of elementary subcompetencys, thus there is a process of their study according to the importance, complexity and extent of understanding; second, subcompetencys are formed also consistently. Therefore, it is Kolmogorov's system of differential equations (7). This system is mathematical model for competency formation process of this pupil. The decision of the given system is similar to the decision of the system (1):

$$\begin{cases} P_1'(t) = -(\mu_{12}+\mu_{13})P_1(t)+\lambda_{21}P_2(t)+\lambda_{31}P_3(t), \\ P_2'(t) = -(\lambda_{21}+\mu_{2,111}+\ldots+\mu_{2,n_1m_1s_1})P_2(t)+\mu_{12}P_1(t)+ \\ \qquad +\lambda_{111,2}P_{111}(t)+\ldots+\lambda_{n_1m_1s_1,2}P_{n_1m_1s_1}(t), \\ P_3'(t) = -(\lambda_{31}+\mu_{34}+\mu_{35})P_3(t)+\mu_{13}P_1(t)+\lambda_{43}P_4(t)+\lambda_{53}P_5(t), \\ P_i'(t) = \mu_{4,i}P_4(t)-\lambda_{i,4}P_i(t), \quad i\in\{111,\ldots,n_2m_2s_2\}, \\ P_j'(t) = \mu_{5,j}P_5(t)-\lambda_{j,5}P_j(t), \quad j\in\{11,\ldots,n_3m_3\}. \end{cases} \tag{7}$$

Thus, the competency matrix is similar to the matrix (5). It is possible to introduce a classification of competences by indications: convexity - concavity, positive - negative definiteness of the corresponding quadratic forms. Competency is steady (is asymptotically steady) if Kolmogorov's system of differential equations is steady (is asymptotically steady). Competency on stability (on asymptotic stability) is investigated using criterion of stability (of asymptotic stability). It is performed in the same way as for the competency formation macrosystem which is considered above. Ontologies are analyzed similarly.

4 Mathematical Model for Description of Textbook Studying

The system for description of studying a textbook (Fig. 3) is considered in [10]. It is shown that it is Markov's process.

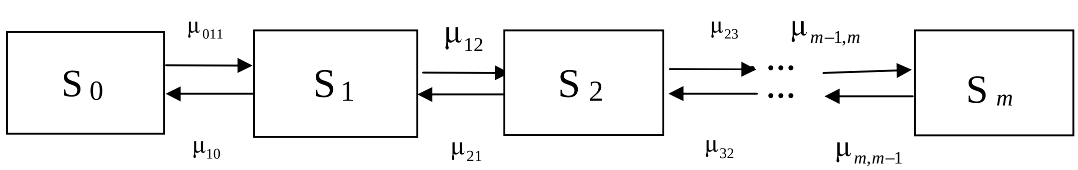

Fig. 3. Studying of a textbook

$\mu_{i,i+1}$ is density of "failures" in understanding of training material $\mu_{i+1,i}$ is density of eliminated "failures".

It is assumed that the corresponding Kolmogorov's system of the differential equations is steady (asymptotically steady). Then textbook studying process may be determined as steady (asymptotically steady). It is possible to introduce a classification of textbooks concerning convexity - concavity positive and negative definiteness of quadratic forms represented the textbook. With use of quadratic forms it is possible to carry out an analysis of stability (asymptotic stability) of textbooks, like it was made for the macrosystem (Fig. 1). The system (8) represents differential model of textbook studying:

$$\begin{cases} P_0'(t) = -\mu_{01}P_0(t) + \mu_{10}P_1(t), \\ P_i'(t) = -(\mu_{i,i-1} + \mu_{i,i+1})P_i(t) + \mu_{i-1,i}P_{i-1}(t) + \mu_{i+1,i}P_{i+1}(t), \quad i = \overline{1, m-1}, \\ P_m'(t) = -\mu_{m,m-1}P_m(t) + \mu_{m-1,m}P_{m-1}(t). \end{cases} \tag{8}$$

5 Conclusion

This article considers differential, matrix and algebraic models of ontologies, textbooks, macro system for competence formation and derives classification of this component, reveals methods of analysis on their stability and on asymptotic stability and method of basic data correction with stability preservation.

This system description of influence of educational process on personality formation and on development of civil society finds a broad application in educational institutions in searching of optimum influencing streams and for definition of such indicators which assume steady functioning of the analyzed systems.

The method of correction of initial parameters of educational process, methods of classification of components, constructed algebraic, matrix and differential models are considered for the first time.

Priority of considered component is a macrosystem, competence, ontology, textbook. For determination of weight coefficients it is necessary to carry out the factor analysis on the basis of statistical data. It is the subject of a separate study.

Competing Interests

Author has declared that no competing interests exist.

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