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A Common Fixed Point Theorem with Three Pairs of Self Mapping in Fuzzy Metric Space Using Property (E.A.) and Implicit Relation

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

The proposed method generalises the implicit relation in fuzzy metric space by applying three pairs of self mapping to satisfy the property (E.A.) and the implicit relation.

Keywords: Fuzzy metric space; common fixed points; occasionally weakly compatible mappings; weakly compatible maps; implicit relation; property (E.A.).

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1 Introduction

In the present world of mathematics, the concept of fuzzy logic aspects in mathematical decision making are applicable which was intially introduced by Zadeh [1] which introduced the foundation of fuzzy mathematics. Fuzzy set thoery is broadly applied in neural network, mathematical modeling and other engineering disciplines.

George and Veeramani [2] modified the concept of fuzzy metric space introduced by Karmosil and Michalek [3]. Manro, Bhatia and Kumar [4] proved the common fixed point theorem in fuzzy metric space and discussed result related to R-weakly commuting type mappings.

Altun and Turkoglu [5] proved some fixed point theorems on fuzzy metric space with implicit relation.

Perlovsky and Leonid[6],discussed the extension of fuzzy logic is inspired by processes in the brain-mind. Cognitive models are presented and experimental data are discussed demonstrating connections of these models to workings of the mind.

Kumar and Fisher [7] proved A common fixed point theorem in fuzzy metric space using properly (E.A.) and implicit relation. The proposed method is generalization of [7] where (A,S), (B,T) and (C,U) are weakly compatible pairs of self mapping satisfying property (E.A.) and implicit relation.

Han and Jianchao [8], discussed the feature subset selection is an important component of knowledge discovery and Rough sets theory provides a mechanism of selecting feature subsets.

Pathak and Rai [9] gave some important and interesting remarks on the concept of occasionally weakly compatible (owc) mappings, which is an active and interesting area of research in present era. In [10], the authors, established a fixed point theorem for two pair of maps satisfying a contractive type condition by using the concept of occasionally weakly compatible maps application, the existence and uniqueness of common solutions for certain systems of functional equations arising in dynamic programming are discussed by using the common fixed.

2 Preliminaries

Definition 2.1[1] A 3-tuple $(X, M, \star)$ is said to a fuzzy metric space if X is an arbitrary set, $\star$ is a continuous t-norm and M is a fuzzy set of $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X, s, t > 0$.

(f_1) $M(x, y, t) > 0$

(f_2) $M(x, y, t) = 1$ if and only if $x = y$

(f_3) $M(x, y, t) = M(y, x, t)$

(f_4) $M(x, y, t) \star M(y, z, s) \leq M(x, z, t + s)$

(f_5) $M(x, y, \cdot) : (0, \infty] \to (0, 1)$ is continuous

Then M is called a fuzzy metric of X and$M\,(x, y, t)$ denotes the degree of nearness between x and y with respect to t.

Definition 2.2 A binary operation $\star : [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-norm if $\star$ satisfies the following conditions:

(i) $\star$ is an commutative and associative

(ii) $\star$ is continuous

(iii) $a \star 1 = a$ for $a \in [0, 1]$

(iv) $a \star b \leq c \star d$ whenever $a \leq c, b \leq d$ for all $a, b, c, d, \in [0, 1]$

Definition 2.3 A pair of self mappings (f, g) of a fuzzy metric space $(X, M, \star)$ is said to be

(i) weakly commuting if

$$M(fgx, gfx, t) \geq M(fx, gx, t) \text{ for all } x \in X \text{ and } t > 0.$$

(ii) R-weakly commuting if there exist some $R > 0$ such that

$$M(fgx, gfx, t) \geq M(fx, gx, t/R) \text{ for all } x \in X \text{ and } t > 0.$$

Definition 2.4 Two self mappings f and g of a fuzzy metric space $(X, M, \star)$ are called reciprocally continuous on X if

$\lim\limits_{n\to\infty} fgx_n = fx \, and \, \lim\limits_{n\to\infty} gfx_n = gx$

Whenever $\{x_n\}$ is a sequence in X such that

$\lim\limits_{n\to\infty} fx_n = \lim\limits_{n\to\infty} gx_n$ for some $x \in X$.

Definition 2.5 Two self mappings f and g of a fuzzy metric space $(X, M, \star)$ are compatible if

$\lim\limits_{n\to\infty} M(fgx_n, gfx_n, t) = 1$

Whenever $\{x_n\}$ is a sequence in X such that

$\lim\limits_{n\to\infty} fx_n = \lim\limits_{n\to\infty} gx_n = x$, for some $x \in X$.

Definition 2.6 Two self mappings f and g of a set X are occasionally weakly compatible (owc) if there exist x in X which is coincidence point of f and g at which f and g commute.

A.Al-Thagafi and Nasseer Shahzad [11] shows that occasionally weakly is weakly compatible but converse is not true.

Definition 2.7 [12] A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, \star)$ is called Cauchy sequence if for every $\epsilon > 0$ and each $t > 0$

There exist $n_0 \in N$ such that

$M(x_n, x_{n+p}, t) > 1 - \epsilon$ for all $n \geq n_0$, $t > 0$

Definition 2.8 A fuzzy metric space in whch every Cauchy sequence is convergent is said to be complete.

Definition 2.9 If for all $x, y, \in X, t > 0$ and for a number $k \in (0, 1)$ such that

$M(x, y, kt) \geq M(x, y, t)$ then $x = y$

through this paper $(X, M, \star)$is considered to be fuzzy metric space with condition:

$f_6 : lim_{t\to\infty} M(x, y, t) = 1\ for all\, x, y, \in X.$

Definition 2.10 Implicit Relation [13]

Let $I = [0, 1], \star$ be a continuous t-norm and F be the set of all real continuous functions $F : I^6 \to R$ satisfying the following conditions

(f_a) F is non increasing in the Fifth and Sixth variable

(f_b) If for some constant $k \in (0, 1)$, we have

(i) $F \left\{u(kt), v(t), v(t), u(t), 1, u(\frac{t}{2}) \star v(\frac{t}{2})\right\} \geq 1\, or$

(ii) $F \left\{u(kt), v(t), u(t), v(t), u(\frac{t}{2}) \star v(\frac{t}{2}), 1\right\} \geq 1$

For any fixed $t > 0$ and any non decreasing function $u, v : (0, \infty) \to I$ with $0 \leq u(t), v(t) \leq 1$ then there exist $h \in (0, 1)$ with $u(ht) \geq v(t) \star u(t)$

(f_c) If for some constant $k \in (0, 1)$, we have

$F \{u(kt), u(t), 1, 1, u(t), u(t)\} \geq 1$ for any $t > 0$ and any non decreasing function

$u : (0, \infty) \to I$ then $u(kt) \geq u(t)$

3 Main Results

In [5] Altun and Turkoglu proved the following.

Theorem 1. Let $(X, M, \star)$ be a complete fuzzy metric space with

$a \star b = min\{a, b\}$ for all $a, b, \epsilon I$ and A, B, S and T be maps from X into itself satisfying the conditions.

(3.1) $A(X) \subseteq T(X), B(X) \subseteq S(X)$

(3.2) One of the maps $A, B, S\, and\, T$ is continuous

(3.3) $(A, S), (B, T)$ are compatible of type $(\propto)$

(3.4) there exist $k \in (0, 1)$ and $F \in \digamma$ such that

$$F \left\{ \begin{array}{l} M(Ax, By, kt)M(Sx, Ty, t), M(Ax, Sx, t) \\ M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t) \end{array} \right\} \geq 1$$

For all $x, y, \in X$ and $t > 0$

Then A, B, S and T have a unique common fixed point in X.

Theorem 2. Let $(X, M, \star)$ be a fuzzy metric space with $a \star b = min\{a, b\}$ for all $a, b, \in I$ further let $(A, S)(B, T)$ and (C, U) be weakly compatible pairs of self mappings of X satisfying $(3.1), (3.2), (3.3), (3.4)$ and $(A, S), (B, T)$ and (C, U) the satisfies the property (E, A).

If the range of one of the maps A, B, C, S, T and U is complete subspace of X then A, B, C, S, T and U have a unique common fixed point in X.

Proof: If the pair (B, T) satisfies the properly (E.A) then there exist a sequence $\{x_n\}$ in X such that

$Bx_n \to z$ and $Tx_n \to z \, as \, n \to \infty$ for some $z \in X$

Since $B(X) \subseteq S(X)$ there exist a sequence $\{y_n\}$ in X such that $Bx_n = Sy_n$
hence $Sy_n \to z \, as \;\; n \to \infty$

And $A(X) \subseteq T(X)$ there exist a sequence $\{y_n'\}$ in X such that $Ay_n' = Tx_n$ hence $Ay_n' \to z \, as \, n \to \infty$

Suppose $S(X)$ is a complete sub-space of X then $Sa = z$ for some $a \in X$

$$\begin{aligned} &Ay_n' \to Sa, Bx_n \to Sa, \; Cx_n \to Sa \\ &\quad Sy_n \to Sa, Tx_n \to Sa, \; Ux_n \to Sa -------------- (1) \\ &\quad Sx_n \to Sa, Ty_n \to Sa, \; Uy_n \to Sa \end{aligned}$$

As $n \to \infty$ then by (3.4) we have

$$F \begin{Bmatrix} M(Ax, By, Cz, kt), M(Sx, Ty, t), M(Ty, Uz, t), M(Ax, Sx, t)M(By, Ty, t) \\ F\{M(Cz, Uz, t), M(Ax, Ty, T), M(By, Uz, t), M(By, Sx, t), M(Cz, Ty, t) \end{Bmatrix} \geq 1$$

As $n \to \infty$ then by (3.4) we have $x \to a, y \to x_n, z \to x_n$

$$F \begin{Bmatrix} M(Aa, Bx_n, Cx_n, kt), M(Sa, Tx_n, t), M(Tx_n, Ux_n, t), M(Aa, Sa, t) \\ M(Bx_n, Tx_n, t), M(Cx_n, Ux_n, t), M(Aa, Tx_n, t), \\ M(Bx_n, Ux_n, t), M(Bx_n, Sa, t), M(Cx_n, Tx_n, t), \end{Bmatrix} \geq 1$$

Taking $n \to \infty$ we have

$$F \begin{Bmatrix} M(Aa, Sa, Sa, kt), M(Sa, Sa, t), M(Sa, Sa, t), M(Aa, Sa, t), M(Sa, Sa, t) \\ M(Sa, Sa, t), M(Aa, Sa, t), M(Sa, Sa, t), M(Sa, Sa, t), M(Sa, Sa, t \end{Bmatrix} \geq 1$$

That is

$$F \left\{ M(Aa, Sa, kt)1, 1, M(Aa, Sa, t), 1, 1, M(Aa, Sa, t), 1, 1, 1 \right\} \geq 1$$

On the other hand since

$$M(Aa, Sa, t) \geq M\left(Aa, Sa, \frac{t}{2}\right) = M\left(Aa, Sa, \frac{t}{2}\right) \star 1$$

And F is non increasing in the fifth variable we have any $t > 0$

$$F\left\{ M(Aa, Sa, Sa, kt)1, 1, M(Aa, Sa, t), 1, 1, M\left(Aa, Sa, \frac{t}{2}\right) \star 1, 1, 1, 1 \right\}$$

$$\geq F\left\{ M(Aa, Sa, Sa, kt)1, 1, M(Aa, Sa, t), 1, 1, M(Aa, Sa, t), 1, 1, 1 \right\} \geq 1$$

Which implies by $(f_b) Aa = Sa$

The weak compatiblity of A and S implies that $ASa = SAa$ and

$AAa = ASa = SAa = SSa - - - - - - - - - - - - (2)$

On the other hand since $A(X) \subseteq T(X)$

Then there exist $b \in X$ such that $Aa = Tb$

We now show that $Tb = Bb$

By (3.4) we have

$$F\left\{\begin{matrix} M(Ax,By,Cz,kt), M(Sx,Ty,t), M(Ty,Uz,t), M(Ax,Sx,t) \\ M(By,Ty,t), M(Cz,Uz,t), M(Ax,Ty,t), M(By,Uz,t), \\ M(By,Sx,t), M(Cz,Ty,t) \end{matrix}\right\} \geq 1$$

$$x \to a,\ y \to b,\ z \to b$$

$$F\left\{\begin{matrix} M(Aa,Bb,Cb,kt), M(Sa,Tb,t), M(Tb,Ub,t), \\ M(Aa,Sa,t), M(Bb,Tb,t), M(Cb,Ub,t), M(Aa,Tb,t), \\ M(Bb,Ub,t), M(Bb,Sa,t), M(Cb,Tb,t) \end{matrix}\right\} \geq 1$$

$By(1)$

$$F\left\{\begin{matrix} M(Tb,Bb,Tb,kt), M(Tb,Tb,t), M(Tb,Tb,t), M(Tb,Tb,t) \\ M(Bb,Tb,t), M(Cb,Tb,t), M(Tb,Tb,t), M(Bb,Tb,t), \\ M(Bb,Tb,t), M(Tb,Tb,t) \end{matrix}\right\} \geq 1$$

That is

$$F\left\{\begin{matrix} M(Tb,Bb,Tb,kt), 1, 1, 1, M(Bb,Tb,t), 1, 1 \\ M(Bb,Tb,t), M(Bb,Tb,t), 1 \end{matrix}\right\} \geq 1$$

On the other hand since

$M(Bb,Tb,t) \geq M\left(Bb,Tb,\frac{t}{2}\right) = M\left(Bb,Tb,\frac{t}{2}\right) \star 1$

And F is non increasing in the sixth variable, we have any $t > 0$

$F\left\{M(Tb,Bb,Tb,kt), 1, 1, 1, M(Bb,Tb,t), 1, 1, M(Bb,Tb,t), M\left(Bb,Tb,\frac{t}{2}\right) \star 1, 1\right\}$

$$\geq F\left\{\begin{matrix} M(Tb,Bb,Tb,kt), 1, 1, 1, M(Bb,Tb,t), \\ 1, 1, M(Bb,Tb,t), M(Bb,Tb,t) \end{matrix}\right\} \geq 1$$

Which implies by $(f_b) Bb = Tb$ this implies that

$Aa = Ua = Sa = Cb = Ub = Bb = Tb - - - - - - - - - - (3)$

The weak compatibility of B and T implies that

$BTb = TBb$ and $TTb = TBb = BTb = BBb$

Let us show that Aa is a common fixed point of A, B, C, S, T and U

In view of (3.4) we have

$$F\left\{\begin{array}{c} M(Ax,By,Cz,kt),M(Sx,Ty,t),M(Ty,Uz,t),M(Ax,Sx,t) \\ M(By,Ty,t)M(Cz,Uz,t),M(Ax,Ty,T),M(By,Uz,t) \\ M(By,Sx,t),M(Cz,Ty,t) \end{array}\right\} \geq 1$$

$$x \to Aa, y \to b, z \to b$$

This is

$$F\left\{\begin{array}{c} M(AAa,Bb,Cb,kt),M(SAa,Tb,t),M(Tb,Ub,t) \\ ,M(AAa,SAa,t)M(Bb,Tb,t),M(Cb,Ub,t) \\ M(AAa,Tb,t),M(Bb,Ub,t),M(Bb,SAa,t),M(Cb,Tb,t), \end{array}\right\} \geq 1$$

Using {3}we have

$$F\left\{\begin{array}{c} M(AAa,Aa,Aa,kt),M(SAa,Aa,t),M(Aa,Aa,t) \\ ,M(AAa,SAa,tM(Aa,Aa,t),M(Aa,Aa,t) \\ ,M(AAa,Aa,t),M(Aa,Aa,t),M(Aa,SAa,t),M(Aa,Aa,t), \end{array}\right\} \geq 1$$

Thus from (f_c) and (2) we have

$M(AAa,Aa,Aa,kt) \geq M(AAa,Aa,Aa,t)$

By (2.9) we get $AAa = Aa$

Therefore $AAa = Aa = SAa = ASa$, by (2)

Similarly we can prove Cb is a common fixed point of C and U

In view of (3.4) it follows

$$F\left\{\begin{array}{c} M(Ax,By,Cz,kt),M(Sx,Ty,t),M(Ty,Uz,t),M(Ax,Sx,t) \\ M(By,Ty,t)M(Cz,Uz,t),M(Ax,Ty,T),M(By,Uz,t), \\ M(By,Sx,t)M(Cz,Ty,t) \end{array}\right\} \geq 1$$

$$x \to a,\ y \to b,\ z \to b$$

That is

$$F\left\{\begin{array}{c} M(Aa,Bb,CCb,kt),M(Sa,Tb,t),M(Tb,UCb,t),M(Aa,Sa,t) \\ M(Bb,Tb,t),M(CCb,UCb,t),M(Aa,Tb,t),M(Bb,UCb,t) \\ M(Bb,Sa,t),M(CCb,Tb,t) \end{array}\right\} \geq 1$$

Using (3) we have

$$F\left\{\begin{array}{c} M(Aa,Aa,CCb,kt),M(Aa,Aa,t),M(Aa,UCb,t),M(Aa,Aa,t) \\ M(Aa,Aa,t),M(CCb,UCb,t),M(Aa,Tb,t), \\ M(Aa,UCb,t),M(Aa,Aa,t),M(CCb,Aa,t) \end{array}\right\} \geq 1$$

Thus from (f_c) and (3)

$$M(Aa,Cb,CCb,kt) \geq M(Aa,Cb,CCb,t)$$

$$\Rightarrow Aa = Cb = CCb$$

Similarly, if the pair (C, U) satisfies the property (E.A.) then there exist a sequence $\{x_n\}$ in X such that $Cx_n \to z$ and $Ux_n \to z$ for some $z \in X$ as $n \to \infty$.

Since $C(X) \subseteq T(X)$ then there exist a sequence $\{y_n\}$ in X such that $Cx_n = Ty_n$ hence $Ty_n \to z$ as $n \to \infty$.

Also since $B(X) \subseteq U(X)$ then there exist a sequence $\{y'_n\}$ in X such that $By'_n = Ux_n$ hence $By'_n = z$ hence $By'_n \to z$ as $n \to \infty$

Suppose that $U(X)$ is a complete subspace of X then $Ua = z$ for some $a \in X$ subsequently we have

$Ay'_n \to Ua,\ Bx_n \to Ua,\ Cx_n \to Ua$

$Sx_n \to Ua,\ Tx_n \to Ua,\ Ux_n \to Ua$

$Sy_n \to Ua,\ Ty_n \to Ua,\ Uy_n \to Ua - - - - - - - -$ (4)

as $n \to \infty$ then by (3.4) we have

$$F\left\{\begin{matrix} M(Ax, By, Cz, kt), M(Sx, Ty, t), M(Ty, Uz, t), M(Ax, Sx, t) \\ M(By, Ty, t), M(Cz, Uz, t), M(Ax, Ty, t), M(By, Sx, t), \\ M(By, Uz, t), M(Cz, Ty, t) \end{matrix}\right\} \geq 1$$

$$x \to a,\ y \to x_n,\ z \to x_n$$

That is

$$F\left\{\begin{matrix} M(Aa, Bx_n, Cx_n, kt), M(Sa, Tx_n, t), M(Tx_n, Ux_n, t) \\ M(Aa, Sa, t), M(Bx_n, Tx_n, t), M(Cx_n, Ux_n, t), M(Aa, Tx_n, t) \\ , M(Bx_n, Sa, t,)M(Bx_n, Ux_n, t), M(Cx_n, Tx_n, t) \end{matrix}\right\} \geq 1$$

As $n \to \infty$ we have

$$F\left\{\begin{matrix} M(Aa, Ua, Ua, kt), M(Sa, Ua, t), M(Ua, Ua, t), M(Aa, Sa, t) \\ M(Ua, Ua, t), M(Ua, Ua, t), M(Aa, Ua, t), M(Ua, Sa, t) \\ M(Ua, Ua, t), M(Ua, Ua, t), \end{matrix}\right\} \geq 1$$

That is

$$F\left\{\begin{matrix} M(Aa, Ua, Ua, kt), M(Sa, Ua, t), 1, M(Aa, Sa, t) \\ 1, 1, M(Aa, Ua, t), M(Ua, Sa, t), 1, 1 \end{matrix}\right\} \geq 1$$

On the other hand side

$M(Aa, Ua, t) \geq M(Aa, Ua, \frac{t}{2}) = M(Aa, Ua, \frac{t}{2}) \star 1$

And F is non increasing in the fifth variable, we have for any t>0

$$F\left\{\begin{matrix} M(Aa, Ua, Ua, kt), M(Sa, Ua, t), 1, M(Aa, Sa, t), 1, 1 \\ M(Aa, Ua, \frac{t}{2}) \star 1, M(Ua, Sa, t), 1, 1 \end{matrix}\right\}$$

$$\geq F\left\{\begin{matrix} M(Aa, Ua, Ua, kt), M(Sa, Ua, t), 1, M(Aa, Sa, t), 1, 1 \\ M(Aa, Ua, t), M(Ua, Sa, t), 1, 1 \end{matrix}\right\} \geq 1$$

Using [7]

Which implies by $(f_b) Aa = Ua$

The weak compatibility of A and U implies that

$AUa = UAa$ and $AAa = AUa = UAa = UUa$

On the other hand

Since $B(X) \subseteq U(X)$ there exist $b \in X$ such that $Ba = Ub$

Now we show that $Ub = Cb$

By (3.4) we have

$$F\left\{\begin{matrix} M(Ax, By, Cz, kt), M(Sx, Ty, t), M(Ty, Uz, t), M(Ax, Sx, t) \\ M(By, Ty, t), M(Cz, Uz, t), M(Ax, Ty, t) \\ M(By, Sx, t), M(By, Uz, t), M(Cz, Ty, t) \end{matrix}\right\} \geq 1$$

$$x \to a,\ y \to b,\ z \to b \text{ and } (1)$$

That is

$$F\left\{\begin{matrix} M(Aa, Bb, Cb, kt), M(Sa, Tb, t), M(Tb, Ub, t), M(Aa, Sa, t) \\ M(Bb, Tb, t), M(Cb, Ub, t), M(Aa, Tb, t) \\ M(Bb, Sa, t), M(Bb, Ub, t), M(Cb, Tb, t) \end{matrix}\right\} \geq 1$$

That is

$$F\left\{\begin{matrix} M(Ua, Ua, Cb, kt), M(Sa, Ua, t), M(Ua, Ub, t), M(Ua, Sa, t) \\ M(Ua, Ua, t), M(Cb, Ub, t), M(Ua, Ua, t) \\ M(Ua, Sa, t), M(Ua, Ub, t), M(Cb, Ua, t) \end{matrix}\right\} \geq 1$$

That is

$$F\left\{\begin{matrix} M(Ua, Ua, Cb, kt), M(Ua, Ua, t), M(Ua, Ub, t), M(Ua, Sa, t) \\ M(Ua, Ua, t), M(Cb, Ub, t), M(Ua, Ua, t) \\ M(Ua, Sa, t), M(Ua, Ub, t), M(Cb, Ua, t) \end{matrix}\right\} \geq 1$$

That is

$$F\left\{\begin{matrix} M(Ua, Ua, Cb, kt), 1, M(Ua, Ub, t), 1, 1, M(Cb, Ub, t), \\ 1, 1, M(Ua, Ub, t), M(Cb, Ub, t) \end{matrix}\right\} \geq 1$$

That is

$$F\left\{\begin{matrix} M(Ua, Ua, Cb, kt), 1, M(Cb, Ub, t), 1, 1, 1 \\ M(Cb, Ub, t) \end{matrix}\right\} \geq 1$$

On other hand since

$M(Cb, Ub, t) \geq M(Cb, Ub, \frac{t}{2}) = M(Cb, Ub, \frac{t}{2}) \star 1$

And F is non increasing in the sixth variable, we have for any $t > 0$

$$F\left\{\begin{matrix} M(Ua, Ua, Cb, kt), 1, 1, 1, M(Cb, Ub, t), 1, 1, 1 \\ M(Cb, Ub, \frac{t}{2}) \star 1 \end{matrix}\right\}$$

$$\geq F\left\{\begin{matrix} M(Ua, Ua, Cb, kt), 1, 1, 1, M(Cb, Ub, t), 1, 1, 1 \\ M(Cb, Ub, t) \end{matrix}\right\} \geq 1$$

Which implies by (f_b)

$$Cb = Ub$$

This implies that $Aa = Ua = Cb = Ub - - - - - - (5)$

The weak compatibility of C and U implies that

$$CUb = UCb \ and \ UUb = UCb = CUb = CCb$$

Similarly we can prove Bb is a common fixed point of B and T.

Since $Aa = Cb = Bb$

We conclude that Aa is common fixed point of $A, B, C, S, T \, and \, U$

For uniqueness

Since $A(X) \subseteq T(X), B(X) \subseteq S(X)$

And $B(X) \subseteq U(X), C(X) \subseteq T(X)$

If $Aa = Ba = Ca = Sa = Ta = Ua = a - - - - - - (6)$

And $Ab = Bb = Cb = Sb = Tb = Ub = b$

Let w is another point such that

$$Cw = w = Uw$$

then by (3.4) we have

$$F \left\{ \begin{array}{c} M(Ax, By, Cz, kt), M(Sx, Ty, t), M(Ty, Uz, t), M(Ax, Sx, t) \\ M(By, Ty, t), M(Cz, Uz, t), M(Ax, Ty, t) \\ M(By, Uz, t), M(By, Sx, t), M(Cz, Ty, t) \end{array} \right\} \geq 1$$

$$x \to a, y \to b, z \to w$$

That is

$$F \left\{ \begin{array}{c} M(Aa, Bb, Cw, kt), M(Sa, Tb, t), M(Tb, Uw, t), M(Aa, Sa, t) \\ M(Bb, Tb, t), M(Cw, Uw, t), M(Ax, Tb, t) \\ M(Bb, Uw, t), M(Bb, Sa, t), M(Cw, Tb, t) \end{array} \right\} \geq 1$$

That is using (6)

$$F \left\{ \begin{array}{c} M(a, b, w, kt), M(a, b, t), M(b, w, t), M(a, a, t), M(b, b, t), \\ M(w, w, t), M(a, b, t), M(b, w, t), M(b, a, t), M(w, b, t) \end{array} \right\} \geq 1$$

Thus from (f_c) and using (2.9), (f_6) we have

$M(a, b, w, kt) \geq M(a, b, w, t)$

Then by (2.9) we have

$$a = b = w$$

Therefore common fixed point is unique.

Competing Interests

The authors declare that they have no competing interests.

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